

Stats 2 - June 2008

① a) 2 by 2, so need Yates correction

		Expected		Yates $X_2$		$\frac{( O - E  - 0.5)^2}{E}$	
		A	No A	A	No A	A	No A
H	44	66	1.2784	0.8523			
L	36	54	1.5625	1.0417			

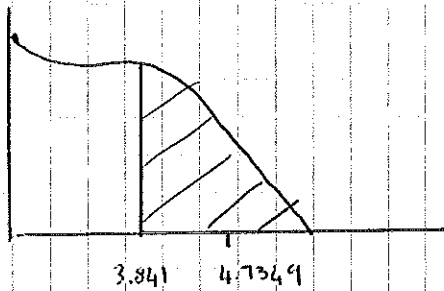
Total  $X_2 = 4.7349$  (TEST STATISTIC)

$H_0$ : No association between Asthma + Traffic (Independent)

$H_1$ : Association between Asthma + Traffic (Associated)

$v = (2-1) \times (2-1) = 1$

CRITICAL VALUE  $X_{2, (1) 5\%} = 3.841$



$4.7349 > 3.841$

$\therefore$  Reject  $H_0$

Evidence at 5% level to suggest an association between accidents & Asthma and the traffic volume.

b) 44 were expected, 52 were recorded  
 $\therefore$  More than expected had Asthma.

② a)  $X \sim P_0(b)$

$P(X = 8) = \frac{e^{-b} (b^8)}{8!} = 0.10325...$

b)  $Y \sim P_0(3^2)$

i)  $\lambda = 3^2 = 9$

ii)  $P(Y > 9) = 1 - P(Y \leq 9)$   
 $= 1 - 0.5874$  (from tables)  
 $= 0.4126$

c) i)  $T \sim P_0(15)$

ii)  $P(T \leq 20) = 0.9170$  (from tables)

iii)  $P(T \geq 21) = 1 - P(T \leq 20) = 1 - 0.9170 = 0.083$

For 4 out of 6, use Binomial

$\rightarrow 6 \binom{6}{4} \times (0.083)^4 \times (0.917)^2 = 0.0005996...$

③  $H_0: \mu = 34.5$

$H_1: \mu \neq 34.5$  (2 tailed test)

$\sigma = 2.5$

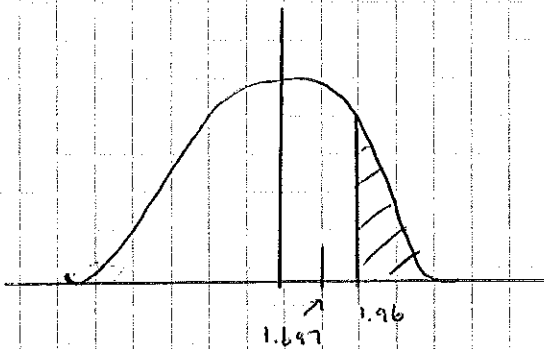
$\bar{x} = 35.1$

$n = 50$

we know  $\sigma$  and  $n = 50$ , so use Z distribution due to CLT

TEST STATISTIC:  $\frac{35.1 - 34.5}{\frac{2.5}{\sqrt{50}}} = 1.697...$

CRITICAL VALUE: Z, 5%, 2 tailed =  $\pm 1.96$

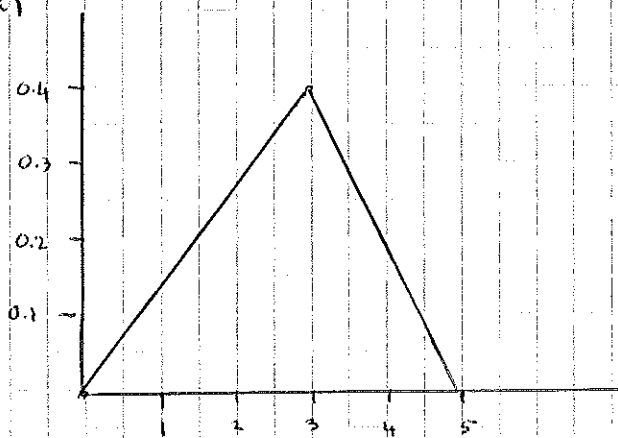


$1.697 < 1.96$

$\therefore$  Accept  $H_0$

Not enough evidence at 5% level to suggest mean weight has changed.

④ a)



b) i)  $P(T \leq 2)$

Use area of  $\Delta = \frac{1}{2} \times 2 \times \frac{4}{15} = \frac{4}{15}$

ii)  $P(2 < T < 4)$

$P(T < 2) = \frac{4}{15}$

$P(T > 4) = \frac{1}{2} \times 1 \times \frac{1}{5} = \frac{1}{10}$

$\therefore P(2 < T < 4) = 1 - (\frac{4}{15} + \frac{1}{10}) = \frac{10}{30}$

$$\begin{aligned}
 c) E(T) &= \int_0^3 t f(t) + \int_3^5 t f(t) \\
 &= \int_0^3 \frac{2t^2}{15} + \int_3^5 t - \frac{t^2}{15} \\
 &= \left[ \frac{2t^3}{45} \right]_0^3 + \left[ \frac{t^2}{2} - \frac{t^3}{15} \right]_3^5 \\
 &= \left( \frac{18}{45} - 0 \right) + \left( \frac{25}{2} - \frac{125}{15} - \frac{9}{2} + \frac{27}{15} \right) \\
 &= \frac{54}{45} + \frac{22}{15} = \frac{8}{3} = 2\frac{2}{3}
 \end{aligned}$$

5) a) i)  $\sum x = 31.9$   $\bar{x} = 31.9/10 = 3.19$   
 $\sum (x - \bar{x})^2 = 1.869$   $s^2 = 1.869/9 = 0.20766...$   
 $n = 10$   
 $v = 10 - 1 = 9$

t value (9) 99% (2 tailed) = 3.250

$$\begin{aligned}
 99\% \text{ CI} &= 3.19 \pm 3.250 \times \sqrt{\frac{0.20766}{10}} \\
 &= 3.19 \pm 0.46583 \\
 &= (2.724, 3.656)
 \end{aligned}$$

ii) Claim seems reasonable as 3.5g is comfortably in the confidence interval

b)  $0.01 \times 200 = 2$

6)  $H_0: \mu = 3.8$   
 $H_1: \mu > 3.8$  (1 tailed test)

Don't know  $\sigma$ , and  $n=7$   
 so use t

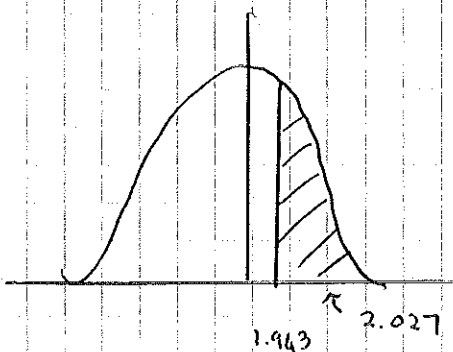
Data (from calc):

$$\begin{aligned}
 \sum x &= 28.7 \\
 \sum x^2 &= 118.59 \\
 n &= 7 \\
 v &= 7 - 1 = 6
 \end{aligned}$$

$$\begin{aligned}
 \bar{x} &= 4.1 \\
 s &= 0.39157... \\
 s^2 &= 0.15333...
 \end{aligned}$$

TEST STATISTIC:  $t = \frac{4.1 - 3.8}{\frac{0.30157}{\sqrt{7}}} = 2.027..$

CRITICAL VALUE:  $t_{(6)} 5\%, 1 \text{ tailed} = 1.943$



$2.027 > 1.943$

∴ Reject  $H_0$

There is evidence at 5% level to support belief that level of cholesterol is higher than manager's claim

Assumption: Cholesterol levels normally distributed

① a) i) Mean:  $E(X) = 5 \times 0.1 + 15 \times 0.2 + 25 \times 0.3 + 35 \times 0.4 = 25$

$E(X^2) = 5^2 \times 0.1 + 15^2 \times 0.2 + 25^2 \times 0.3 + 35^2 \times 0.4 = 725$

Variance =  $725 - 25^2 = 100$

∴  $SD = \sqrt{100} = 10$

ii)  $E(C) = 10 \times E(X) + 5 = 10 \times 25 + 5 = 255p$

b)  $Var(X) = 75.25 - 8.35^2 = 5.5275$

$Var(T) = 0.4^2 \times 5.5275 = 0.8844$

② a)  $P(X < 0) = F(0) = \frac{0+1}{k+1} = \frac{1}{k+1}$

b) For LQ ( $q_1$ ),  $F(q_1) = 0.25$

$\frac{q_1 + 1}{k+1} = 0.25$

→  $q_1 = 0.25(k+1)$

→  $q_1 = 0.25(k+1) - 1$

c)  $f(x) = F(x)$  differentiated

∴  $y = \frac{x+1}{k+1}$ ,  $\frac{d}{dx} y = \frac{1}{k+1} + \frac{1}{k+1}$

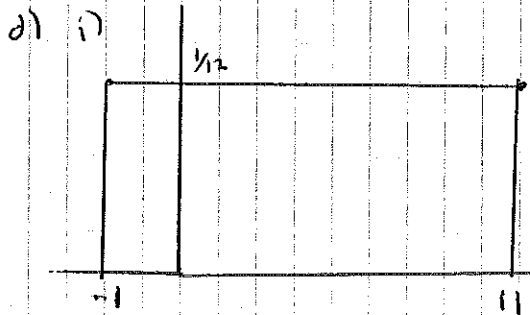
$$d) \beta(x) = \frac{1}{k+1} = \beta(x)$$

For other values  $F(x) = 0$

$$\rightarrow \beta(x) = \begin{cases} \frac{1}{k+1} \\ 0 \end{cases}$$

$$-1 \leq x \leq k$$

otherwise



$$ii) E(x) = \frac{11 + (-1)}{2} = 5$$

$$\text{Var}(x) = \frac{1}{12} (11 - (-1))^2 = 12$$

$$iii) P(q_1 < x < E(x))$$

$$q_1 = -1 + \frac{1}{4}(11 - (-1)) = 2$$

$$E(x) = 5$$

$$P(2 < x < 5) = (5 - 2) \times \frac{1}{12} = 0.25$$

↑  
From Diagram