Stats 2 - January 2008

① $H_0: \mu = 5$ mins

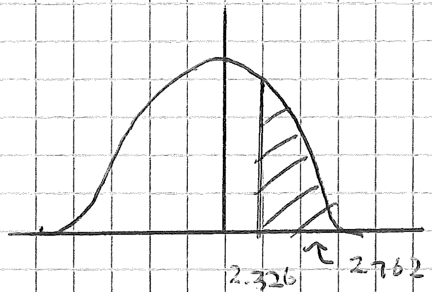
$H_1: \mu > 5$ mins

$n = 40$, so use Z distribution because of CLT

$\bar{x} = 5.5 \qquad s^2 = 1.31$

TEST STATISTIC: $\dfrac{5.5 - 5}{\sqrt{\dfrac{1.31}{40}}} = 2.762\ldots$

CRITICAL VALUE: 1%, 1 tailed test : 2.3263



Reject H_0 $\qquad 2.7262 > 2.3263$

Evidence at 1% level suggests average waiting times are ~~grea~~ not 5 mins

② a) $X \sim Po(9)$

i) $SD = \sqrt{9} = 3$

ii) $P(6 < X < 12) = P(X \leq 11) - P(X \leq 6)$

$= 0.8030 - 0.2068$

$= 0.5962$

b) i) $T \sim Po(11.5)$

ii) $P(T < 2) = P(T \leq 1) = P(T = 0) + P(T = 1)$

$= \dfrac{e^{-11.5} \times 11.5^0}{0!} + \dfrac{e^{-11.5} \times 11.5^1}{1!}$

$= 0.000127\ldots$

c) From calculator:

$n = 10 \qquad \bar{x} = 12$

$\Sigma x = 120 \qquad s = 4.3969\ldots$

$\Sigma x^2 = 1614 \qquad s^2 = 19\,1/3$

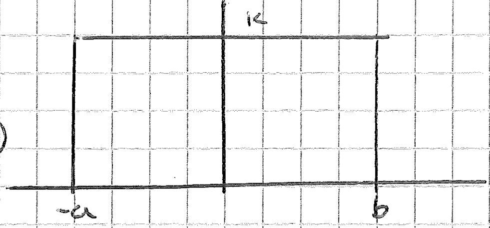
Mean and variance are very different.

$\therefore$ $P(12.0)$ not suitable model

③ a) i) $K = \frac{1}{b-(-a)} = \frac{1}{a+b}$

ii) $E(T) = \int_{-a}^{b} t(K)\,dt$

$\int x f(x)$



$= \left[\frac{Kt^2}{2}\right]_{-a}^{b}$

$= \left(\frac{Kb^2}{2}\right) - \left(\frac{K(-a)^2}{2}\right)$

$= \frac{K}{2}\left(b^2 - a^2\right)$

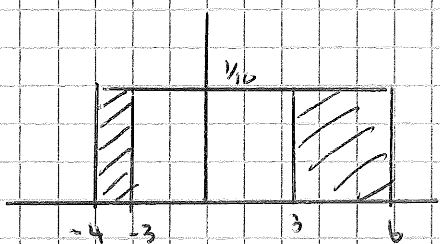
$K = \frac{1}{a+b}$

$= \frac{1}{2} \times \frac{1}{a+b} (b+a)(b-a)$

$= \frac{1}{2}(b-a)$

b) i) $E(T) = \frac{1}{2}(6-(-4)) = \frac{1}{2}(2) = 1$

ii)



$P(T < -3) = \frac{1}{10}$

$P(T > 3) = \frac{3}{10}$

$\therefore P(T < -3 \text{ or } T > 3)$

$= \frac{1}{10} + \frac{3}{10} = \frac{4}{10}$ or 0.4

④ a) $\sum v = 1179$ $\quad \bar{x} = 1179/10 = 117.9$

$\sum (v-\bar{v})^2 = 1014.9$ $\quad s^2 = 1014.9/9 = 112.766...$

$n = 10$ $\quad s = 10.6191...$

No σ^2, so must use t distribution

$v = 10 - 1 = 9$, 99% CI, $t = 3.250$ (look 0.995)

95% CI $= 117.9 \pm 3.250 \times \frac{10.6191}{\sqrt{10}}$

$= 117.9 \pm 10.9136...$

$= (106.98, 128.82)$

Assumption: speeds from Normal Distribution

b) Claim unlikely as 130mph lies outside confidence interval

(5) a) $P(X \geq 5) = P(X=5) + P(X=6)$

$= 5/20 + 6/24 = 1/2$

b) i)

$1/x$	1	1/2	1/3	1/4	1/5	1/6
$P(X=x)$	1/20	2/20	3/20	4/20	5/20	6/24

$E(1/X) = 1 \times (1/20) + 1/2 \times (2/20) + 1/3(3/20) + 1/4(4/20)$
$+ 1/5(5/20) + 1/6(6/24) = 7/24$

ii) $E(1/X^2) = 1^2 \times (1/20) + (1/2)^2(2/20) + (1/3)^2(3/20)$
$+ (1/4)^2(4/20) + (1/5)^2(5/20) + (1/6)^2(6/24)$
$= 109/900$

$\therefore Var(X) = 109/900 - (7/24)^2 = 173/4800$
$= 0.036$ (3dp)

c) $Area = (X+3)(1/x)$
$= 1 + 3/x$

Mean $E(1 + 3/x) = 1 + 3E(1/x)$
$= 1 + 3(7/24) = 15/8$ or $1\,7/8$

Variance $Var(1 + 3/x) = Var(1) + 3^2 Var(1/x)$
$= 0 + 9 \times (0.036)$
$= 0.32437$ or $519/1600$

(6) a) H_0: Salary not associated with uni (INDEPENDENT)
H_1: Salary is associated with uni (NON-INDEPENDENT)

2 by 2, so must use Yate's Correction, $\nu = 1$

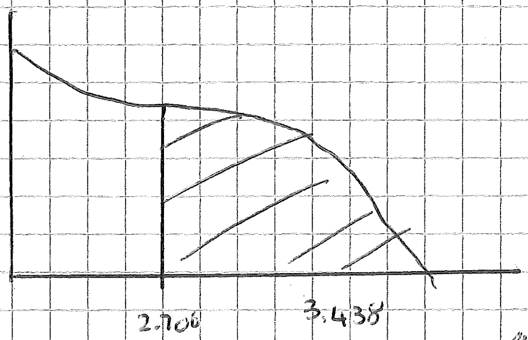
Expected

	< 30,000	> 30,000
Uni	59.8	70.2
No uni	55.2	64.8

Yates $= |O - E| - 0.5$
$= |78 - 70.2| - 0.5$
$= 7.3$

χ^2 $\quad \dfrac{(|O - E| - 0.5)^2}{E}$

	< 30,000	> 30,000
Uni	0.8411	0.7591
no-Uni	0.9654	0.8224



$\chi^2 = 3.4380$

(Test statistic)

CRITICAL VALUE: 10%, $v = 1$

$\chi^2_{(10\%)}(1) = 2.706$

$3.438 > 2.706$

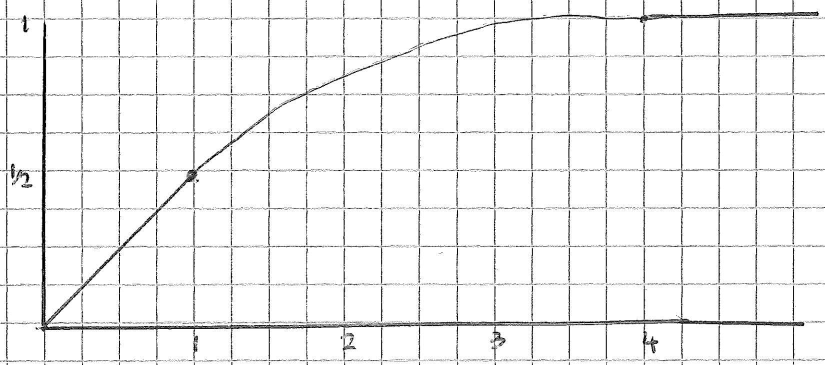
$\therefore$ Reject H_0

Evidence at 10% sig level suggests an association between salary and going to university.

b) Type I error = Rejecting H_0 when H_0 is correct

So, saying there is an association between salary and university when there is not.

7) a) i)



ii) For LQ, $F(x) = 0.25$

$\frac{1}{2}x = 0.25 \Rightarrow x = 0.5$

iii) Use $1 \leq x \leq 4$ group

$F(1.6) = \frac{1}{54}(1.6^3 - 12(1.6)^2 + 48(1.6) - 10) = 0.744$

$F(1.7) = 0.775$

For UQ, $F(x) = 0.75$, $\therefore$ $1.6 < q < 1.7$

b) $f(x) = F'(x)$ [differentiate]

$F(x) = 1/2\,(x) \qquad 0 \le x \le 1$

so $f(x) = 1/2 = \alpha$

$F(x) = 1/54\,(x^3 - 12x^2 + 48x - 10) \qquad 1 \le x \le 4$

so $f(x) = 1/54\,(3x^2 - 24x + 48)$

$= 3/54\,(x^2 - 8x + 16)$

$= 3/54\,(x-4)^2$

so $\beta = 3/54 = 1/18$

ii) $E(X) = \int_0^1 x\, f(x) + \int_1^4 x\, f(x)$

$= \int_0^1 1/2\, x + \int_1^4 1/18\, x(x-4)^2$

$= 1/2 \int_0^1 x + 1/18 \int_1^4 x^3 - 8x^2 + 16x$

$= 1/2 \left[x^2/2 \right]_0^1 + 1/18 \left[x^4/4 - 8x^3/3 + 8x^2 \right]_1^4$

$= (1/2)(1/2) - 0 + 1/18 \left[\left(256/4 - 512/3 + 128 - 1/4 + 8/3 + 8 \right) \right]$

$= 1/4 + 1/18\,(63/4)$

$= 1/4 + 7/8 = 1\,1/8$