Stats 2 - June 2007

① Ho: No association between outcome & treatment (Independent)

H_1: Association between outcome & treatment (non-Independent)

2 by 2, so need to use Yate's correction

~~Expected~~ Observed

	P	D	
Imp	20	46	66
Not	55	29	84
	75	75	150

Expected

	P	D
Imp	33	33
Not	42	42

$$X^2 \quad \frac{(|O - E| - 0.5)^2}{E}$$

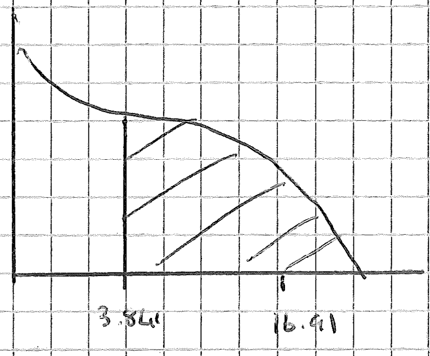
$$|O - E| - 0.5 = 12.5$$

	P	D
Imp	4.7348	4.7348
Not	3.7202	3.7202

$X^2 = 16.91$ (Test Statistic)

CRITICAL VALUE: $v = (2-1)(2-1) = 1$

$X^2_{5\%}(1) = 3.841$



$16.91 > 3.841$

Reject Ho

Evidence at 5% level suggests outcome (condition) may be dependent on treatment received

② a) i) $X \sim Po(3.5)$

$$P(X = 3) = \frac{e^{-3.5} \times (3.5)^3}{3!} = 0.216$$

ii) $Y \sim Po(6)$

$$P(Y \geq 5) = 1 - P(Y \leq 4)$$

$= 1 - 0.2851$ (from tables)

$= 0.7149$

b) i) $T \sim Po(9.5)$.

ii) $P(7 \le T \le 10) = P(T \le 10) - P(T \le 6)$

$= 0.6453 - 0.1649 = 0.4804$

iii) $p = (0.4804)^3 = 0.11086\ldots$

(3) $H_0: \mu = 36$

$H_1: \mu < 36$

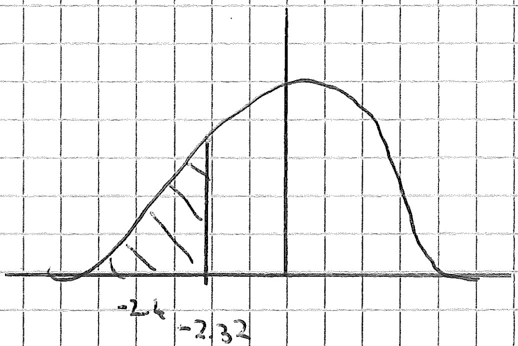
$n = 50$ $\qquad$ $\bar{x} = 1730/50 = 34.6$

$\Sigma x = 1730$ $\qquad$ $s^2 = 784/49 = 16$

$\Sigma(x - \bar{x})^2 = 784$ $\qquad$ $s = 4$

Test Statistic: $\dfrac{34.6 - 36}{4/\sqrt{50}} = -2.4748\ldots$

Critical Value: Z, 1%, 1 tailed test $= -2.3263$



$-2.47 < -2.32$

Reject H_0

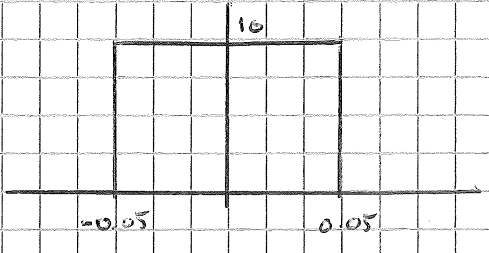
Sufficient evidence at 1% level to suggest average number of puts has decreased.

(4) a) Nearest 1/10 in m has bounds -0.05 to 0.05

For rectangular distribution $f(x) = \begin{cases} 1/(b-a) & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$

$-0.05 \le x \le 0.05$

$\rightarrow 1/(b-a) = 1/0.05-(-0.05) = 10$



Area $= 10 \times 1/10 = 1$

b) From diagram $= 0.03 \times 10 = 0.3$

c) Mean $= \dfrac{0.05 + (-0.05)}{2} = 0$

Variance $= 1/12\,(0.05 - (-0.05))^2 = 1/12\,(0.01)$

$\therefore$ sd $= \sqrt{1/12(0.01)} = 0.02886\ldots$

5) a) From calculator:

$n = 10$ $\qquad\qquad \bar{x} = 35.6$

$\Sigma x = 356$ $\qquad\qquad s = 6.1860\ldots$

$\Sigma x^2 = 13{,}018$ $\qquad\qquad s^2 = 38.2666\ldots$

99% CI. No σ^2, so need t distribution,

$v = 10 - 1 = 9 \rightarrow t = 3.250$ (look up 0.995)

$$99\%\text{ CI} = 35.6 \pm 3.250 \times \frac{6.1860}{\sqrt{10}}$$

$$= 35.6 \pm 6.357\ldots$$

$$= (29.2,\ 42.0)$$

b) Confidence interval includes 30mph

BUT 8/10 cars in sample exceed it

$\therefore$ speed limit not being stuck to by most motorists

6) a) i) $E(1/x) = \int_0^1 1/x\ f(x)\ dx$

$$= \int_0^1 1/x\,(3x^2) = 3\int_0^1 x\ dx$$

$$= \left[3x^2/2\right]_0^1 = 3/2 \text{ or } 1.5$$

ii) $E(1/x^2) = \int 1/x^2\ f(x)\ dx$

$$= \int_0^1 1/x^2\,(3x^2)\ dx = \int_0^1 3$$

$$= \left[3x\right]_0^1 = 3$$

$$\mathrm{Var}(1/x) = E(1/x^2) - \left[E(1/x)\right]^2$$

$$= 3 - 1.5^2 = 0.75$$

b) MEAN: $E\left(\frac{5}{x} + 2(x)\right) = E\left(\frac{5}{x}\right) + E(2)$

$$= 5E\left(\frac{1}{x}\right) + 2$$

$$= 5(1.5) + 2 = 9.5$$

VARIANCE: $Var\left(\frac{5+2x}{x}\right) = Var\left(\frac{5}{x}\right) + Var(2)$

$= 5^2\, Var(1/x) + 0$

$= 25 \times 0.75 = 18.75$

(7) a) i)

x	4	-1
$P(X=x)$	1/5	4/5

ii) $E(X) = 4(1/5) + (-1)(4/5) = 0$

b)

x	4	-1
$P(X=x)$	1/3	2/3

$E(X) = 4(1/3) + (-1)(2/3) = 2/3$

out of $24 = 24 \times 2/3 = 16$ $24E(X)$

(8) a) $H_0: \mu = 230$

$H_1: \mu \neq 230$ (2 tailed)

No σ^2, so must use t-distribution, $\nu = 8 - 1 = 7$

From calculator:

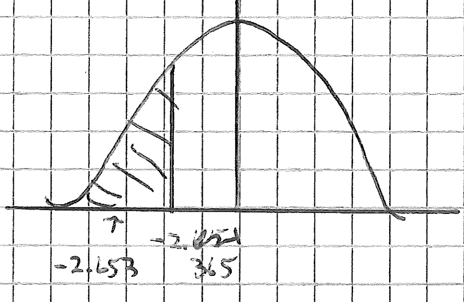
$n = 8$ $\bar{x} = 225.25$

$\Sigma x = 1802$ $s = 5.0638..$

$\Sigma x^2 = 406{,}080$ $s^2 = 25.642..$

TEST STATISTIC: $\frac{225.25 - 230}{5.0638/\sqrt{8}} = -2.6531..$

CRITICAL VALUE: $t_{5\%}(7)$, 2 tailed test $= \pm 2.365$



$-2.653 < -2.365$

$\therefore$ Reject H_0

Evidence at 5% level suggests that average weight in jar is not 230g

b) Rejected H_0 when H_0 was true = Type 1 Error.