Stats 2 - January 2007

① Don't know σ^2 so must use t

From calculator:

$n = 8$ $\quad \bar{x} = 39.5$

$\Sigma x = 316$ $\quad s = 4.8403...$

$\Sigma x^2 = 12,646$ $\quad s^2 = 23.4285...$

$\nu = 8 - 1 = 7$

$t_{(7)}$ for 95% $= 2.365$ (look up 0.975)

95% CI $= 39.5 \pm 2.365 \times \frac{4.8403...}{\sqrt{8}}$

$= 39.5 \pm 4.0472...$

$= (35.5, 43.5)$

② a) i) $A \sim Po(3.5)$

$P(A=4) = \frac{e^{-3.5} \times (3.5)^4}{4!} = 0.18891...$

ii) $B \sim P(5)$

$P(B \leq 6) = 0.7622$ from tables

iii) $A + B \sim Po(8.5)$

$P(A+B < 10) = P(A+B \leq 9)$

$= 0.6530$

b) Let $(A+B) = X \rightarrow X \sim B(5, 0.6530)$

$P(X \geq 4) = P(X=4) + P(X=5)$

$= {}^5C_4 \times (0.653)^4 (0.347) + {}^5C_5 \times (0.653)^5$

$= 0.31547 + 0.11873$

$= 0.434$ (3dp)

c) i) From calc: $\bar{x} = 9.2$ $\quad s = 3.04776$ $\quad s^2 = 9.2898...$

iii) Mean and variance have similar values (9.2, 9.28...) which suggests a Poisson may be appropriate.

(3) $H_0: \mu = 85.9$

$H_1: \mu \neq 85.9$ (2 tailed)

$\Sigma x = 8350$ $\quad n = 100$

$\Sigma (x - \bar{x})^2 = 15\,321$

$\bar{x} = \frac{8350}{100} = 83.5$

$s^2 = \frac{15\,321}{99} = 154.7575...$ $\quad s = 12.440...$

Over 30 people, so use Z distribution because of CLT

5%, 2 tailed test

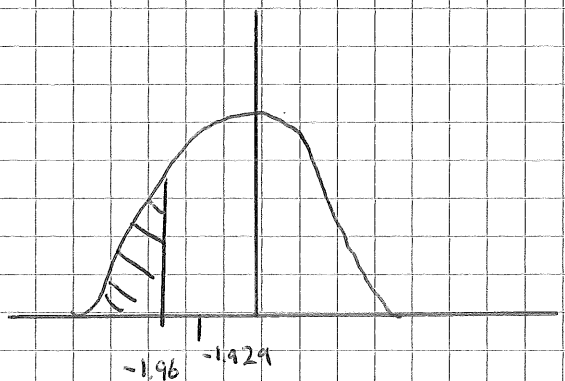
TEST STATISTIC: $Z = \frac{83.5 - 85.9}{12.440... / \sqrt{100}} = -1.929$

CRITICAL VALUE: 5%, 2 tailed test = ± 1.96

$-1.929 > -1.96$

$\therefore$ Accept H_0

Not enough evidence at 5% level to suggest average golf scores have changed.



(4) a) Probs must sum to 1

$0.62 + K = 1 \rightarrow K = 0.38$

b) i) $E(X) = 1 \times 0.01 + 2 \times 0.05 + 3 \times 0.14 + 4 \times 0.3 + 5 \times 0.38 + 6 \times 0.12 = 4.35$

ii) $E(X^2) = 1^2 \times 0.01 + 2^2 \times 0.05 + 3^2 \times 0.14 + 4^2 \times 0.3 + 5^2 \times 0.38 + 6^2 \times 0.12 = 20.09$

$\text{Var}(X) = E(X^2) - E(X)^2$

$= 20.09 - (4.35)^2 = 1.1675$

c) i) $E(4) = 5E(X) + 2$

$= 5 \times 4.35 + 2 = 23.75$

ii) $Var(Y) = 5^2 \ Var(X)$

$= 25 \times 1.1675 = 29.1875$

$\therefore \ sd(Y) = \sqrt{29.1875} = 5.4025...$

(5) a) $H_0: \mu = 30$

$H_1: \mu > 30$

Small sample, no population variance, so use t

From calculator:

$n = 10$ $\bar{x} = 33.5$

$\Sigma x = 335$ $s = 4.2491...$

$\Sigma x^2 = 11385$ $s^2 = 18.0555...$

$v = 10 - 1 = 9$

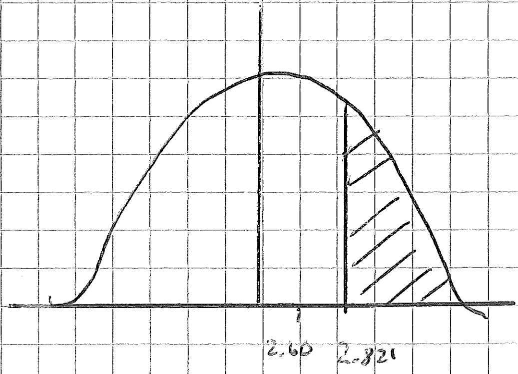
TEST STATISTIC: $t = \dfrac{33.5 - 30}{4.2491/\sqrt{10}} = 2.6047...$

CRITICAL REGION: (VALUE) $t_{(1\%)}$ (9) 1 Tailed: $= 2.821$

$2.60 < 2.821$

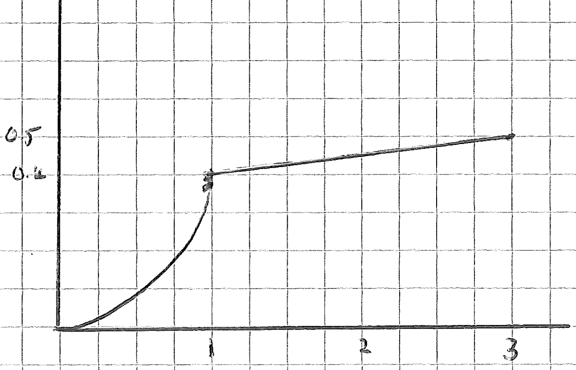
Do not Reject Ho

Not enough evidence at 1% level to suggest the teacher is underestimating H/W times



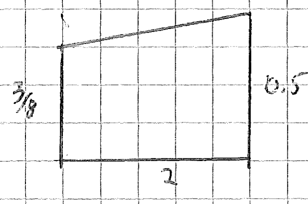
b) H/W times are Normally Distributed

(6) a)



b) $P(T \geq 1)$

= Area of Trapezium



$= \frac{1}{2}\left(\frac{3}{8} + 0.5\right) \times 2$

$= \frac{7}{8}$

c) i) From b), $F(1) = 1 - 7/8 = 1/8$

This needs adding on!

Need: $\int_1^t 1/16\,t + 5/16\,dt$

$= \left[1/32\,t^2 + 5/16\,t \right]_1^t$

$= 1/32\,t^2 + 5/16\,t - 1/32 - 5/16$

So: $F(t) = 1/32\,t^2 + 5/16\,t - 1/32 - 5/16 + 1/8$

$= 1/32\,t^2 + 5/16\,t - 7/32$

$= 1/32\,(t^2 + 10t - 7)$

ii) For median $F(t) = 0.5$

$\therefore 1/32\,(t^2 + 10t - 7) = 0.5$

$t^2 + 10t - 7 = 16$

$t^2 + 10t - 23 = 0$

Does not factorise: $t = \dfrac{-10 \pm \sqrt{10^2 - 4 \times 1 \times (-23)}}{2}$

$t = \dfrac{-10 \pm \sqrt{192}}{2} = -5 \pm \sqrt{48}$

As t between 0 and 3

median $= -5 + \sqrt{48} = 1.9282\ldots$

⑦ a) H_0: no association between maths performance at KS3 & GCE (Independent)

H_1: there is an association (Non-Independent)

Expected

	A	B	C	Below C
8	63.55	51.25	46.33	43.87
7	44.64	36	32.54	30.82
6	46.81	37.75	34.13	32.31

χ^2 $\dfrac{|O_i - E_i|^2}{E_i}$

	A	B	C	Below C
8	0.1483	0.2744	0.0097	0.0173
7	2.4043	0.6644	0.0733	0.7527
6	0.9907	0.007	0.0222	1.0005

$\chi^2 = 6.1897$ (Test statistic)

CRITICAL VALUE: $\nu = (4-1)(3-1) = 6$, $\chi^2_{(90\%)}(6) = 10.645$

$6.1897 < 10.645$

Do NOT reject H_0

Not enough evidence at 90% level to suggest an association between KS3 & GCE performance

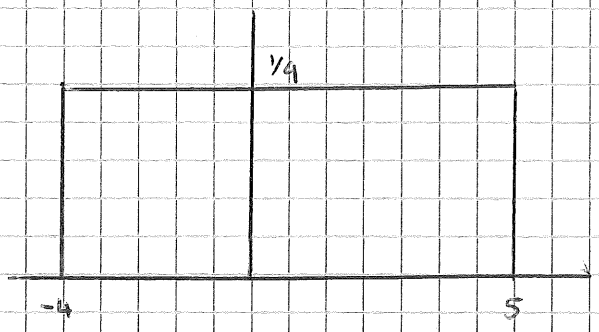
b) More students who got Level 7 achieved and A than expected.

(8) a) $x \leq -4 \rightarrow 0$ and $x \geq 5 \rightarrow 0$

Let $y = \frac{x+4}{9}$, $dy/dx = 1/9$

$$\therefore f(x) = \begin{cases} 1/9 & -4 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

b)



c) $P(X > 2) = (5-2) \times 1/9$

$= 1/3$

d) Mean $= \frac{5 + (-4)}{2} = 1/2$

Variance $= 1/12 (b-a)^2$

$= 1/12 (-4-5)^2$

$= 6.75$