

## Stats 2 - May/June 2006

① a)  $X \sim \text{Po}(3)$

For a 5 year period:

$$Y \sim \text{Po}(15)$$

$$P(Y > 18) = 1 - P(Y \leq 18)$$

$$= 1 - 0.8195$$

From Tables!

$$= 0.1805$$

b)  $Y \sim \text{Po}(7)$

i)  $(X + Y) \sim \text{Po}(10)$

$$P(X + Y) \geq 15 = P(X + Y) \leq 14$$

$$= 0.9165$$

From Tables

ii) That Maths grades and English grades are independent

② a) From calculator:

$$n = 5$$

$$\bar{x} = 50.8$$

don't know pop variance  
so must use t

$$\sum x = 254$$

$$s = 4.5497$$

$$\sum x^2 = 12986$$

$$s^2 = 20.7$$

$$v = 5 - 1 = 4$$

$$t \text{ critical value (95\%)} = 2.776$$

$$\text{Confidence interval: } 50.8 \pm 2.776 \times \frac{4.5497}{\sqrt{5}}$$

$$= (45.151, 56.449)$$

b) 5% or 0.05 as 95% CI

③ a)  $E(X) = 1 \times \left(\frac{1}{16}\right) + 2 \times \left(\frac{5}{16}\right) + 3 \times \left(\frac{3}{16}\right) + 4 \times \left(\frac{1}{16}\right)$   
 $= \frac{15}{8} = \text{mean}$

$$E(X^2) = 1^2 \times \left(\frac{1}{16}\right) + 2^2 \times \left(\frac{5}{16}\right) + 3^2 \times \left(\frac{3}{16}\right) + 4^2 \times \left(\frac{1}{16}\right)$$
$$= \frac{35}{8}$$

$$\text{Variance} = E(X^2) - [E(X)]^2 = \frac{35}{8} - \left(\frac{15}{8}\right)^2$$
$$= \frac{55}{64}$$

b) i)  $32 \times (\frac{3}{16}) + 32 \times (\frac{1}{16}) = 8$  games

ii) when score at least 3 =  $0.9 \times 8 = \frac{36}{5}$

when score 2 goals =  $0.5 \times [32 \times (\frac{5}{16})] = 5$

when score 1 goal =  $0.2 \times [32 \times (\frac{7}{16})] = \frac{14}{5}$

TOTAL = 15

4) a) i)

	A	B	
22-34	21	32	(53)
35-39	13	36	(108)
40-59	27	12	(39)
	(120)	(80)	(200)

ii)  $H_0$ : No association between area and age (Independent)

$H_1$ : Association between area and age (Non-Independent)

Expected

	A	B
22-34	31.8	21.2
35-39	64.8	43.2
40-59	23.4	15.6

$\chi^2$  values

	A	B
22-34	3.6679	5.5019
35-39	0.8	1.2
40-59	0.5539	0.8368

$\sum \chi^2 = 12.554$  (CRITICAL VALUE) (TEST STATISTIC)

~~TEST STATISTIC: X~~

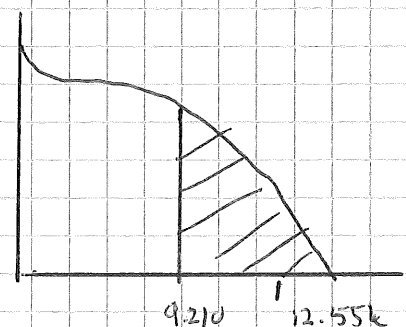
$v = (3-1)(2-1) = 2$

CRITICAL VALUE:  $\chi^2_{1\%}(2) = 9.210$

$12.554 > 9.210$

$\therefore$  Reject  $H_0$

There is evidence of an association between area and age profile



- b) In school A, fewer staff employed than expected in 22-34 group  
 In school B, more staff employed than expected in 22-34 group

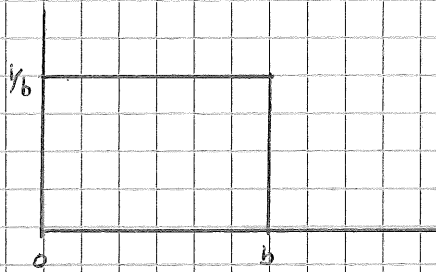
⑤ a) i)  $E(X) = b/2$

ii)  $E(X^2) = \int x^2 f(x)$

$= \int_0^b x^2 (1/b)$

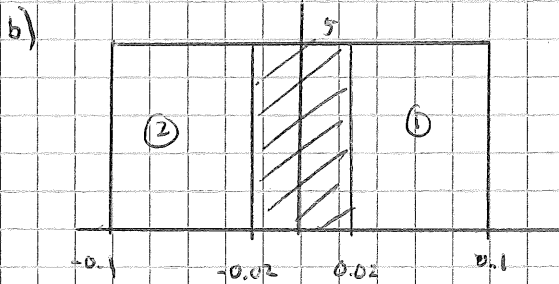
$= 1/b \int_0^b x^2$

$= 1/b [x^3/3]_0^b = 1/b [b^3/3 - 0] = b^2/3$



$Var(X) = E(X^2) - E(X)^2$

$= b^2/3 - (b/2)^2 = b^2/3 - b^2/4 = 1/12 b^2$



$P(|T| > 0.02) = (1) + (2) \text{ or } (1) \times 2$   
 $= [5 \times (0.1 - 0.02)] \times 2$   
 $= 4/5 \text{ or } 0.8$

- ⑥ a)  $H_0: \mu = 100$   
 $H_1: \mu < 100$  (1-tailed)

Use t-distribution as we don't know pop variance

Data from Calc:

$n = 5$

$\bar{x} = 94.2$

$\sum x = 471$

$s = 6.0580$

$\sum x^2 = 44,515$

$s^2 = 36.7$

$v = 5 - 1 = 4$

TEST STATISTIC:

$\frac{94.2 - 100}{6.0580 / \sqrt{5}} = -2.140$

~~CRITICAL VALUE~~

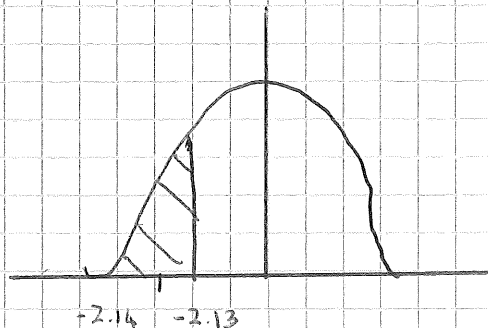
CRITICAL VALUE:

$t_{5\% (4)} \text{ 1-tailed} = -2.132$

$-2.14 < -2.13$

$\therefore$  Reject  $H_0$

There is evidence at 5% level to reject  $H_0$  and support the view that the average life of batteries is less than 100 hours



b) As the sample is now  $n = 80$ , we can use the  $Z$  distribution because of Central Limit Theorem

$H_0: \mu = 100$

$H_1: \mu \neq 100$  (2 tailed test)

$$\bar{x} = \frac{8080}{80} = 101$$

$$s^2 = \frac{6399}{79} = 81 \quad s = 9$$

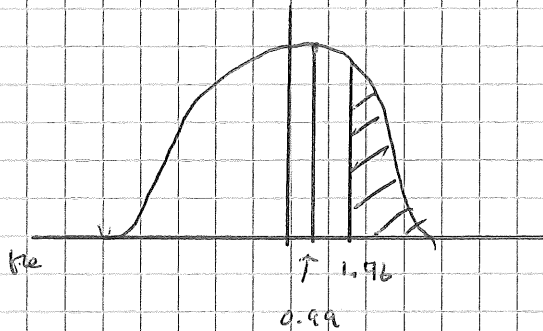
Test Statistic:  $Z = \frac{101 - 100}{9/\sqrt{80}} = 0.9938...$

CRITICAL VALUE: 5% (2 TAILED) =  $\pm 1.96$

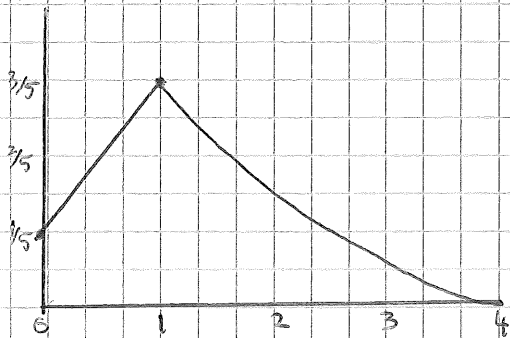
$0.99 < 1.96$

$\therefore$  Accept  $H_0$

There is enough evidence at 5% level to accept the manufacturer's claim that mean battery life is 100 hours



⑦ a)



b) i) 
$$F(x) = \int_0^x f(x) dx$$

$$= \int_0^x \frac{1}{5}(2x+1) dx$$

$$= \frac{1}{5} \int_0^x (2x+1) dx$$

$$= \frac{1}{5} [x^2 + x]_0^x$$

$$= \frac{1}{5} x^2 + \frac{1}{5} x - 0$$

$$= \frac{1}{5} x(x+1)$$

ii)  $P(X \leq 1) = F(1) = \frac{1}{5}(1)(1+1) = \frac{2}{5}$

iii)  $P(X \geq x) = 1 - P(X \leq x)$

so  $\frac{17}{20} = 1 - P(X \leq x)$

so  $P(X \leq x) = \frac{3}{20}$

so  $F(x) = \frac{3}{20}$

$\frac{1}{5} x(x+1) = \frac{3}{20}$

$$x(x+1) = 15/20$$

$$x^2 + x = 15/20$$

$$20x^2 + 20x = 15$$

$$20x^2 + 20x - 15 = 0$$

$$4x^2 + 4x - 3 = 0$$

$$(2x - 1)(2x + 3) = 0$$

↓

$$x = 1/2 \quad \text{or} \quad x = -3/2$$

$$\text{So } x = 1/2 \quad \text{as} \quad 0 \leq x \leq 4$$

iv) As  $F(1) = 0.4$ , the LQ is  $0 \leq x \leq 1$

$$F(x) = 0.25$$

$$\frac{1}{5}x^2 + \frac{1}{5}x = 0.25$$

$$x^2 + x - 1.25 = 0$$

Doesn't Factorise so use formula:

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1.25)}}{2}$$

$$= \frac{-1 \pm \sqrt{6}}{2} = \frac{1}{2}(-1 + \sqrt{6}) \quad \text{as}$$

$x$  must be positive.