Stats 2 – January 2006

① $X \sim Po(1.5)$

a) i) $P(X=2) = \dfrac{e^{-1.5} \times (1.5)^2}{2!} = 0.251$ (3dp)

ii) $P(X=2)^3 = 0.251^3 = 0.0158$

b) i) $Y \sim Po(9)$ $[6 \times 1.5]$

ii) $P(Y \geq 12) = 1 - P(Y \leq 11) = 1 - 0.8030 = 0.197$

c) The bug attacks patients randomly and independently

② a) H_0: choice of sport is independent of gender

H_1: choice of sport is affected by gender (non-independent)

Observed

	Squash	Badminton	Archery	Hockey	
Male	5	16	30	19	70
Female	4	20	33	53	110
	9	36	63	72	180

Expected

	Squash	Badminton	Archery	Hockey
Male	3.5	14	24.5	28
Female	5.5	22	38.5	44

$\dfrac{\text{Row} \times \text{Collum}}{\text{Total}}$

As expected value for squash < 5, must combine

Combine Squash and Badminton as similar categories:

Expected

	Squash + Bad	Archery	Hockey
Male	17.5	24.5	28
Female	27.5	38.5	44

$\dfrac{(O-E)^2}{E}$

X^2 Values

	SB	A	H
M	0.7	1.2347	2.8928
F	0.4455	0.7857	1.8409

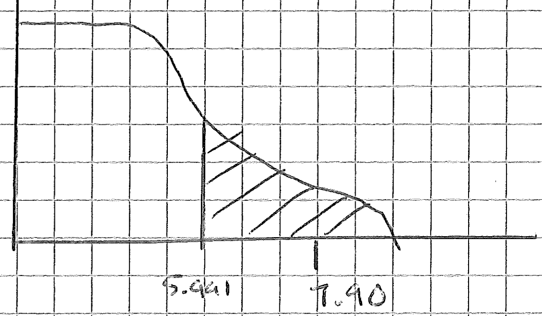
Sum of X^2 values = 7.90 = Test Statistic

Degrees of freedom (v) = $(3-1) \times (2-1) = 2$

Critical Value : $X^2_{5\%}(2) = 5.991$

Reject Ho as $7.90 > 5.991$

There is enough evidence at 5% level of significance to suggest an association between sport and gender



b) More females and fewer males choose hockey than would be expected if choices independent of gender.

(3) a) Don't know population variance, so use t-distribution

From calc: $n = 9$ $\bar{x} = 8$

$\Sigma x = 72$ $s = 2.1213$

$\Sigma x^2 = 612$ $s^2 = 4.5$

Degrees of freedom $= 9 - 1 = 8$

t value for 90% = 1.860 (look up 0.95)

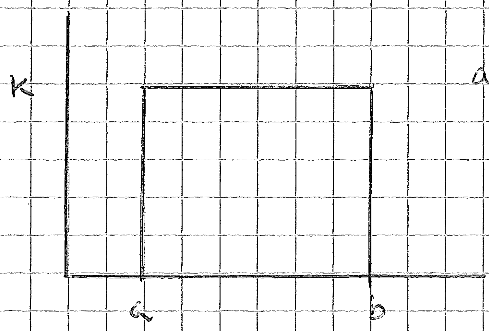
Confidence interval $= 8 \pm 1.86 \times \left(\frac{2.1213}{\sqrt{9}}\right)$ $\frac{s}{\sqrt{n}}$

$= 8 \pm 1.315$

$= (6.68, 9.32)$

b) Head teachers' claim not supported by evidence at 10% level as 5 lies outside of the confidence interval

(4)



a) i) $K \times (b-a) = 1 \rightarrow K = 1/b-a$

ii) $E(X) = \int x f(x)\,dx$

$= \int_a^b xK\,dx$

K = just a number

$= \left[\frac{x^2}{2}K\right]_a^b$

$$= \left[\left(\frac{x^2}{2}\right)\left(\frac{1}{b-a}\right)\right]_a^b$$

$$= \left(\frac{1}{2(b-a)}\right)\left[x^2\right]_a^b$$

$$= \frac{1}{2(b-a)}\left[b^2 - a^2\right]$$

$$= \frac{1}{2(b-a)}\left((b+a)(b-a)\right)$$

$$= \tfrac{1}{2}(a+b)$$

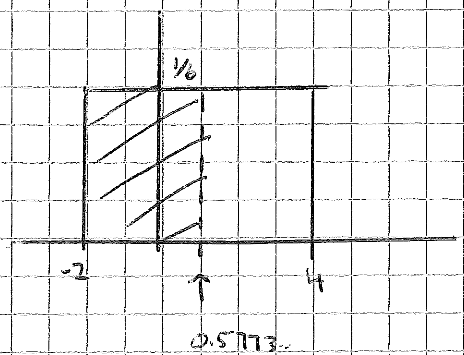
b) i) $\mu = \tfrac{1}{2}(-2+4) = 1$

ii) For rectangular distribution $\sigma^2 = \tfrac{1}{12}(b-a)^2$

$$= \tfrac{1}{12}(4+2)^2 = 3$$

$$\therefore \sigma = \sqrt{3}$$

iii) $P\left(X < \frac{2-\mu}{\sigma}\right) = P\left(X < \frac{1}{\sqrt{3}}\right) = P(X < 0.5773..)$



$$= \tfrac{1}{6} \times (2 + 0.5773..)$$

$$= 0.430$$

(5) a) Mean $= 40 \times 0.3 + 45 \times 0.24 + 55 \times 0.36 + 74 \times 0.1$

$$= 50 = E(X)$$

$E(X^2) = 40^2 \times 0.3 + 45^2 \times 0.24 + 55^2 \times 0.36 + 74^2 \times 0.1$

$$= 2602.6$$

$\text{Var}(X) = E(X^2) - E(X)^2$

$$= 2602.6 - 50^2 = 102.6$$

$\therefore sd(X) = \sqrt{102.6} = 10.129...$

b) Mean $= 10 \times 50 + 250 = 750$

SD $= 10 \times 10.129.. = 101.29...$

(6) a) $H_0: \mu = 65$

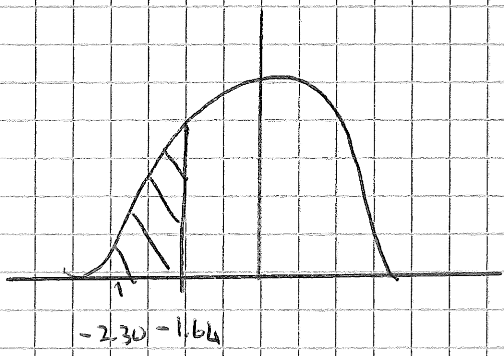
$H_1: \mu < 65$ (1TT)

{We know population sd so use z}

If H_0 true: $\bar{X} \sim N(65, 9^2/35)$

Test Statistic: $z = \dfrac{61.5 - 65}{9/\sqrt{35}} = -2.30$

Critical value: 5%, 1 tailed test, $= -1.6449$



$-2.30 < -1.6449$

Reject H_0

Evidence at 5% level suggests students may be under-achieving

b) Type I error = Reject H_0 when True

→ Conclude students are underachieving when they are not

(7) a) $E(T) = \int t f(t) = \int_0^1 t \cdot 4t(1-t^2)\, dt$

$= \int_0^1 4t^2 - 4t^4\, dt$

$= \left[\frac{4}{3}t^3 - \frac{4}{5}t^5\right]_0^1$

$= \frac{4}{3} - \frac{4}{5} - 0 = \frac{8}{15}$

b) i) $F(t) = P(T \leq t)$

$= \int_0^t f(t) = \int_0^t 4t(1-t^2)\, dt$

$= \int_0^t 4t - 4t^3\, dt$

$= \left[2t^2 - t^4\right]_0^t = 2t^2 - t^4$

ii) $P(\mu < T < med) = P(T < med) - P(T < mean)$

$= F(med) - F(\mu)$ using cum distribution

F(med) always 0.5

$= \cancel{F(m)}\ 0.5 - 2(8/15)^2 - (8/15)^4$

$= 0.5 - 0.4879...$

$= 0.0120...$

(8) $H_0: \mu = 1000$

$H_1: \mu \neq 1000$ (2 tailed)

we don't know population SD, so need t-distribution

From calc:

$n = 12$ $\bar{x} = 1003$

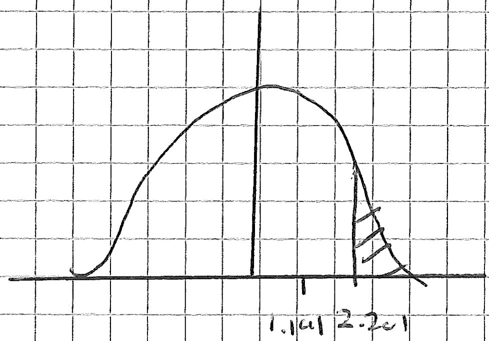
$\Sigma x = 12036$ $s = 5.4439..$

$\Sigma x^2 = 12072434$ $s^2 = 29.636..$

Test statistic $= \dfrac{1003 - 1000}{5.4439 / \sqrt{12}} = 1.91$

Critical Value: $v = 12 - 1 = 11$

2 Tailed: $t_{5\%}(11) = \pm 2.201$



Accept H_0

Not enough evidence at 5% level to suggest a change in the mean content of sherry in a bottle