

STATS ② - June 2005

① a) $P(X=2) = \frac{e^{-2.6} \times (2.6)^2}{2!} = 0.25104...$

b) i) $2.5 \times 5 = 13 \rightarrow Y \sim P_0(13)$

ii) $P(Y \geq 15) = 1 - P(Y \leq 14)$
 $= 1 - 0.6751 = 0.3249$

② H_0 : Time of day has no effect on result (INDEPENDENT)

H_1 : Time of day influences result (NON-INDEPENDENT)

As there are 2 rows = 2 columns, we need YATE'S CORRECTION

$\nu=1$

	Observed			Expected	
	Afternoon	Evening		Afternoon	Evening
Win	30	24	54	25.92	28.08
Loss	18	28	46	22.08	23.92
	48	52	100		

Yates' Correction: $10 - E| - 0.5$

$\chi^2 = \frac{(10 - E| - 0.5)^2}{E}$

	A	E
W	3.58	3.58
L	3.58	3.58

	A	E
W	0.4945	0.4564
L	0.5905	0.5358

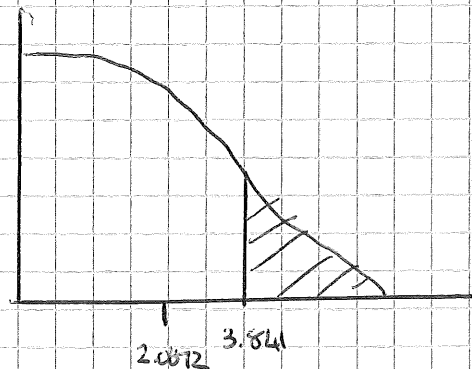
Calculated value of χ^2 (Test Statistic) = 2.0672

Critical Value: $\chi^2_{(5\%)}(1) = 3.841$

$2.0672 < 3.841$

\therefore Do not reject H_0

Not enough evidence at the 5% level to suggest the time of day has an effect on the outcome of a game of snooker.



③ a) From calculator:

$$\sum x = 15.8, \quad \sum x^2 = 25.0592, \quad n = 10$$

$$\bar{x} = \frac{15.8}{10} = 1.58$$

$$s^* = 0.10282$$

$$s^2 = 0.01057$$

b) $n = 10$, so $v = 9$

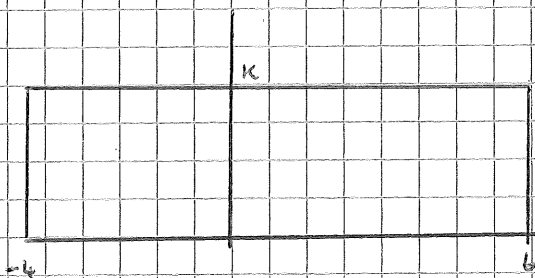
For 90% CI, look up 0.95, $t_9 = 1.833$

$$CI = 1.58 \pm 1.833 \times \frac{0.10282}{\sqrt{10}}$$

$$= 1.58 \pm 0.0595$$

$$= (1.52, 1.64)$$

④



a) $k = 1/10$

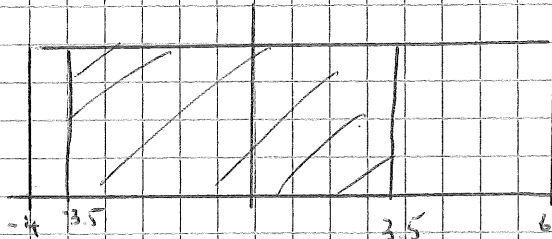
b) $E(X) = \frac{b + (-4)}{2} = 1$

c) $P(X > 0) = 6 - 0 = 0.6$

d) $P(X > 3.5)$

$$= 1 - P(X < 3.5)$$

$$= 1 - 0.7 = 0.3$$



⑤ a) $E(R) = 1 \times (1/4) + 2 \times (1/2) + 4 \times (1/4) = 2.25$

$$\text{Var}(R) = E(R^2) - [E(R)]^2$$

$$E(R^2) = 1 \times (1/4) + 2^2 \times (1/2) + 4^2 \times (1/4) = 6.25$$

$$\therefore \text{Var}(R) = 6.25 - 2.25^2 = 1.1875$$

b) i)

x	1	$1/4$	$1/16$
$P(X=x)$	$1/4$	$1/2$	$1/4$

 $x = 1/R^2$

$$E(X) = 1 \times (1/4) + 1/4 \times (1/2) + 1/16 \times (1/4) = 25/64$$

ii) Area = $8/R(R + 8/R) = 8 + 64/R^2$

$$E(\text{Area}) = 8 + 64 E(1/R^2) = 8 + 64 \times 25/64 = 33$$

(b) $H_0: \mu = 568$

$H_1: \mu < 568$ (1-tailed)

σ as don't know
 σ^2

1% sig level, 1-tailed test, $v = 8 - 1 = 7$

• CRITICAL VALUE = -2.998

$\bar{x} = \frac{4510}{8} = 563.75$

$n = 8$
 $\sum x = 4510$
 $\sum x^2 = 254256.8$

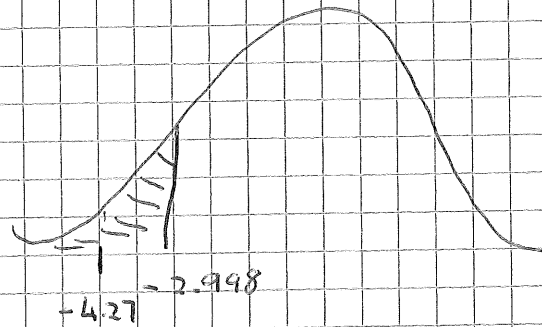
$s^2 = 2.816 \rightarrow S^2 = 7.929$

• TEST STATISTIC =

$t = \frac{563.75 - 568}{\frac{2.816}{\sqrt{8}}} = -4.27$

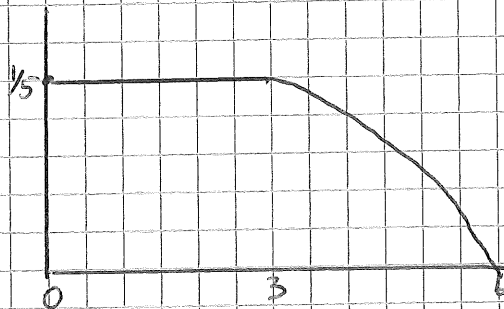
$-4.27 < -2.998$

∴ Reject H_0



There is evidence at the 1% significance level to suggest the average contents of the tins have been reduced.

(7) a)



b) $P(T=3) = 0$
as cumulative

c) $P(T \geq 3) = 1 - P(T < 3)$
 $= 1 - (3 \times 1/5)$ From graph
 $= 2/5$

d) As $P(T \leq 3) = 3/5$, median must be between 0 and 3

$\int_0^m 1/5 dt = 0.5$

$[t/5]_0^m = 0.5$

$m/5 = 0.5 \rightarrow m = 2.5$

e) Need Mean = $E(T) = \int_a^b x f(x)$

$= \int_0^3 1/5 t dt + \int_3^6 1/45 t^2 (6-t)$

$$\begin{aligned}
 &= \int_0^3 \frac{1}{5} t \, dt + \int_3^6 \left(\frac{2}{15} t^2 - \frac{1}{45} t^3 \right) dt \\
 &= \left[\frac{1}{10} t^2 \right]_0^3 + \left[\frac{2}{45} t^3 - \frac{1}{180} t^4 \right]_3^6 \\
 &= \frac{1}{10} (9) + \frac{2}{45} (216) - \frac{1}{180} (1296) - \frac{2}{45} (27) + \frac{1}{180} (81) \\
 &= 2.55
 \end{aligned}$$

$P(\text{median} < T < \text{mean})$

$$= P(2.5 < T < 2.55)$$

$$= 0.05 \times 15 \quad (\text{From graph})$$

$$= 0.01$$

⑧ a) $H_0: \mu = 35$

$H_1: \mu \neq 35$ (2 tailed)

Z as we know
 $\sigma^2 = 12^2$

2 tailed test, 1% sig level, $n = 100$

CRITICAL VALUE = ± 2.5758

$$\bar{x} = 37.9$$

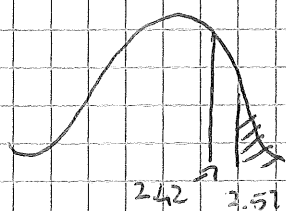
$$\sigma^2 = 12^2$$

$$n = 100$$

$H_0: \bar{X} \sim N(35, \frac{144}{100})$

TEST STATISTIC:

$$Z = \frac{37.9 - 35}{12/\sqrt{100}} = 2.42$$



$$2.42 < 2.5758$$

Do not reject H_0

Not enough evidence at 1% level to reject claim that average age is 35 years

b) Type II error means accept H_0 when it is false

So accept mean age is 35 years when it is not