June 2012

(1) Sample size $n = 20$.

minimum width x cm

minimum thickness y mm

$S_{xx} = 2.030 \quad S_{yy} = 1.498 \quad S_{xy} = -0.410$

a) Product moment correlation.

$$r = \frac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}}$$

$$r = \frac{-0.410}{\sqrt{2.030 \times 1.498}}$$

$$r = -0.23511478$$

$$\underline{\underline{r = -0.235}}$$

b.) Some weak negative correlation between width and thickness of lengths of steel.

(Q2) (a) (i) mode = 23 as it has the highest frequency.

(ii) Median: $\frac{175+1}{2} = \frac{176}{2} = 88\text{th value}$

Median (88th value) = $\underline{\underline{22}}$

IQR: $LQ = \frac{175+1}{4} = \frac{176}{4} = 44\text{th value}$

$= 20$

$UQ = \frac{175+1}{4} \times 3 = 132\text{nd Value}$

$= 23$

IQR. = $UQ - LQ = 23 - 20 = \underline{\underline{3}}$

(b.) Estimated mean $\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{3902.5}{175}$

$= 22.3$

Standard deviation = 6.37 from calculator.

using midpoints

(c) estimated Mean from Royal mail $= 22.3$

private courier : estimated mean $= \frac{280}{175} = 1.6$.

Mean $= 22.3 + 1.6 = 23.9$.

③ (a) $b = \dfrac{S_{xy}}{S_{xx}}$ and $a = \bar{y} - b\bar{x}$

$a = 4.1698 = 4.170$ from calculator

$b = 2.271$

$$\underline{\underline{y = 4.170 + 2.271x}}$$

(b.) See graph

$x = 10$ $y = 26.878$

$x = 50$ $y = 117.708$

$x = 70$ $y = 163.122$

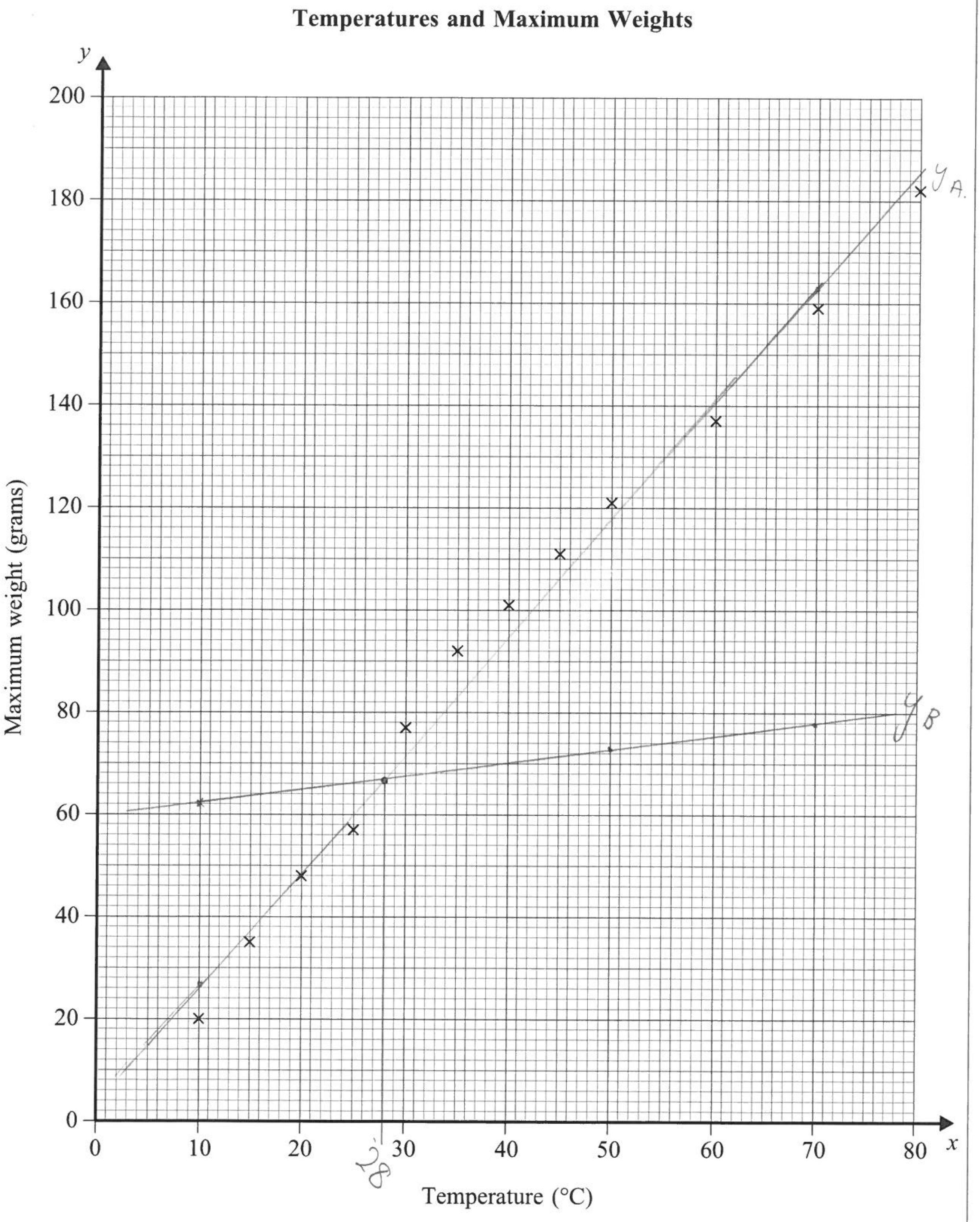
(c) (i) See graph

(ii) See graph, point where both lines intersect.

Water temperature $= 28$.

(iii) At low temperatures (below 28 °c) more of Salt B dissolves than Salt A

The amount of Salt that dissolves increases more rapidly for A (higher gradient).


Temperatures and Maximum Weights
y
200
180
160
140
120
100
80
60
40
20
0
Maximum weight (grams)
0
10
20
30
40
50
60
70
80
x
Temperature (°C)
y_A
y_B
28

(Q4.) (a) (i) $P(B=3) = \dfrac{7+72+99+16}{640} = \dfrac{194}{640}$

$= \underline{\underline{0.303}}$

(ii) $P(T \geq 2) = 1 - P(T=1)$

$= 1 - \dfrac{77}{640} = \underline{\underline{0.880}}$

(iii) $P(B=3 \ \& \ T \geq 2) = \dfrac{72+99+16}{640} = \dfrac{187}{640}$

$= \underline{\underline{0.292}}$

(iv) $P(B \leq 3 \mid T=2) = \dfrac{P(B \leq 3 \ \& \ T=2)}{P(T=2)}$

$= \dfrac{\frac{77+67+14}{640}}{\frac{172}{640}}$

$= \dfrac{153}{172} = \underline{\underline{0.890}}$

(b) $P(a \cap b) = P(a) \times P(b)$

if two events are independent.

$P(B = 3)\ \dfrac{97}{320}$

$P(T \geq 2) = \dfrac{563}{640}$

$P(B = 3 \cap T \geq 2) = \dfrac{187}{640} \neq \dfrac{97}{320} \times \dfrac{563}{640}$

$0.292 \neq 0.267.$

(c) $P(2T \cap 3T \cap \geq 4T \mid B = 3)$

$$= \frac{72}{194} \times \frac{99}{193} \times \frac{16}{192}$$

$$= \frac{297}{18721} = 0.01586$$

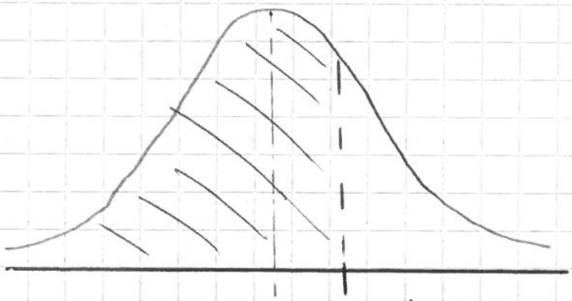
There are different ways to choose the houses but each will give the same answer of 0.01586

$0.01586 \times 6 = \underline{\underline{0.09516}}$

June 2012

5. a.) $W \sim N(2.75, 0.15^2)$

(i) $P(W < 2.8)$

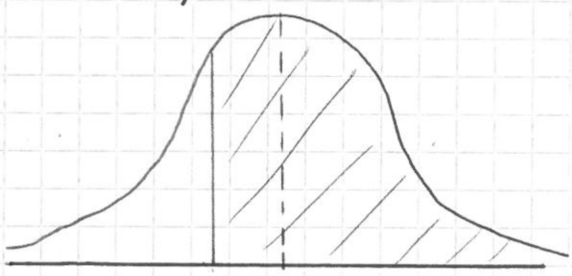


W	2.75	2.8
Z	0	0.33

$$\frac{2.8 - 2.75}{0.15} = \frac{1}{3}$$

$P(Z < 0.33) = \underline{\underline{0.62930}}$ from tables.

(ii) $P(W > 2.5)$



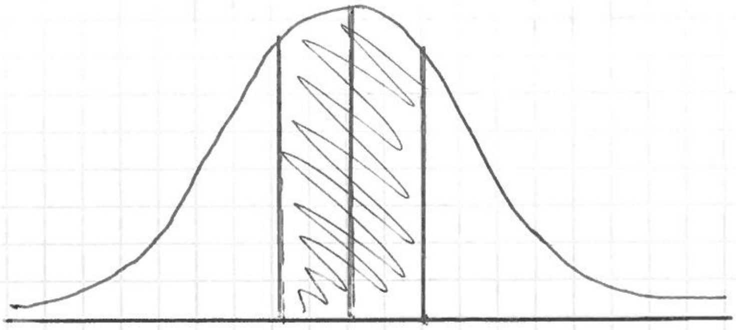
W	2.5	2.75
Z	-1.67	0

$$\frac{2.5 - 2.75}{0.15} = -1.67$$

$P(Z > -1.67) = P(Z < 1.67)$

$= \underline{\underline{0.95254}}$

(b.) $X \sim N(5.25, 0.20)$



X	5.1	5.25	5.3
Z	-0.75	0	0.25

$$\frac{5.1 - 5.25}{0.2} = -0.75 \qquad \frac{5.3 - 5.25}{0.2}$$

$$P(Z < 0.25) - P(Z < -0.75)$$

$$= P(Z < 0.25) - (1 - P(Z < 0.75))$$

$$= 0.59871 - (1 - 0.77337)$$

$$= 0.59871 - 0.22663$$

$$= 0.37208$$

$$= \underline{\underline{0.372}} \ (3.d.p)$$

(ii) four 5kg bags is selected

$$P(5.1 < X < 5.3) = 0.372$$

$$P(\text{0 out of 4}) = [1 - 0.372]^4$$

$$= \underline{\underline{0.1556}}$$

(c) 10 kg bags

$$Y \sim N(10.75, 0.50^2)$$

$$P(\bar{Y}_6 < 10.5)$$

$$\text{Variance } \bar{Y}_6 = \frac{0.50^2}{6} = 0.0416$$

$$\text{SD } \bar{Y}_6 = \frac{0.50}{\sqrt{6}} = 0.204$$

$$P(\bar{Y}_6 < 10.5) \Rightarrow P\left(Z < \frac{10.5 - 10.75}{\sqrt{0.0416}}\right)$$

$$= P(Z < -1.2257)$$

$$= 1 - P(Z < 1.23)$$

$$= 1 - 0.89065$$

$$= \underline{\underline{0.109}}$$

(6.) (a)(i) $P \sim B(30, 0.13)$

$$P(P=2) = \binom{30}{2}(0.13)^2(1-0.13)^{28}$$

$$= \underline{\underline{0.1489}}$$

(ii) $p = 0.22 + 0.13 = 0.35$

$$R \cup P \sim B(30, 0.35)$$

$$P(R \cup P > 10) = 1 - P(R \cup P \leq 10)$$

$$1 - 0.5078$$

$$= \underline{\underline{0.4922}}$$

(iii) $G \sim B(30, 0.20)$

$$P(5 \leq G \leq 10)$$

$$P(5 \leq G) = 1 - P(G \leq 4) = 0.2552.$$

$$P(G \leq 10) = 0.9744.$$

$$0.9744 - 0.2552$$

$$= \underline{\underline{0.7192}}$$

(b.) $n = 100$
$p = 0.22$

mean $= np$

$$= 100 \times 0.22$$

$$\mu = 22.$$

Variance $\sigma^2 = np(1-p)$.

$\sigma^2 = 22(0.78)$

$\sigma^2 = \underline{\underline{17.16}}$.

(ii)(1) Proportion of red paper clips is greater than 0.22.

Reject this claim as the means are similar

$0.22 \approx 0.221$.

(2) Jumbo packets do not contain random samples.

Variance (from above) = 17.16

$sd = \sqrt{17.16} = 4.14$

$4.14 \approx 4.17$ standard deviations are similar

Therefore reject the claim that not random samples.

Q(7)

5 litre bottle Standard deviation = 75 ml

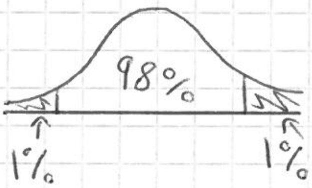
= 0.075 l.

$n = 36$ 181.80 litres in total.

8 bottles contained less than 5 litres

(a.) Sample mean $\bar{x} = \frac{181.80}{36} = 5.05$

using the Central limit theorem. $\bar{X} \sim \left(5.05, \frac{0.075^2}{36}\right)$



98% confidence Interval.

98% multiplier for z (2 tails) = 2.3263

$$5.05 \pm 2.3263 \times \frac{0.075}{\sqrt{36}}$$

$$= \underline{\underline{(5.02, 5.08)}}$$

(b) (1) mean volume exceeds 5 - litre bottle

Accept this claim as Confidence Interval for the mean exceeds 5 litres, for 98% of bottles mean lies between 5.02 and 5.08

(2) 10% of bottles contain less than 5 libres.

10% = 0.10 found that $\frac{8}{36} = 0.22$ contain less than 5

$0.10 \neq 0.22$ reject claim

(c) I made use of the central limit theorem in part (a) because the volume of the bleach was not normally distributed but the Sample size was great enough (≥ 30) to make use of the CLT.