Stats 1 - January 2011

(1) a) i) Strong positive correlation $\rightarrow r = 0.8$

ii) Weak negative correlation $\rightarrow r = -0.3$

b) i) From calculator: $\Sigma x^2 = 614.97$

$r = 0.7570847...$

ii) Strong, positive, linear correlation between the circumference and weight of cricket balls.

(2) a) i) $175/645$

ii) $519/645$

iii) $63/645$

iv) $P(L \mid F) = \dfrac{P(L \cap F)}{P(F)} = \dfrac{94}{126}$

v) $P(M \mid L') = \dfrac{P(M \cap L')}{P(L')} = \dfrac{175 + 54 + 35}{296} = \dfrac{264}{296}$

b) $P(F, F) = \dfrac{94}{126} \times \dfrac{93}{125} = \dfrac{8742}{15750}$ or 0.555

c) $P(L, C, \text{NEITHER}) = \dfrac{349}{645} \times \dfrac{193}{644} \times \dfrac{103}{643} = 0.02597$

6 ways of arranging 3 objects

$\rightarrow 6 \times 0.02597 = 0.15585..$

(3) a) i) True boundaries $= 0.975 - 1.005$

Midpoint $= \dfrac{0.975 + 1.005}{2} = 0.99$

ii) Minimum $= 0.975$, Maximum $= 1.005$

b) From calculator, and using midpoints for x:

$\Sigma x^2 = 112.9662 \qquad \Sigma x = 106.2$

$\rightarrow$ mean $(\bar{x}) = 1.062$

sample standard deviation $(s) = 0.04285$

c) i) $\bar{x} = 1.062$ $s = 0.043$ $n = 100$

99% multiplier (2 tailed) for $z = 2.5758$

99% CI for $\mu = \bar{x} \pm z \times \frac{s}{\sqrt{n}}$

$= 1.062 \pm 2.5758 \times \frac{0.043}{\sqrt{100}}$

$= 1.062 \pm 0.01107...$

$= (1.051, 1.073)$

ii) We know the population is normally distributed

iii) We have used midpoints as the data is in groups.

d) i) The lower bound of the confidence interval (1.051) is greater than 1

ii) 99/100 of the pieces of data in the sample are in this range.

(4) $X \sim B(15, 0.45)$ X = number of times target hit

a) i) $P(X \leq 5) = 0.2608$ (tables)

ii) $P(X > 10) = 1 - P(X \leq 10)$

$= 1 - 0.9745 = 0.0255$

iii) $P(X = 6) = {}^{15}C_6 \times 0.45^6 \times 0.55^9$

$= 0.1914$

iv) $P(5 \leq X \leq 10)$

CAN BE: 5, 6 ... 10

$\rightarrow P(X \leq 10) - P(X \leq 4)$

$= 0.9745 - 0.1204 = 0.8541$

b) i) HITS = $P(H) = 0.85$

$P(H', H) = 0.15 \times 0.8 = 0.12$

$$\left.\begin{array}{r} 0.85 \\ 0.12 \end{array}\right\} +$$
$$\underline{0.97}$$

ii) Let M = number of times the target is missed

$M \sim B(50, 0.03)$ $\qquad P(M) = 1 - 0.97 = 0.03$

$P(\text{HITS} \geq 48)$

HITS :	48	49	50
MISSES :	2	1	0

$= P(M \leq 2) = 0.8108$ (from tables)

iii) Probability she shoots with 2nd = Probability she misses with 1st

$= 1 - 0.85 = 0.15$

$\therefore$ Mean $= np = 80 \times 0.15 = 12$ times

(5) a) Time taken is dependent on leaving time

b) From calculator: $a = 29.9545\ldots$ (intercept)

$b = 1.281818\ldots$ (gradient)

$\rightarrow y = 29.95 + 1.28x$

c) 7.45am $\rightarrow x = 15$

$\rightarrow y = 29.95 + 1.28(15)$

$= 49.15$ minutes

$\rightarrow$ arrival time of 8.34 am

= 26 minutes before

d) i) $x = 85 \rightarrow y = 29.95 + 1.28(85)$

$= 138.75$

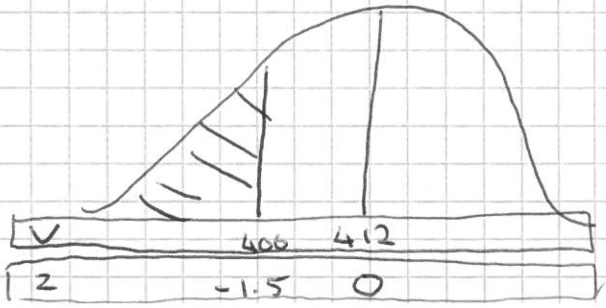
ii) You shouldn't use x values outside of the range of data to make predictions.

(6) a) $V \sim N(412, 8^2)$

i) $P(V < 400)$

$= P\left(Z < \frac{400 - 412}{8}\right)$

$= P(Z < -1.5)$



$= P(Z > 1.5)$

$= 1 - P(Z < 1.5)$

$= 1 - 0.93319 = 0.06681$

ii) $P(V > 420)$

$= P\left(Z > \frac{420 - 412}{8}\right)$

$= P(Z > 1)$

$= 1 - P(Z < 1)$

$= 1 - 0.84134$

$= 0.15866$

iii) $P(V = 410) = 0$

b) i) $P(V < 400) = 0.05$

Z value for $0.95 = 1.6449$

$\therefore$ Z value for $V = 400$

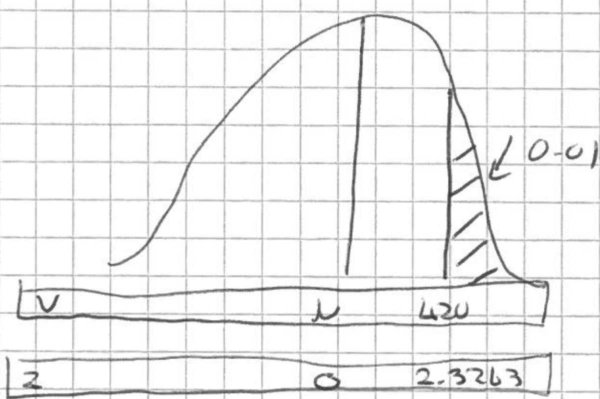
$= -1.6449$

Standardise:

$$\frac{400 - \mu}{\sigma} = -1.6449$$

$\rightarrow \quad 400 - \mu = -1.6449\sigma$

$P(V > 420) = 0.01$



Z value for $0.99 = 2.3263$

$\rightarrow$ Standardise:

$$\frac{420 - \mu}{\sigma} = 2.3263$$

$\rightarrow 420 - \mu = 2.3263\sigma$

ii) (1) $400 - \mu = -1.6449\sigma$

(2) $420 - \mu = 2.3263\sigma$

(2) - (1) $\rightarrow 20 = 3.9712\sigma$

$\rightarrow \sigma = \dfrac{20}{3.9712} = 5.036\ldots$

$\boxed{\mu}$ (1) $400 - \mu = -1.6449(5.036\ldots)$

$\rightarrow 400 - \mu = -8.2841\ldots$

$\rightarrow 400 + 8.2841\ldots = \mu$

$\rightarrow \mu = 408.28\ldots$