Stats 1 - June 2009 (specimen)

1) a) i) $100/160$

ii) $\dfrac{160-32}{160} = \dfrac{128}{160}$

iii) $\dfrac{32+60-18}{160} = \dfrac{74}{160}$

iv) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{30}{100}$

b) $P(C, R, S) = \dfrac{24}{160} \times \dfrac{56}{159} \times \dfrac{32}{158} = \dfrac{43008}{4019520}$

6 ways of arranging C, R, S

$\rightarrow 6 \times \dfrac{43008}{4019520} = \dfrac{1344}{20935} = 0.064198\ldots$

2) a) From calculator: $\Sigma x^2 = 30667$

$r = 0.89319\ldots$

b) Strong, positive, linear correlation between lengths and widths of snakes.

c) See graph on mark scheme

d) i) G & D (closest to top-right of graph)

ii) $r \approx 0.5$ as there appears to be positive, but weaker linear correlation for males

3) $X \sim N(253, \sigma^2)$ $\sigma = 5$

a) i) $P(X < 250)$

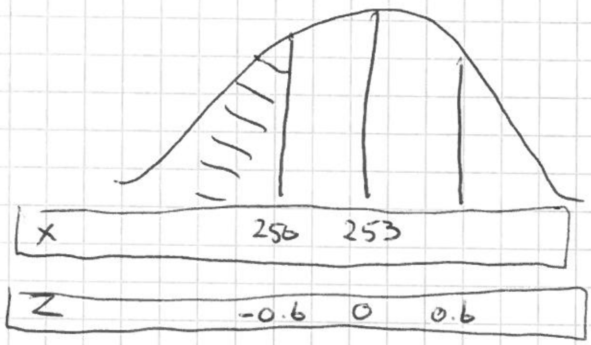
$= P\left(Z < \dfrac{250-253}{5}\right)$

$= P(Z < -0.6)$

$= P(Z > 0.6)$

$= 1 - P(Z < 0.6)$

$= 1 - 0.72575 = 0.27425$



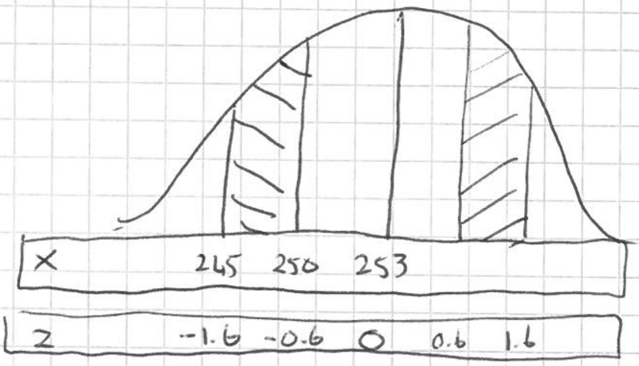
ii) $P(245 < x < 250)$

$= P\left(\frac{245-253}{5} < z < \frac{250-253}{5}\right)$

$= P(-1.6 < z < -0.6)$

$= P(0.6 < z < 1.6)$

(you can see they are same area on the diagram)



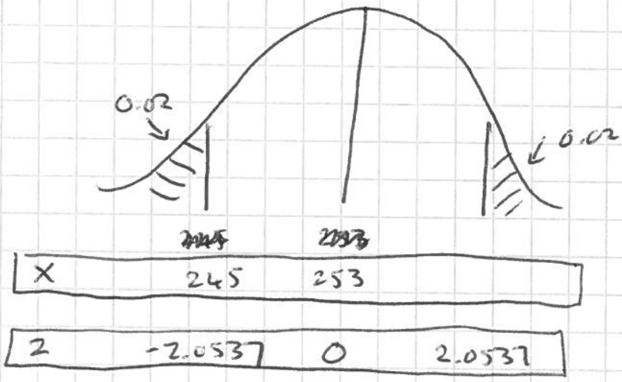
$= P(z < 1.6) - P(z < 0.6)$

$= 0.94520 - 0.72575 = 0.21945$

iii) $P(X = 245) = 0$

b) z value for 98%
$= 2.0537$

$\therefore$ z value for $x = 245$ must be -2.0537



Standardise:

$\frac{245-253}{\sigma} = -2.0537$

$245 - 253 = -2.0537\sigma$

$\sigma = \frac{-8}{-2.0537} = 3.8954\ldots$

(4) a) From calculator: $\Sigma x^2 = 14975$

$a = 49.7982\ldots$ (intercept)

$b = -0.54814\ldots$ (gradient)

$\rightarrow y = 49.7982 - 0.54814x$

b) when $x = 0 \rightarrow y = 49.7982$

$= 50g$ (nearest g)

c) 13 weeks = 91 days $(7 \times 13) \rightarrow x = 91$

$\rightarrow y = 49.7982 - 0.54814(91)$

$= -0.08254g$

Negative weight implies the tablet has dissolved

$\therefore$ claim appears justified

(5) a)

x	1	2	3	4	5	6	7
f	14	35	25	13	9	2	1
cum f	14	49	74	87	96	98	99

i) Median $= \dfrac{99+1}{2}$ = 50th value = 3 children

LQ $= \dfrac{99+1}{4}$ = 25th value = 2 children

UQ $= \dfrac{3(99+1)}{4}$ = 75th value = 4 children

$\therefore$ IQR = $4 - 2 = 2$ children

ii) From calculator: $\Sigma x^2 = 933$ $\quad \Sigma x = 275$

Mean = 2.777... $(\bar{x})$

sd = 1.31362... (s)

b) i) Σx still 275 $\rightarrow$ Mean $= \dfrac{275}{163} = 1.68711...$

ii) sd will increase due to increase in spread of data

iii) Mean = 1.687

New sd must be bigger than 1.313...

Mean $- 2 \times$ sd $\rightarrow$ negative number of children!

And yet, for Normal distribution we would expect some data to be with 2 sds of the mean.

(6) a) From Calculator: $\bar{x} = 47$ $\quad s = 15$ $\quad n = 10$

98% z multiplier (2 tailed) = 2.3263

98% CI for $\mu = \bar{x} \pm z \times \dfrac{s}{\sqrt{n}}$

$\rightarrow \mu = 47 \pm 2.3263 \times \dfrac{15}{\sqrt{10}}$

$\rightarrow \mu = 47 \pm 11.034\ldots$

$\rightarrow (35.965, \quad 58.034)$

b) $\bar{Y} \sim N(108, \ 28^2/40)$

$P(\bar{Y} > 120)$

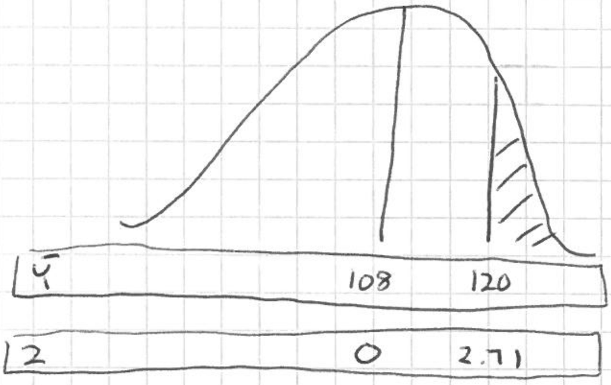
$= P\left(Z > \dfrac{120 - 108}{28/\sqrt{40}}\right)$

$= P(Z > 2.71)$

$= 1 - P(Z < 2.71)$

$= 1 - 0.9964$

$= 0.00336$



(4)

c) Yes. In b) we did not know if the population was normally distributed

(7) a) $R \sim B(50, 0.15)$

i) $P(R < 10) = P(R \leq 9) = 0.7911$ (tables)

ii) $P(5 \leq R \leq 10)$

can be: $5, 6 \ldots 10$

$\rightarrow P(R \leq 10) - P(R \leq 4)$

$= 0.8801 - 0.1121 = 0.768$

b) i) $F \sim B(22, 0.06)$

$P(F = 2) = {}^{22}C_2 \times 0.06^2 \times 0.94^{20} = 0.24125$

ii) $F \sim B(35, 0.06)$

$P(F \geq 1) = 1 - P(F = 0)$

$= 1 - {}^{35}C_0 \times 0.06^0 \times 0.94^{35}$

$= 1 - 0.11467\ldots$

$= 0.8853$

iii) Correctly ! $\rightarrow$ $F \sim B(120, 0.94)$

$\boxed{\text{MEAN}} = np = 120 \times 0.94 = 112.8$

$\boxed{\text{VARIANCE}} = np(1-p) = 120 \times 0.94 \times 0.06 = 6.768$

iv) Means are the same (112.8)

$\therefore$ Agree probability she sorts a letter incorrectly is 0.06

But variances are very different. Much bigger for 20 batches

$\therefore$ Disagree that this is independent from letter to letter.