

Stats 1 - January 2006

① a) From calculator: $a = 2.318181\dots$
 $b = 0.886363\dots$

$$\rightarrow y = 2.32 + 0.886x$$

b) $a =$ average waiting time of 2.32 minutes if there are 0 customers already seated (i.e. $x=0$)

$b =$ the increase in waiting time of 0.886 mins that each additional customer entering the restaurant causes.

c) i) $x=5 \rightarrow y = 2.32 + 0.886(5) = 6.75$ mins

ii) $x=25 \rightarrow y = 2.32 + 0.886(25) = 24.47$ mins

d) i) $x=5$ is in range of data, and -1.1 to $+1.1$ is fairly small residuals, so result is reliable

$x=25$ lies outside data range so result is unreliable (extrapolation).

② a) i) $P(L, L, L) = 0.3 \times 0.4 \times 0.2 = 0.024$

ii) $P(L', L', L') = 0.7 \times 0.6 \times 0.8 = 0.336$

iii) $P(L', L', L) = 0.7 \times 0.6 \times 0.2 = 0.084$

b) i) $P(ZL, WL) = 0.2 \times 0.9 = 0.18$

ii) $P(ZL, WL') = 0.2 \times 0.1 = 0.02$

$$P(ZL', WL) = 0.8 \times 0.25 = 0.20$$

$$\text{Probability of either} = 0.02 + 0.2 = 0.22$$

$$(3) a) \bar{x} = \frac{286.5}{50} = 5.73$$

$$s = \sqrt{\frac{45.16}{49}} = \sqrt{0.921...} = 0.96001...$$

b) 99% (2 tailed) z multiplier = 2.5758

$$\mu = \bar{x} \pm z \times \frac{s}{\sqrt{n}}$$

$$\mu = 5.73 \pm 2.5758 \times \frac{0.96001...}{\sqrt{50}}$$

$$\mu = 5.73 \pm 0.3497...$$

$$\mu = (5.38, 6.08)$$

c) The confidence interval does not include 5 or $6\frac{1}{2}$, so both claims appear invalid.

$$(4) a) \text{ From calculator: } \sum fsc = 8025 \quad n = 100$$

(use midpoints for x)

$$\sum fx^2 = 739975$$

$$\text{Mean } (\bar{x}) = 80.25$$

$$\text{Standard deviation } (s) = 31.134889$$

b) i) Sample size is > 30 , so using Central Limit Theorem

ii) $\mu_{\text{mean}} (\bar{y})$ is same = 80.25

$$\text{Standard error } (\bar{y}) = \frac{s}{\sqrt{n}} = \frac{31.13...}{\sqrt{36}} = 5.1891...$$

$$\text{iii) } \bar{y} \sim N(80.25, 5.1891^2)$$

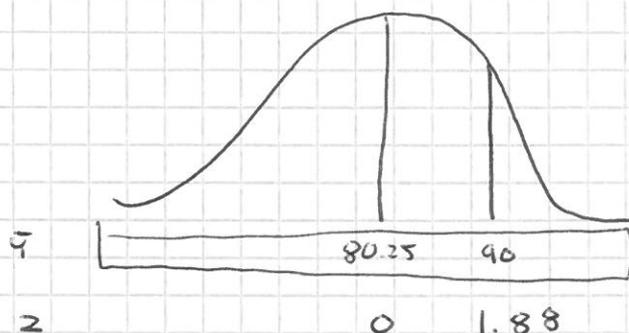
$$1/2 \text{ mins} = 90 \text{ xcs}$$

$$P(\bar{y} < 90)$$

$$= P\left(Z < \frac{90 - 80.25}{5.181}\right)$$

$$= P(Z < 1.8818...)$$

$$\sim P(Z < 1.88) = 0.96995$$



- ⑤ a) See Scatter Diagram (not included here!)
- b) i) ① Positive Linear Correlation between times
 ② Two outliers, C & D
- ii) 0.462 - fairly weak positive

c) C = Free style Champion
 D = Backstroke Champion

d) i) From calculator: $r = 0.91242\dots$

ii) Strong, positive correlation \rightarrow boys are equally good/bad at both strokes.

⑥ $R \sim B(50, 0.2)$

a) i) $P(R \leq 15) = 0.9692$ (from tables)

$$\text{ii) } P(R=10) = {}^{50}C_{10} \times 0.2^{10} \times 0.8^{40} \\ = 0.13981\dots$$

iii) $P(5 < R < 15)$

can be: 6, 7, 8, ..., 14, ~~15~~

so need $P(R \leq 14) - P(R \leq 5)$

$$= 0.9393 - 0.0480 = 0.8913$$

b) **MEAN** Population mean (μ) = $np = 50 \times 0.2 = 10$

Sly claims 10.5

Mean is similar to expected

VARIANCE

Population Variance (σ^2) = $np(1-p)$

$$= 50 \times 0.2 \times 0.8 = 8$$

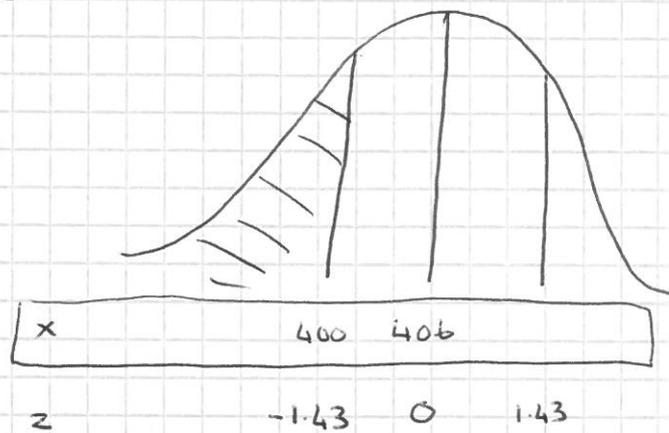
Sly claims 20.41

This is very different from expected

Therefore we have reason to doubt Sly's claims

$$\textcircled{1} \quad X \sim N(406, 4.2^2)$$

$$\begin{aligned} \text{a) i) } P(X < 400) &= P\left(Z < \frac{400 - 406}{4.2}\right) \\ &= P(Z < -1.43) \\ &= P(Z > 1.43) \\ &= 1 - P(Z < 1.43) \\ &= 1 - 0.92364 \\ &= 0.07636 \end{aligned}$$

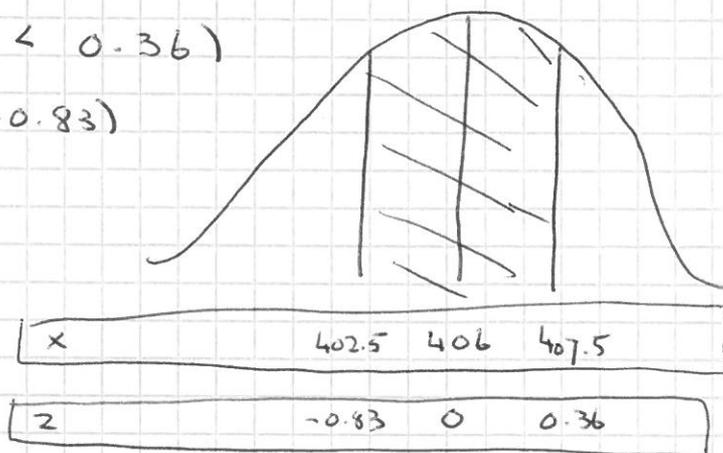


$$\begin{aligned} \text{ii) } P(402.5 < X < 407.5) &= P\left(\frac{402.5 - 406}{4.2} < Z < \frac{407.5 - 406}{4.2}\right) \\ &= P(-0.83 < Z < 0.36) \end{aligned}$$

Need: $P(Z < 0.36) - P(Z < -0.83)$

0.64058

$$\begin{aligned} &= P(Z > 0.83) \\ &= 1 - P(Z < 0.83) \\ &= 1 - 0.79673 \\ &= 0.20327 \end{aligned}$$



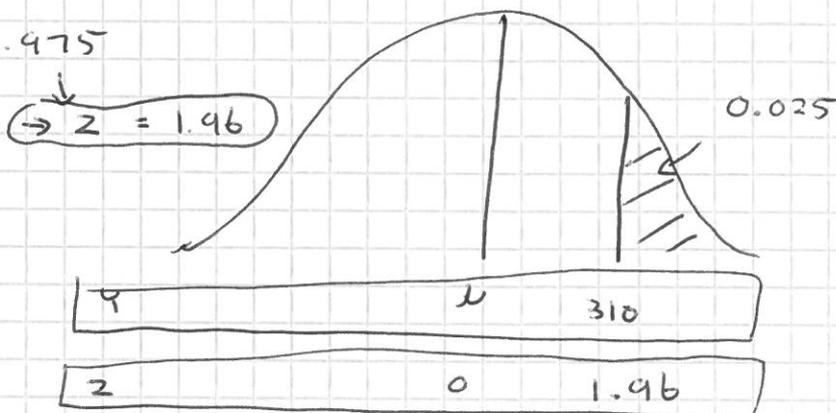
$$\rightarrow 0.64058 - 0.20327 = 0.43731$$

$$\text{b) i) } P(Y < 310) = 0.975$$

Standardize:

$$\frac{310 - \mu}{\sigma} = 1.96$$

$$310 - \mu = 1.96\sigma$$



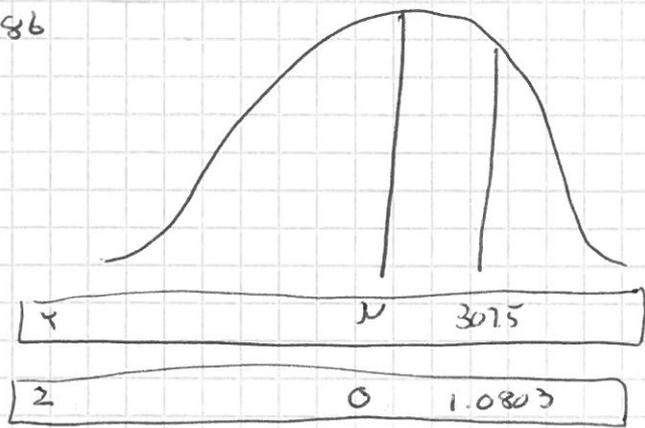
$$\text{ii) } P(Y < 307.5) = 0.86$$

$$0.86 \rightarrow Z = 1.0803$$

Standardise:

$$\frac{307.5 - \mu}{\sigma} = 1.0803$$

$$307.5 - \mu = 1.0803\sigma$$



Simultaneous Equations

$$310 - \mu = 1.96\sigma$$

$$307.5 - \mu = 1.0803\sigma$$

$$2.5 = 0.8797\sigma$$

$$\rightarrow \sigma = 2.5 \div 0.8797 = 2.84 \text{ (2dp)}$$

$$310 - \mu = 1.96(2.8418\dots)$$

$$310 - \mu = 5.570\dots$$

$$\mu = 310 - 5.570\dots$$

$$= 304.43 \text{ (2dp)}$$