M2 June 2011 Written Solutions

① (a) $\frac{1}{2}mu^2 = \frac{1}{2} \times 58 \times 2^2 = 116$ J

(b) altogether $\Delta h = 6 + 1 = 7m$

$\Delta GPE = mg\Delta h = 58 \times 9.8 \times 7 = 3978.8$ J

$E_k = 116 + 3978.8 = 4094.8$

$\frac{1}{2}mv^2 = 4094.8$

$V = \sqrt{\dfrac{2 \times 4094.8}{58}} = 11.8828$

$V = 11.9 \, ms^{-1}$

② A $(9,6)$ 2kg   C $(3,8)$ 8kg

B $(2,4)$ 3kg   D $(6,11)$ 7kg

Total mass $= 2 + 8 + 3 + 7 = 20$ kg

$20\bar{x} = 9 \times 2 + 2 \times 3 + 3 \times 8 + 6 \times 7$

$20\bar{x} = 90$

$\bar{x} = \dfrac{9}{2}$ $(= 4.5)$

$20\bar{y} = 6 \times 2 + 4 \times 3 + 8 \times 8 + 11 \times 7 = 165$

$20\bar{y} = 165 \Rightarrow \bar{y} = \dfrac{33}{4}$ $(= 8.25)$

Com is at $(4.5, 8.25)$

③ (a) $a = \dfrac{dv}{dt} = (-2 \times 4e^{-2t})\underline{i} + (6 - 6t)\underline{j}$

$\qquad a = -8e^{-2t}\underline{i} + 6(1-t)\underline{j}$

(b) (i) $F = ma = -40e^{-2t}\underline{i} + 30(1-t)\underline{j}$

(ii) $t = 0 \qquad F = -40e^{0}\underline{i} + 30(1-0)\underline{j}$

$\qquad\qquad = -40\underline{i} + 30\underline{j}$

$\qquad$ magnitude $= \sqrt{(40)^2 + (30)^2} = 50\ N$

(c) F acts due west $\Rightarrow F_{\underline{j}} = 0$

$\qquad$ so set $\quad 30(1-t) = 0$

$\qquad\qquad \Rightarrow t = 1s$

(d) $r = \int \underline{v}\ dt$

$\qquad = \int 4e^{-2t}\ dt\ \underline{i} + \int 6t - 3t^2\ dt\ \underline{j}$

$\qquad = \left(\dfrac{-4}{2}e^{-2t}\right)\underline{i} + (3t^2 - t^3)\underline{j} + \underline{c}$

$\qquad r = -2e^{-2t}\underline{i} + (3t^2 - t^3)\underline{j} + \underline{c}$

$t = 0 \qquad r = 6\underline{i} + 5\underline{j}$

$6 = -2e^{-2 \times 0} + c_i \qquad\qquad\qquad 5 = (3(0) - 0) + c_j$

$6 = -2 + c_i \qquad\qquad\qquad\qquad\qquad \Rightarrow 5 = c_j$

$\Rightarrow c_i = 8$

$r = (-2e^{-2t} + 8)\underline{i} + (3t^2 - t^3 + 5)\underline{j}$

④ (a)
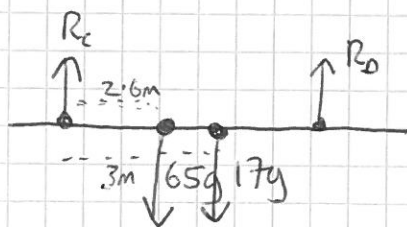
(b) $D_0 = 44g$

Take moments about C.

$$65g \times 2.6 + 17g \times 3 = 44g \times S$$

$$220 = 44S \qquad \Rightarrow \underline{\underline{S = 5}}$$

~~D~~ D is 5m from C.

so $5 + 1$ metres $= 6m$ from A

Riverbank is $1 + 0.6 + 4 = 5.6m$ from A

$\Rightarrow$ D is $0.4m$ from nearest riverbank

(c) I have assumed that centre of mass of the plank is directly in the middle. of the plank.

⑤ (a) $90 \, km h^{-1}$

$(\times 1000) = 90000 \, mh^{-1}$

$(\div 3600) = 25 \, ms^{-1}$

(b) At max speed $\Rightarrow$ max power output and $\underline{\underline{a = 0}}$.

$F_0 = $ total resistive forces.

$= 400 \times 5 + 3000 = 5000 \, N$

$P = F_0 v = 5000 \times 25 = 125000 \, W$

$= 125 \, kW$

⑥ (a) $F = -2m v^{\frac{5}{4}}$

$a = \dfrac{F}{m} = -2v^{\frac{5}{4}}$

$\Rightarrow \dfrac{dv}{dt} = -2v^{\frac{5}{4}}$

(b) $\dfrac{dv}{dt} = -2v^{\frac{5}{4}}$

$\int \dfrac{1}{v^{\frac{5}{4}}} \, dv = -\int 2 \, dt \qquad \Rightarrow \qquad \int v^{-\frac{5}{4}} \, dv = -\int 2 \, dt$

So $\quad \dfrac{v^{-\frac{1}{4}}}{-\frac{1}{4}} = -2t + c$

$-4v^{-\frac{1}{4}} = -2t + c \qquad (\times -1)$

$4v^{-\frac{1}{4}} = 2t + d$

$t = 0 \quad v = 16$

$4(16)^{-\frac{1}{4}} = 0 + d$

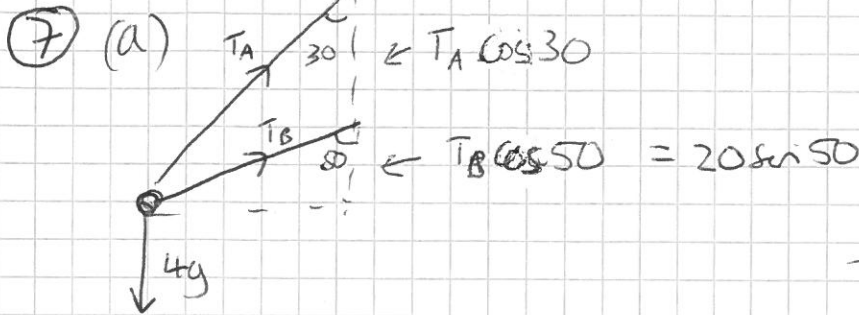$\dfrac{4}{2} = d \qquad \Rightarrow d = 2$

$4v^{-\frac{1}{4}} = 2t + 2$

$v^{-\frac{1}{4}} = \frac{1}{2}t + \frac{1}{2}$

$\dfrac{1}{v^{\frac{1}{4}}} = \frac{1}{2}t + \frac{1}{2} \qquad \Rightarrow \quad v^{\frac{1}{4}} = \dfrac{1}{\frac{1}{2}t + \frac{1}{2}} \qquad \dfrac{\times 2}{\times 2}$

$v^{\frac{1}{4}} = \dfrac{2}{t + 1}$

$\Rightarrow \quad v = \left(\dfrac{2}{t+1}\right)^4$

⑦ (a)

resolve vertically

$\leftarrow T_A \cos 30$

$\leftarrow T_B \cos 50 = 20 \sin 50$

$4g = T_A \cos 30 + 2 \cos 50$

$$\frac{4g - 20 \cos 50}{\cos 30} = T_A$$

$$\Rightarrow T_A = 30.4197$$
$$= 30.4 N$$

(b) Net force inwards $= T_A \sin 30 + T_B \sin 50$

$= 30.4197 \times \sin 30 + 20 \sin 50$
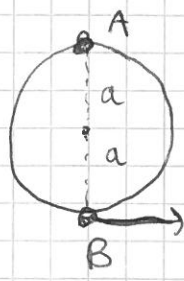
$= 30.5307 N$

$F = \frac{mv^2}{r}$

$F = \frac{4 \times 5^2}{r}$ $\Rightarrow$ $r = \frac{4 \times 25}{F} = \frac{100}{30.5307}$

$r = 3.275387$

$r = 3.28 m$

⑧ (a) it needs to have a high enough initial E_h so that it can gain enough GPE to reach the top of the circle.

$\Delta h = 2a$

$mg\Delta h = 0.3 \times g \times 2a = 0.6ag$
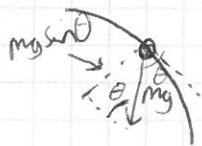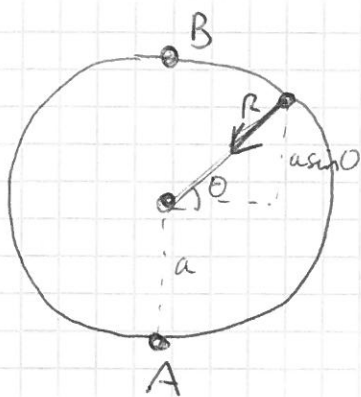
so $0.6ag$ is the required vertical E_h

Let u be min initial speed $\therefore$ $0.6ag = \frac{1}{2}mu^2$

$1.2ag = 0.3u^2$

$\Rightarrow \sqrt{\frac{1.2ag}{0.3}} = u^2$ $\Rightarrow u^2 = \sqrt{4ag} = \sqrt{4}\sqrt{ag} = 2\sqrt{ag}$

so $u \geq 2\sqrt{ag}$ acc

b(c)

Net force (inwards radially) $= R + mg\sin\theta$

① $mg\sin\theta + R = \dfrac{mv^2}{r}$

E_k at $B = \frac{1}{2}mu^2 - \Delta GPE$

$\Delta h = a + a\sin\theta$ (between A and B)
     $= a(1 + \sin\theta)$

so $\Delta GPE = mg\Delta h = mga(1 + \sin\theta)$

so $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \sin\theta)$

$v^2 = u^2 - 2ag(1 + \sin\theta)$

$v^2 = \frac{9}{2}ag - 2ag - 2ag\sin\theta$

$\quad = \frac{5}{2}ag - 2ag\sin\theta$

$\left(v = \sqrt{\frac{5}{2}ag - 2ag\sin\theta} \right)$

① $mg\sin\theta + R = \dfrac{m}{a}\left(\frac{5}{2}ag - 2ag\sin\theta\right)$

$mg\sin\theta + R = \frac{5}{2}mg - 2mg\sin\theta$

$R = \frac{5}{2}mg - 3mg\sin\theta$

$m = 0.3$

$\quad = \frac{3}{4}g - \frac{9}{10}g\sin\theta$

$R = g\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)$

(ii) $R = 0$

$$0 = g\left(\frac{3}{4} - \frac{9}{10}\sin\theta\right)$$

$$\frac{9}{10}\sin\theta = \frac{3}{4}$$

$$\sin\theta = \frac{0.75}{0.9} \qquad \Rightarrow \qquad \theta = \sin^{-1}\left(\frac{0.75}{0.9}\right)$$

$$= 56.4427$$

$$\theta = 56.4^\circ$$

(above the horizontal.)

(9) $L = 6m \qquad \lambda = 1800N$

(a) $EPE = \dfrac{\lambda x^2}{2L} \qquad x = 10 - 6 = 4m$

$$= \frac{1800 \times 4^2}{2 \times 6} = 2400J$$

(b) Initial $E_k = \frac{1}{2}mu^2 = \frac{1}{2} \times 200 \times 8^2 = \underline{6400J}$

$$E_k \to EPE$$

$$6400 = \frac{\lambda x^2}{2L} = \frac{1800x^2}{2 \times 16}$$

$$6400 = \frac{1800x^2}{12}$$

$$\frac{128}{3} = x^2$$

$$\Rightarrow x = 6.532m$$

distance from $O = 6 + x = 12.5m \quad (3SF)$

(c) Now $\quad E_k \rightarrow GPE + $ work done against resistive forces.

$$6400 = \frac{1800 x^2}{2 \times 6} + Fs$$

where $\quad F = 800N \quad$ and $\quad s = $ distance moved $= L + x$
$$= 6 + x$$

so $\qquad 6400 = 150 x^2 + 800(6+x)$

$\qquad\qquad 6400 = 150x^2 + 4800 + 800x$

$\Rightarrow \qquad 0 = 150x^2 + 800x - 1600$

$(\div 50) \qquad 0 = 3x^2 + 16x - 32$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(3)(-32)}}{2 \times 3} = \frac{-16 \pm \sqrt{640}}{6}$$

$x = 1.5497 \qquad$ or $\qquad x = -6.88$

$\downarrow$

distance from $0 = 6 + x$

$$= \underline{\underline{7.55\,m}}$$
$$\underline{(3\,SF)}$$