M2 June 09 written solutions

① (a) $\underline{a} = \frac{dv}{dt} = (3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}$

(b)(i) $F = ma = 4[(3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}]$

$= (12t^2 - 60)\underline{i} + (24 - 8t)\underline{j}$ as required.

(ii) $t = 2$ $\quad \underline{F} = (12(2)^2 - 60)\underline{i} + (24 - 8(2))\underline{j}$

$= (48 - 60)\underline{i} + 8\underline{j}$

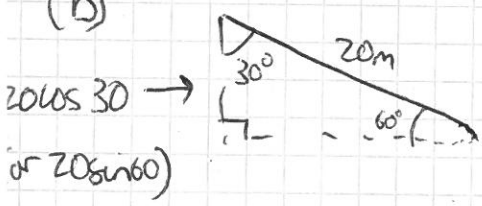
$= -12\underline{i} + 8\underline{j}$

$|F| = \sqrt{12^2 + 8^2} = 4\sqrt{13}$

$= 14.4N$

② (a) $\frac{1}{2}mu^2 = \frac{1}{2} \times 55 \times 3^2 = 247.5\,J$

(b)



$h = 20\cos 30 = 10\sqrt{3}$

$\Delta GPE = mg\Delta h = 55 \times 9.8 \times 10\sqrt{3}$

$= 5390\sqrt{3}$

$E_{k\,final} = E_{k\,initial} + \Delta GPE$

$= 247.5 + 5390\sqrt{3}$

$\frac{1}{2}mv^2 = 247.5 + 5390\sqrt{3}$

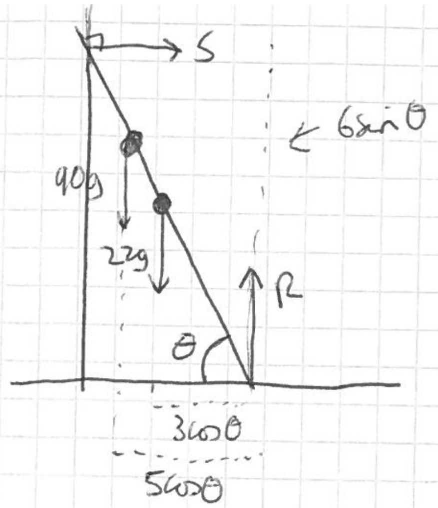
$v = \sqrt{\frac{2 \times (247.5 + 5390\sqrt{3})}{55}}$

$= 18.6677$

$v = 18.7\,ms^{-1}$

(c) No air resistance

(3)



(a) Resolve vertically

$R = 90g + 22g = 112g$

$F = \mu R = 0.6 \times 112g = 67.2g$

$= 658.56$

$= 659$ (3SF)

(b) Take moments about A

Moments must "balance".

$S \times 6\sin\theta = 22g \times 3\cos\theta + 90g \times 5\cos\theta$

$S = F = 67.2g$

$\Rightarrow 6 \times 67.2g \sin\theta = 516g\cos\theta$

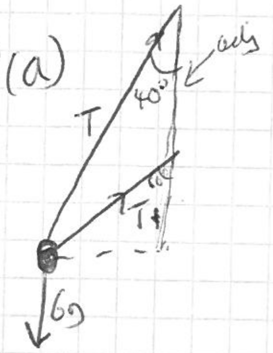
$\frac{\sin\theta}{\cos\theta} = \frac{516g}{403.2g} = \frac{215}{168}$

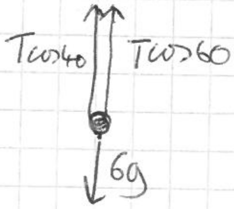
$\Rightarrow \tan\theta = \frac{215}{168}$

$\Rightarrow \theta = 51.996$

$\underline{\theta = 52.0^\circ}$ (3SF)

④ (a)





$T(\cos 40 + \cos 60) = 6g$

$T = \dfrac{6g}{\cos 40 + \cos 60} = 46.4439$

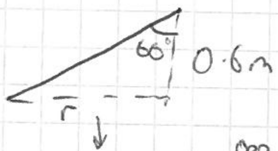
$T = 46.4$ (3SF) as required

(b) $F = ma$

$= \dfrac{mv^2}{r}$

$F = T\sin 40 + T\sin 60$

$T(\sin 40 + \sin 60) = \dfrac{mv^2}{r}$



$\tan\theta = \dfrac{opp}{adj}$

$\tan 60 = \dfrac{opp}{0.6}$

$r = 0.6\tan 60$

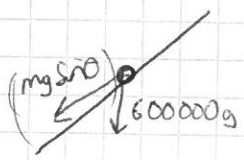
save T into calc. memory to avoid rounding errors.

$\Rightarrow T(\sin 40 + \sin 60) = \dfrac{6v^2}{0.6\tan 60}$

$\Rightarrow v = 3.48387$

$v = 3.48\,ms^{-1}$

⑤ Constant speed $\Rightarrow a = 0 \Rightarrow$ Net force $= 0$.



Total force opposing motion

$= mg\sin\theta + 200\,000$

$= 600000 \times 9.8 \times \dfrac{1}{40} + 200\,000$

$= 347000\,N$

F_D (driving force) $\Rightarrow$ must be 347000 $(F=0)$

$F_D = \dfrac{P}{v} \Rightarrow P = F_D v = 347000 \times 24 = 8328000\,W$

$= 8328\,kW$

(6) $l = 1.2m \quad \lambda = 180N$

(a) $x = 2 - 1.2 = 0.8$ $\quad EPE = \frac{\lambda x^2}{2l} = \frac{180 \times 0.8^2}{2 \times 1.2} = 48J$

(b) No wd against friction
String will be slack so no EPE

All of the 48 J is converted to Ek

$\frac{1}{2}mv^2 = 48$

$\frac{1}{2} \times 5v^2 = 48 \Rightarrow v = \sqrt{\frac{2 \times 48}{5}} = \frac{4\sqrt{30}}{5} = 4.38178$

$= 4.38 ms^{-1}$

(3SF).

(c) Just as it reaches the wall so $E_k = 0$

All of the 48 J is "Lost" as wd against friction.

$wd = F \times s \Rightarrow F = \frac{wd}{s}$

$\frac{48}{2} = F = 24 N$

$F = \mu R = 24 N \qquad R = mg = 5 \times 9.8 = 49 N$

$24 = \mu R = 49\mu$

$\mu = 0.4897954$

$\mu = 0.490$ (3SF).

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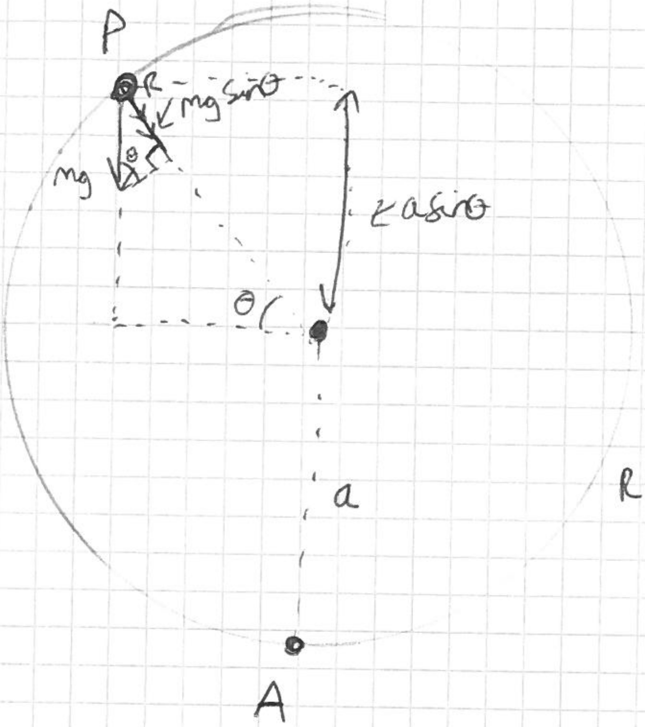

Net force (radially)

$= R + mg\sin\theta$

$$R + mg\sin\theta = \frac{mv^2}{r} \quad (1)$$

Consider Ek at P.

Gained GPE = $mg\Delta h$

gained height (from A) is $a + a\sin\theta$

$= a(1+\sin\theta)$

$\Delta GPE = mga(1+\sin\theta)$

so $\frac{1}{2}\cancel{m}v^2 = \frac{1}{2}\cancel{m}u^2 - \cancel{m}ga(1+\sin\theta)$

$\frac{1}{2}mv^2$ ↓ Ek at P

$\frac{1}{2}mu^2$ ↓ Ek at A

$\Rightarrow \quad v^2 = u^2 - 2ga(1+\sin\theta)$

$\frac{mv^2}{r}$ ↓

(1) $R + mg\sin\theta = \frac{m}{a}\left(u^2 - 2ga(1+\sin\theta)\right)$

$R + mg\sin\theta = \frac{mu^2}{a} - 2mg(1+\sin\theta)$

$R + mg\sin\theta = \frac{mu^2}{a} - 2mg - 2mg\sin\theta$

$R = \frac{mu^2}{a} - 2mg - 3mg\sin\theta$ as required.

(b) $\underline{u^2 = 3ag}$

when it leaves the surface

$$\underline{\underline{R = 0}}$$

$$0 = \frac{\cancel{m}u^2}{a} - 3\cancel{m}g\sin\theta - 2\cancel{m}g$$

$$0 = \frac{3\cancel{a}\cancel{g}}{\cancel{a}} - 3\cancel{g}\sin\theta - 2\cancel{g}$$

$$0 = 3 - 3\sin\theta - 2 \quad \Rightarrow \quad 0 = 1 - 3\sin\theta$$

$$\sin\theta = \frac{1}{3}$$

$$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19 \cdot 47^\circ$$

$$\underline{\underline{19 \cdot 5^\circ}} \quad (3SF)$$

⑧(a) $F_R = -\lambda m v^{\frac{3}{2}}$ use $F=ma \Rightarrow a = \frac{F}{m}$

$a = -\lambda v^{\frac{3}{2}}$

$a = \frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$

(b) $\frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$ Separate variables

$\int \frac{1}{v^{\frac{3}{2}}}\, dv = -\int \lambda \, dt \Rightarrow \int v^{-\frac{3}{2}}\, dv = -\int \lambda \, dt$

$\Rightarrow \frac{v^{-\frac{1}{2}}}{\frac{-1}{2}} = -\lambda t + c$

$-2v^{-\frac{1}{2}} = -\lambda t + c$ $(\times -1)$

$\frac{2}{\sqrt{v}} = \lambda t + d$

$\underline{t=0}$ $\underline{v=9}$ $\Rightarrow \frac{2}{\sqrt{9}} = 0 + d$

so $\underline{d = \frac{2}{3}}$

$\frac{2}{\sqrt{v}} = \lambda t + \frac{2}{3}$

$\frac{\sqrt{v}}{2} = \frac{1}{\lambda t + \frac{2}{3}} \Rightarrow \sqrt{v} = \frac{2}{\lambda t + \frac{2}{3}}$ $\frac{\times 3}{\times 3}$

$\sqrt{v} = \frac{6}{3\lambda t + 2}$

$\Rightarrow v = \frac{36}{(2+3\lambda t)^2}$

(c) use $\sqrt{v} = \frac{6}{3\lambda t + 2}$

$\underline{v=4}$ $\sqrt{4} = \frac{6}{3\lambda t+2} \Rightarrow 2 = \frac{6}{3\lambda t + 2} \Rightarrow 3 = 3\lambda t + 2$

$\Rightarrow 1 = 3\lambda t$ so $t = \frac{1}{3\lambda}$