M2 Jan 2009 Written Solutions

(1) $\int v \, dt = \int 4t^3 - 8\sin 2t + 5 \, dt$

$s = t^4 + \frac{8}{2}\cos 2t + 5t + c = t^4 + 4\cos 2t + 5t + c$

$s=0$ when $t=0$ $\Rightarrow$ $0 = 0 + 4\cos(0) + 0 + c$

$0 = 4 + c \Rightarrow \underline{\underline{c = -4}}$

$s = t^4 + 4\cos 2t + 5t - 4$

(2) (a) $E_k = \frac{1}{2}mu^2 = \frac{1}{2} \times 6 \times 12^2 = 432\,J$

(b)(i) As it rises $E_k \to GPE$. As it then falls $GPE \to E_k$ so since we ignore work done against air resistance, when it reaches its <u>initial position</u> it will have 432 J of E_k again

Final $E_k = \underbrace{\text{initial } E_k}_{432} + \underbrace{\text{loss in GPE}}_{mg \times 4}$

$\frac{1}{2}mv^2 = 432 + 6 \times 9.8 \times 4$

$= 667.2$

$\Rightarrow v = \sqrt{\frac{2 \times 667.2}{6}} = 14.913$

$\Rightarrow \underline{\underline{v = 14.9\,ms^{-1}}}$

3)(a) $\underline{v} = \frac{dr}{dt} = \left(\frac{1}{2} \times 2e^{\frac{1}{2}t} - 8\right)\underline{i} + (2t - 6)\underline{j}$

$= \left(e^{\frac{1}{2}t} - 8\right)\underline{i} + (2t - 6)\underline{j}$

)(i) $\underline{v} = \left(e^{\frac{1}{2} \times 3} - 8\right)\underline{i} + (2 \times 3 - 6)\underline{j}$

$= \left(e^{\frac{3}{2}} - 8\right)\underline{i} + 0\underline{j}$

$\Rightarrow$ velocity $= e^{\frac{3}{2}} - 8 = -3.518 \Rightarrow$ Speed $= 3.52\,ms^{-1}$

(ii) remember → $\underline{j}$ component was 0.

it is in the negative $\underline{i}$ direction ⇒ **West**.

(c) $t = 3$

$$a = \frac{d\underline{v}}{dt} = \left(\tfrac{1}{2}e^{\frac{1}{2}t}\right)\underline{i} + 2\underline{j}$$

$$\Rightarrow a = \left(\tfrac{1}{2}e^{\frac{3}{2}}\right)\underline{i} + 2\underline{j}$$

(d) $m = 7kg$

$$F = ma$$
$$= 7\left(\tfrac{1}{2}e^{\frac{3}{2}}\underline{i} + 2\underline{j}\right)$$
$$= \tfrac{7}{2}e^{\frac{3}{2}}\underline{i} + 14\underline{j}$$

Magnitude $= \sqrt{\left(\tfrac{7}{2}e^{\frac{3}{2}}\right)^2 + 14^2} = 21.0N$

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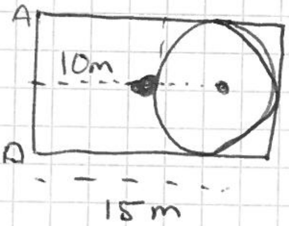
(a)



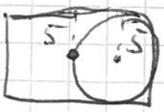
x-coordinate only

Total mass = $8 + 2 = 10kg$

$$10\bar{x} = \underbrace{15 \times 2}_{\text{from circle com}} + \underbrace{10 \times 8}_{\text{from rectangle com}}$$

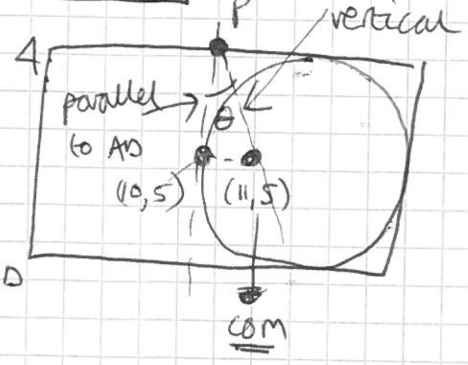
$\Rightarrow 10\bar{x} = 110 \quad \Rightarrow \bar{x} = 11cm$

(b)



Must be 5cm

(c)



$\Rightarrow \tan\theta = \frac{1}{5}$ and $\theta = \tan^{-1}\left(\frac{1}{5}\right)$
$= 11.3°$

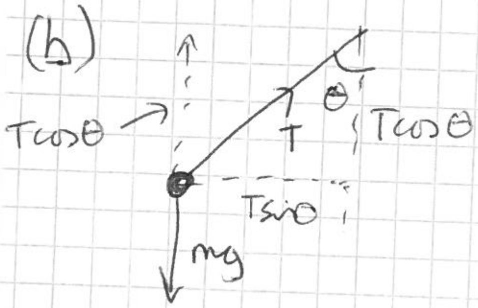
(d) com is in the centre of the individual shapes (circle & rectangle).

5 (a) 40 rev min^{-1}

$40 \times 2\pi = 80\pi \Rightarrow \omega = 80\pi \text{ rad min}^{-1}$

so $\div 60$ $\omega = \frac{4}{3}\pi \text{ rad s}^{-1}$

(b)



resolve vertically

$T\cos 30 = mg$

$T = \dfrac{6g}{\cos 30} = 67.896$

$\Rightarrow T = 67.9\text{ N}$ (3SF)

(c) Net force $= T\sin\theta$

Use $F = ma \Rightarrow T\sin\theta = mr\omega^2$

$67.896 \times \sin 30 = 6\left(\frac{4\pi}{3}\right)^2 r$

$r = \dfrac{67.896 \times \frac{1}{2}}{6 \times \frac{16\pi^2}{9}} = \underline{\underline{0.322\text{ m}}}$

6 $m = 60$ tonnes $= 60000$ kg $P = 800000$ W

$v = 40\text{ ms}^{-1}$

(a)

Driving force F_D

$F_D = \dfrac{P}{v} = \dfrac{800000}{40}$

$= 20000$ N

Max speed $\Rightarrow a = 0$

$\Rightarrow$ Net force $= 0$ and so $r = F_D$

resistive force $= 20000$ N

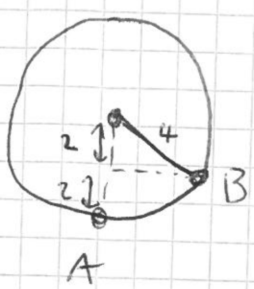
(b) $40\text{ ms}^{-1} \to 36\text{ ms}^{-1}$

$\Delta E_k = \frac{1}{2}m(40^2 - 36^2) = 30000 \times 304 = 9120000\text{ J}$

$\Rightarrow$ This must be "lost" as work done against resistive force

$wd = F \times s \Rightarrow s = \dfrac{wd}{F} = \dfrac{9120000}{20000} = \underline{456\text{ m}}$

⑦



(a) E_k at $A = \frac{1}{2} \times 6 \times 8^2 = 192\text{J}$

Gain in GPE from A to B $= mg\Delta h$

$= 6g \times 2$

$= 117 \cdot 6\text{J}$

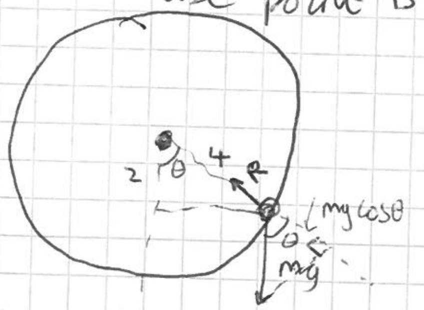
so E_k at $B = 192 - 117\cdot6 = \underline{\underline{74\cdot4\text{J}}}$

$\frac{1}{2}mv^2 = 74\cdot4 \quad \Rightarrow \quad v = \sqrt{\frac{74\cdot4 \times 2}{6}} = \sqrt{24\cdot8}$

$= 4\cdot97996$

$\underline{v = 4\cdot98\text{ms}^{-1}}$

(b) use point B



$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Consider component of mg acting radially $= mg\cos\theta$

Net force (radially) $= R - mg\cos\theta$

$= R - 6g\cos 60$

$= R - 3g$

use $F = ma = \frac{mv^2}{r}$

$R - 3g = \frac{6 \times 24\cdot8}{4}$ ← v^2 at $B = 24\cdot8$

$\Rightarrow \quad R - 3g = 37\cdot2$

so $\underline{\underline{R = 66\cdot6\text{ N}}}$

(8) $m = 0.05\,kg$ $\quad F = -0.08v^2$

(a) $a = \frac{F}{m} = \frac{-0.08v^2}{0.05} = -1.6v^2$

so $a = \frac{dv}{dt} = -1.6v^2$

(b) $\frac{dv}{dt} = -1.6v^2$

Separate variables $\Rightarrow \int \frac{1}{v^2}\,dv = -\int 1.6\,dt$

$$\int v^{-2}\,dv = -\int 1.6\,dt$$

$$\Rightarrow \quad -v^{-1} = -1.6t + c$$

$$\frac{-1}{v} = -1.6t + c$$

$(\times -1)$

$$\frac{1}{v} = 1.6t + d.$$

when $t=0 \quad v=3$

$$\Rightarrow \quad \frac{1}{3} = 0 + d \quad \Rightarrow \quad d = \frac{1}{3}$$

so $\frac{1}{v} = 1.6t + \frac{1}{3}$

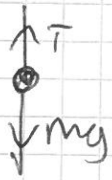
$$v = \frac{1}{1.6t + \frac{1}{3}} \times \frac{15}{15}$$

$$= \frac{15}{24t+5}$$

So $v = \frac{15}{5+24t}$ as required.

(9) $l = 16$ $\quad \lambda = 784N$

(a)



$\underline{\underline{a = 0}} \Rightarrow F = 0$

so $T = mg$

$$T = \frac{\lambda x}{l} = \frac{784x}{16}$$

$T = mg$ so $\dfrac{784x}{16} = 80 \times 9.8$

$$x = \frac{80 \times 9.8 \times 16}{784} = 16m$$

Extension $= 16$ m $\quad$ so length $= l + e$

$= 16 + 16 = \underline{\underline{32m}}$

(b) It means that it has <u>extended</u> x when it comes to rest.

$EPE = \dfrac{\lambda x^2}{2l}$ $\quad$ and $\quad \Delta GPE = -mg(16+x)$

$= -80g(16+x)$

(loss in GPE)

GPE lost is converted into EPE.

so $\quad 80g(16+x) = \dfrac{\lambda x^2}{2l}$

$$12544 + 784x = \frac{784x^2}{2 \times 16}$$

$$12544 + 784x = \frac{49}{2}x^2$$

$\Rightarrow \quad 0 = \dfrac{49}{2}x^2 - 784x - 12544 \qquad \div \dfrac{49}{2}$

$0 = x^2 - 32x - 512$ $\quad$ as required

(ii) $x^2 - 32x - 512 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{32 \pm \sqrt{32^2-4(1)(-512)}}{2\times 1}$$

$$= \frac{16 \pm \sqrt{3072}}{2} = 16 \pm 16\sqrt{3}$$

$16 + 16\sqrt{3}$ is when it first comes to rest

$(= 43\cdot 71)$

So distance below start $= (16+16\sqrt{3}) + l$

$= 16+16\sqrt{3} + 16$

$= 32 + 16\sqrt{3}$

$= 59\cdot 7128$

$65 - 59\cdot 7128 = 5\cdot 287$

$= 5\cdot 29\,m$

$\downarrow$

we need distance above ground