Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	Solutions									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MM1B

Unit Mechanics 1B

Monday 16 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A car is travelling along a straight horizontal road. It is moving at $14\,\text{m s}^{-1}$ when it starts to accelerate. It accelerates at $0.8\,\text{m s}^{-2}$ for 12 seconds.

(a) Find the speed of the car at the end of the 12 seconds.

[3 marks]

(b) Find the distance travelled during the 12 seconds.

[3 marks]

(c) The mass of the car is $1400\,\text{kg}$. A horizontal forward driving force of $1600\,\text{N}$ acts on the car during the 12 seconds. Find the magnitude of the resistance force that acts on the car.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

$s = ?$ $u = 14$ $v = ?$ $a = 0.8$ $t = 12$

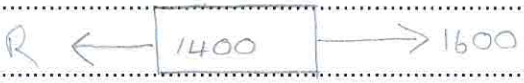
(a) $v = u + at$

$\Rightarrow v = 14 + (0.8 \times 12) = 23.6\ \text{m s}^{-1}$

(b) $s = ut + \frac{1}{2}at^2$

$\Rightarrow s = (14 \times 12) + \frac{1}{2} \times 0.8 \times 12^2 = 225.6\ \text{m}$

(c)



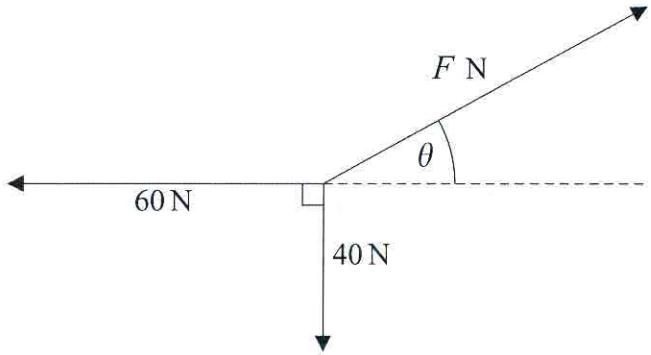
$\Rightarrow F = ma$

$\Rightarrow 1600 - R = 1400 \times 0.8$

$\Rightarrow 1600 - 1120 = R$

$\Rightarrow R = 480\ \text{N}$

2 Three forces are in equilibrium in a vertical plane, as shown in the diagram. There is a vertical force of magnitude $40\,\text{N}$ and a horizontal force of magnitude $60\,\text{N}$. The third force has magnitude F newtons and acts at an angle θ above the horizontal.



(a) Find F.

[2 marks]

(b) Find θ.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

Forces in Equilibrium

$\Rightarrow F\cos\theta = 60$

$F\sin\theta = 40$

$\Rightarrow \frac{F\sin\theta}{F\cos\theta} = \frac{40}{60}$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{2}{3} \Rightarrow \tan\theta = \frac{2}{3}$

$\Rightarrow \theta = \tan^{-1}(2/3)$

$= 33.69 = 33.7$

$\Rightarrow F = \frac{40}{\sin\theta} = \frac{40}{\sin(33.69\ldots)} = 72.11$

$= 72.1\,\text{N}$

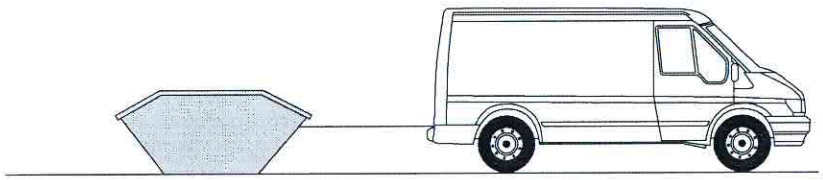
$F = \frac{60}{\cos\theta}$

$\Rightarrow \frac{60}{\cos\theta} \times \sin\theta = 40$

$= \frac{\sin\theta}{\cos\theta} = \frac{40}{60}$

$\Rightarrow \tan\theta = \frac{2}{3}$

3 A skip, of mass $800\,\text{kg}$, is at rest on a rough horizontal surface. The coefficient of friction between the skip and the ground is 0.4. A rope is attached to the skip and then the rope is pulled by a van so that the rope is horizontal while it is taut, as shown in the diagram.



The mass of the van is $1700\,\text{kg}$. A constant horizontal forward driving force of magnitude P newtons acts on the van. The skip and the van accelerate at $0.05\,\text{m s}^{-2}$.

Model both the van and the skip as particles connected by a light inextensible rope. Assume that there is no air resistance acting on the skip or on the van.

(a) Find the speed of the van and the skip when they have moved 6 metres.

[3 marks]

(b) Draw a diagram to show the forces acting on the skip while it is accelerating.

[1 mark]

(c) Draw a diagram to show the forces acting on the van while it is accelerating. State one advantage of modelling the van as a particle when considering the vertical forces.

[2 marks]

(d) Find the magnitude of the friction force acting on the skip.

[3 marks]

(e) Find the tension in the rope.

[3 marks]

(f) Find P.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 3

(a) $s=6 \quad u=0 \quad v=? \quad a=0.05 \quad t=-$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 0.05 \times 6$

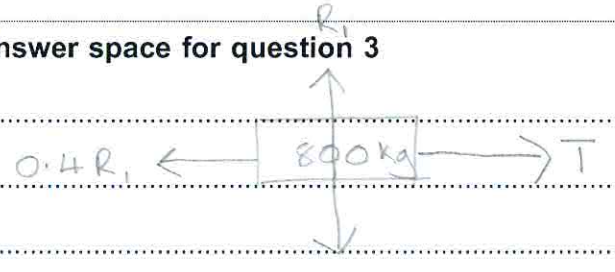
$v = \sqrt{0.6} = 0.774597$

$= 0.775\,\text{ms}^{-1}$

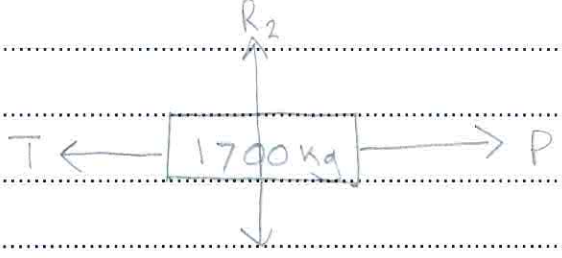
QUESTION PART REFERENCE

Answer space for question 3

(b)



(c)



The normal reaction acts at a point, so there is no need to consider where the normal reaction acts.

(d) Consider the skip only Res ↑ $R_1 = 800g$

$\Rightarrow F_r = \mu R = 0.4 \times 800g = 3136 = \underline{3140N}$

(e) look at the skip $\Rightarrow$ $T - 3136 = 800 \times 0.05$

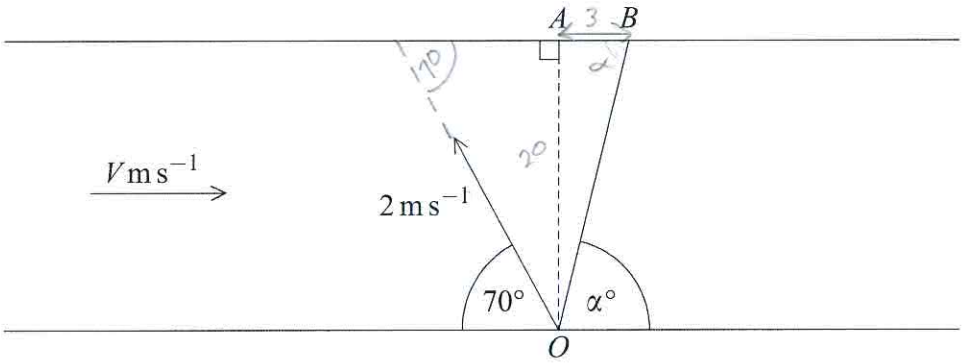
$\Rightarrow T = 3176 = \underline{3180N}$

looking at the car

(f) $P - 3176 = 1700 \times 0.05$

$\Rightarrow P = 3261 = \underline{3260N}$

4 A boat is crossing a river, which has two parallel banks. The width of the river is 20 metres. The water in the river is flowing at a speed of $V\,\mathrm{m\,s^{-1}}$. The boat sets off from the point O on one bank. The point A is directly opposite O on the other bank. The velocity of the boat relative to the water is $2\,\mathrm{m\,s^{-1}}$ at an angle of $70°$ to the bank. The boat lands at the point B which is 3 metres from A. The angle between the actual path of the boat and the bank is $\alpha°$. The river and the velocities are shown in the diagram.



(a) Find the time that it takes for the boat to cross the river.

[3 marks]

(b) Find α.

[2 marks]

(c) Find V.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 4

$\tan\alpha = \frac{20}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{20}{3}\right)$

$\Rightarrow \alpha = 81.5$

(a) Vert component of speed =

$\Rightarrow 2\sin 70$

$\Rightarrow$ time $= \frac{\text{Dist}}{\text{speed}} = \frac{20}{2\sin 70} = 10.64$

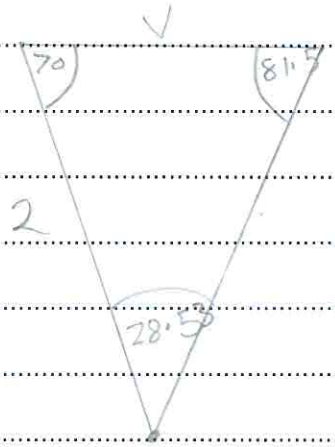
$= 10.6\,\mathrm{ms^{-1}}$

(b) $\tan\alpha = \frac{20}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{20}{3}\right) = 81.464$

$= 81.47$

QUESTION PART REFERENCE

Answer space for question 4



Using Sine Rule

$\frac{V}{\text{Sine } 28.53} : \frac{2}{\text{Sin } 81.47}$

$\Rightarrow V = \frac{2 \times \text{Sin } 28.53}{\text{Sin } 81.47} = 0.9659$

$= 0.966 \text{ ms}^{-1}$

5 Two particles, A and B, have masses of m and km respectively, where k is a constant. The particles are moving on a smooth horizontal plane when they collide and coalesce to form a single particle. Just before the collision the velocities of A and B are $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ and $(6\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ respectively. Immediately after the collision the combined particle has velocity $(5.2\mathbf{i} - 0.4\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find k.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 5

$4i + 2j$ $\quad$ $6i - 2j$

(m) $\quad$ (km)

$5.2i - 0.4j$

(m+km)

Conservation of momentum

$m(4i+2j) + km(6i-2j) = (m+km)(5.2i-0.4j)$

Equate i components

$4m + 6km = (m+km)5.2$

$\Rightarrow 4m + 6km = 5.2m + 5.2km$

$6k - 5.2k = 5.2 - 4$

$0.8k = 1.2 \quad \Rightarrow k = 1.5$

check by equating j

$\Rightarrow 2m - 2km = -0.4m - 0.4km$

$2 + 0.4 = (2 - 0.4)k$

$\Rightarrow k = \frac{2.4}{1.6} = 1.5$

6 A bullet is fired from a rifle at a target, which is at a distance of 420 metres from the rifle. The bullet leaves the rifle travelling at $V\,\mathrm{m\,s^{-1}}$ and at an angle of $2°$ above the horizontal. The centre of the target, C, is at the same horizontal level as the rifle. The bullet hits the target at the point A, which is on a vertical line through C. The bullet takes 1.8 seconds to reach the point A.

(a) Find V, showing clearly how you obtain your answer.

[3 marks]

(b) Find the distance between A and C.

[4 marks]

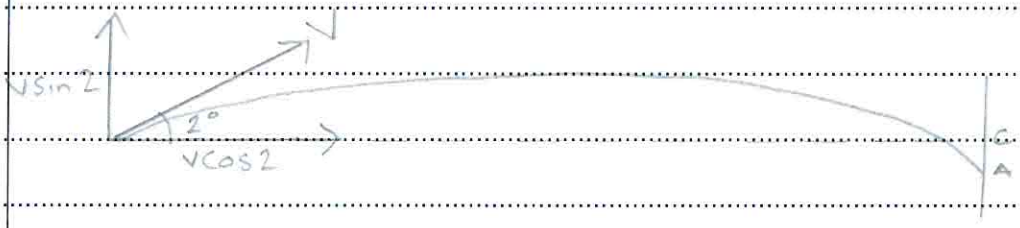
(c) State one assumption that you have made about the forces acting on the bullet.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 6

(a)



Horizontal Distance = Horizontal velocity x time

$V\cos 2 \times 1.8 = 420$

$V = \frac{420}{1.8 \times \cos 2} = 233.47 = 233\,\mathrm{m}$

(b) $s = ?$

$u = 233 \times \sin 2$ $\Rightarrow s = ut + \frac{1}{2}at^2$

$v = -$ $= 233.47 \times \sin 2 \times 1.8 - \frac{1}{2} \times 9.8 \times 1.8^2$

$a = -9.8$

$t = 1.8$ $\Rightarrow s = -1.209$

$\Rightarrow$ A is 1.21m below C

(c) No air resistance.

7 Two particles, A and B, move on a horizontal surface with constant accelerations of $-0.4\mathbf{i}\,\mathrm{m\,s^{-2}}$ and $0.2\mathbf{j}\,\mathrm{m\,s^{-2}}$ respectively. At time $t = 0$, particle A starts at the origin with velocity $(4\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. At time $t = 0$, particle B starts at the point with position vector $11.2\mathbf{i}$ metres, with velocity $(0.4\mathbf{i} + 0.6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find the position vector of A, 10 seconds after it leaves the origin.

[2 marks]

(b) Show that the two particles collide, and find the position vector of the point where they collide.

[9 marks]

QUESTION PART REFERENCE

Answer space for question 7

(a) $r = ut + \frac{1}{2}at^2$

$= (4i + 2j) \times 10 + \frac{1}{2}(-0.4i) \times 10^2$

$= 40i + 20j - 50 \times 0.4i \Rightarrow 40i + 20j - 20i$

$= 20i + 20j$

(b) $r_A = (4i + 2j)t + \frac{1}{2}(-0.4i)t^2$

$= 4i + 2j - 0.2it^2$

$r_B = 11.2i + (0.4i + 0.6j)t + \frac{1}{2} \times 0.2jt^2$

$= 11.2i + (0.4i + 0.6j)t + 0.1jt^2$

$r_A = r_B$ for the particles to collide

$4it + 2jt - 0.2it^2 = 11.2i + 0.4it + 0.6jt + 0.1jt^2$

equate i components

$4t - 0.2t^2 = 11.2 + 0.4t$

$\Rightarrow 0 = 0.2t^2 - 3.6t + 11.2$

$\Rightarrow 0 = 2t^2 - 36t + 112$ 56×1

$= t^2 - 18t + 56$ 28×2

$0 = (t - 14)(t - 4)$ 14×4

$\Rightarrow t = 14, 4$

QUESTION PART REFERENCE

Answer space for question 7

Equating j components

$2t = 0.6t + 0.1t^2$

$0 = 0.1t^2 - 1.4t$

$0 = t^2 - 14t$

$0 = t(t-14) \Rightarrow t = 0, 14$

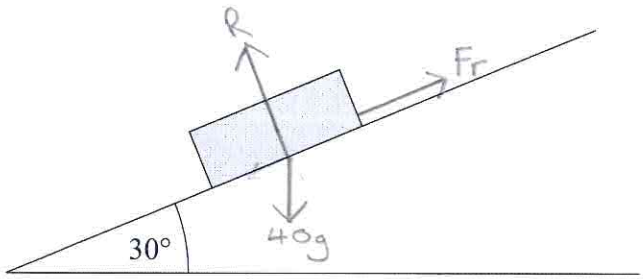
$\Rightarrow t = 14$

$r = (4i + 2j) \times 14 - 0.2 \times 14^2 i$

$= 56i + 28j - 39.2i$

$= 16.8i + 28j$

8 A crate, of mass $40\,\text{kg}$, is initially at rest on a rough slope inclined at $30°$ to the horizontal, as shown in the diagram.

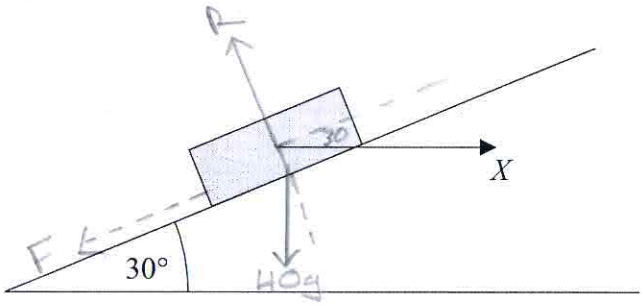


The coefficient of friction between the crate and the slope is μ.

(a) Given that the crate is on the point of slipping down the slope, find μ.

[5 marks]

(b) A horizontal force of magnitude X newtons is now applied to the crate, as shown in the diagram.



(i) Find the normal reaction on the crate in terms of X.

[2 marks]

(ii) Given that the crate accelerates up the slope at $0.2\,\text{m s}^{-2}$, find X.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 8

(a) $F_r = 40 \times 9.8 \times \cos 60$ Res ↗

$= 196$

Res ↑ $R = 40 \times 9.8 \times \cos 30 = 339.48$

$\Rightarrow F = \mu R \Rightarrow 196 = \mu = 339.48$

$\Rightarrow \mu = \frac{196}{339.48} = 0.577$

QUESTION PART REFERENCE

Answer space for question 8

(b)(i) Res ↑ $R - 40g\cos 30 - X\cos 60 = 0$

Res → $F = ma$

$X\cos 30 - 40g\cos 60 - F = 40 \times 0.2$

$R = X\cos 60 + 40g\cos 30$

$\Rightarrow F = 0.577 \times \left(\frac{X}{2} + 392 \times \frac{\sqrt{3}}{2}\right) = \ldots$

$F = X \times 0.2885 + 196$

$\Rightarrow X \times \frac{\sqrt{3}}{2} - 0.2885X - 196 = 8 + 196$

$X\left(\frac{\sqrt{3}}{2} - 0.2885\right) = \frac{400}{\left(\frac{\sqrt{3}}{2} - 0.2885\right)} = 692.6$

$= 693$