Centre Number						Candidate Number				
Surname										
Other Names	Solutions									
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
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6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MM1B

Unit Mechanics 1B

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A car travels on a straight horizontal race track. The car decelerates uniformly from a speed of $20\,m\,s^{-1}$ to a speed of $12\,m\,s^{-1}$ as it travels a distance of 640 metres. The car then accelerates uniformly, travelling a further 1820 metres in 70 seconds.

(a) (i) Find the time that it takes the car to travel the first 640 metres. *(3 marks)*

(ii) Find the deceleration of the car during the first 640 metres. *(3 marks)*

(b) (i) Find the acceleration of the car as it travels the further 1820 metres. *(3 marks)*

(ii) Find the speed of the car when it has completed the further 1820 metres. *(3 marks)*

(c) Find the average speed of the car as it travels the 2460 metres. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

a(i) $s = 640$

$u = 20$

$v = 12$

$a = -$

$t = ?$

$s = \left(\frac{u+v}{2}\right)t$

$\Rightarrow 640 = \frac{(20+12)}{2}t$

$\Rightarrow t = \frac{1280}{32} = 40s$

(ii) $v = u + at$

$12 = 20 + a \times 40$

$-8 = a \Rightarrow a = -\frac{1}{5}$ or $-0.2\,s \Rightarrow$ deceleration $= 0.2\,ms^{-2}$

40

QUESTION PART REFERENCE

Answer space for question 1

b(i) $S = 1820 \quad u = 12 \quad v = ? \quad a = ? \quad t = 70$

$S = ut + \frac{1}{2}at^2 \Rightarrow 1820 = 12 \times 70 + \frac{1}{2} \times a \times 70^2$

$\Rightarrow 980 = \frac{1}{2} \times a \times 70^2$

$\Rightarrow a = \frac{1960}{70^2} = 0.4\ ms^{-2}$

(ii) $v = u + at$

$v = 12 + 0.4 \times 70 = 40\ ms^{-1}$

(c) Average speed = $\frac{\text{total distance}}{\text{total Time}} = \frac{2460}{40 + 70}$

$= 22.364$

$= 22.4\ ms^{-1}$

2 Three forces act on a particle. These forces are $(9\mathbf{i} - 3\mathbf{j})$ newtons, $(5\mathbf{i} + 8\mathbf{j})$ newtons and $(-7\mathbf{i} + 3\mathbf{j})$ newtons. The vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) Find the resultant of these forces. *(2 marks)*

(b) Find the magnitude of the resultant force. *(2 marks)*

(c) Given that the particle has mass 5 kg, find the magnitude of the acceleration of the particle. *(2 marks)*

(d) Find the angle between the resultant force and the unit vector $\mathbf{i}$. *(3 marks)*

QUESTION PART REFERENCE

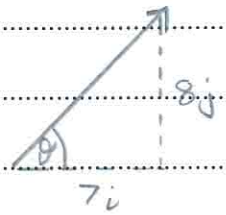
Answer space for question 2

(a) $-7i + 3j$, $5i + 8j$, $9i - 3j$

$\Rightarrow$ Resultant (i direction) $= -7 + 5 + 9 = 7i$

(j direction) $= -3 + 3 + 8 = 8j$

$\Rightarrow$ Resultant $= 7i + 8j$



(b) Magnitude $= \sqrt{7^2 + 8^2} = \sqrt{113} = 10.6\text{N}$

(c) $F = Ma \Rightarrow a = \frac{F}{M} = \frac{\sqrt{113}}{5} = 2.126 = 2.13\,\text{ms}^{-2}$

(d) $\tan\theta = \frac{8}{7} \Rightarrow \theta = \tan^{-1}\left(\frac{8}{7}\right) = 48.8^\circ$

3 A box, of mass 3 kg, is placed on a rough slope inclined at an angle of 40° to the horizontal. It is released from rest and slides down the slope.

(a) Draw a diagram to show the forces acting on the box. *(1 mark)*

(b) Find the magnitude of the normal reaction force acting on the box. *(2 marks)*

(c) The coefficient of friction between the box and the slope is 0.2. Find the magnitude of the friction force acting on the box. *(2 marks)*

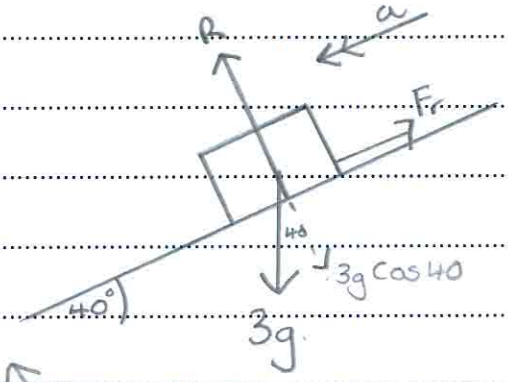
(d) Find the acceleration of the box. *(3 marks)*

(e) State an assumption that you have made about the forces acting on the box. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 3

(a)



(b) Res ⊥ $\Rightarrow R = 3g\cos 40 = 22.5\ N$

(c) $F_r = \mu R = 0.2 \times R = 4.50\ N$

(d) Res ← $\Rightarrow 3g\sin 40 - F_r = 3 \times a$

$\Rightarrow a = \dfrac{18.898 - 4.5043}{3} = 4.7979$

$= 4.80\ ms^{-2}$

(e) No Air Resistance / No other Forces acting on box

No turning effect due to Forces / or something similar.

4 A tractor, of mass 3500 kg, is used to tow a trailer, of mass 2400 kg, across a horizontal field. The trailer is connected to the tractor by a horizontal tow bar. As they move, a constant resistance force of 800 newtons acts on the trailer and a constant resistance force of R newtons acts on the tractor. A forward driving force of 2500 newtons acts on the tractor. The trailer and tractor accelerate at $0.2\,\text{m}\,\text{s}^{-2}$.

(a) Find R. *(3 marks)*

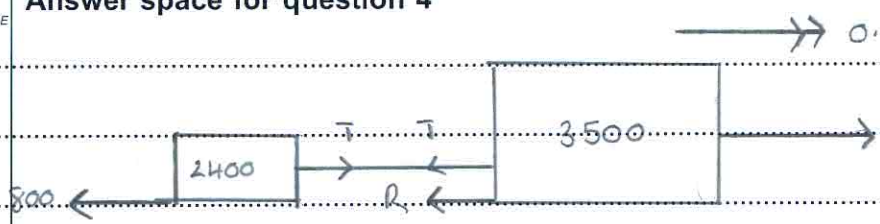
(b) Find the magnitude of the force that the tow bar exerts on the trailer. *(3 marks)*

(c) State the magnitude of the force that the tow bar exerts on the tractor. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

(a)



Taking the system as a whole:

$F = Ma \Rightarrow 2500 - R - 800 = 5900 \times 0.2$

$\Rightarrow 1700 - 1180 = R \Rightarrow R = 520\ \text{N}$

(b) Trailor

$\Rightarrow T - 800 = 2400 \times 0.2$

$\Rightarrow T = 480 + 800$

$= 1280\ \text{N}$

(c) 1280 N

5 Two particles, A and B, are moving towards each other along the same straight horizontal line when they collide. Particle A has mass 5 kg and particle B has mass 4 kg. Just before the collision, the speed of A is $4\,\text{m s}^{-1}$ and the speed of B is $3\,\text{m s}^{-1}$. After the collision, the speed of A is $0.6\,\text{m s}^{-1}$ and both particles move on the same straight horizontal line.

Find the two possible speeds of B after the collision. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 5

A B

(5) (4)

4 → ← 3 Before

0.6 → v → after (1)

← 0.6 v → after (2)

+ve

First → $(5\times4)+(4\times-3)=(0.6\times5)+(4\times v)$

$20-12-3=4v$

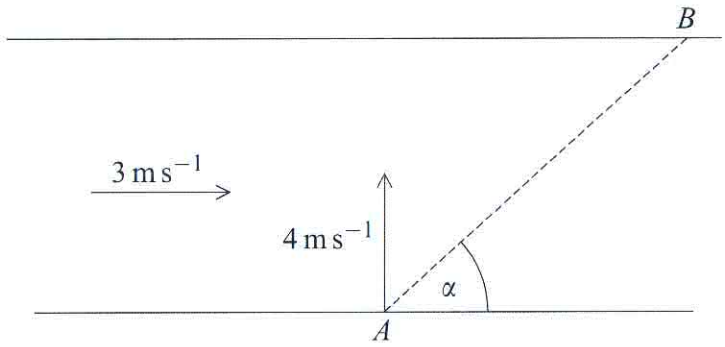
$\Rightarrow v=\frac{5}{4}=1.25\,\text{m s}^{-1}$

Second → $(5\times4)+(4\times-3)=(-0.6\times5)+(4\times v)$

$20-12+3=4v$

$\Rightarrow v=\frac{11}{4}=2.75\,\text{m s}^{-1}$

6 A river has straight parallel banks. The water in the river is flowing at a constant velocity of $3\,\text{m s}^{-1}$ parallel to the banks. A boat crosses the river, from the point A to the point B, so that its path is at an angle α to the bank. The velocity of the boat relative to the water is $4\,\text{m s}^{-1}$ perpendicular to the bank. The diagram shows these velocities and the path of the boat.

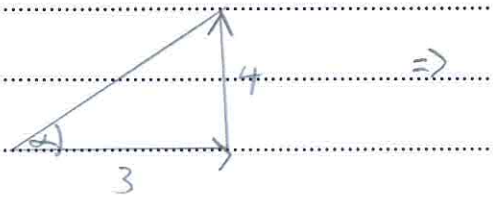


(a) Show that $\alpha = 53.1°$, correct to three significant figures. *(2 marks)*

(b) The boat returns along the same straight path from B to A. Given that the speed of the boat relative to the water is still $4\,\text{m s}^{-1}$, find the magnitude of the resultant velocity of the boat on the return journey. *(6 marks)*

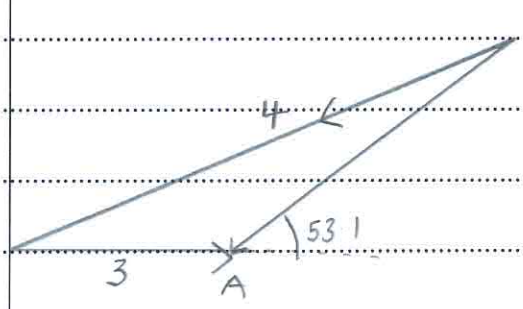
QUESTION PART REFERENCE

Answer space for question 6



$\Rightarrow \tan\alpha = \frac{4}{3} \Rightarrow \alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow \alpha = 53.1^{\circ}$



Using cosine rule

$4^2 = 3^2 + v^2 - 2\times3\times v\times\cos(126.9)$

$0 = v^2 + 3.6v + 9 - 16$

$\Rightarrow v^2 + 3.6v - 7 = 0$

$a = 1 \quad b = +3.6 \quad c = -7$

$v = \frac{-3.6 \pm \sqrt{(-3.6)^2 - 4\times1\times(-7)}}{2} = 1.40 \text{ or } -5.00$

7 A particle is initially at the point A, which has position vector $13.6\mathbf{i}$ metres, with respect to an origin O. At the point A, the particle has velocity $(6\mathbf{i} + 2.4\mathbf{j})\,\mathrm{m\,s^{-1}}$, and in its subsequent motion, it has a constant acceleration of $(-0.8\mathbf{i} + 0.1\mathbf{j})\,\mathrm{m\,s^{-2}}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find an expression for the velocity of the particle t seconds after it leaves A. *(2 marks)*

(b) Find an expression for the position vector of the particle, with respect to the origin O, t seconds after it leaves A. *(3 marks)*

(c) Find the distance of the particle from the origin O when it is travelling in a north-westerly direction. *(7 marks)*

QUESTION PART REFERENCE

Answer space for question 7

(a) Using $v = u + at$

$\Rightarrow v = (6i + 2.4j) + (-0.8i + 0.1j)t$

(b) $r = (6i + 2.4j)t + \frac{1}{2}(-0.8i + 0.1j)t^2 + 13.6i$

(c) If it is travelling in north westerly direction for the velocity vector $j = -i$

$\Rightarrow v = (6i + 2.4j) + (-0.8i + 0.1j)t$

$\Rightarrow v = (6 - 0.8t)i + (2.4 + 0.1t)j$

$\Rightarrow -(6 - 0.8t) = 2.4 + 0.1t$

$-6 + 0.8t = 2.4 + 0.1t$

$0.7t = 8.4 \Rightarrow t = \frac{8.4}{0.7} = 12\,s$

QUESTION PART REFERENCE

Answer space for question 7

(c) Substitute $t = 12$ into the expression for the position vector

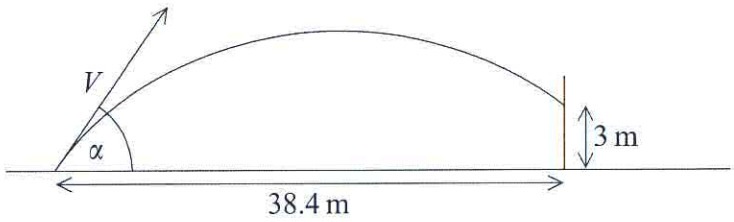
$$\mathbf{r} = (6\mathbf{i} + 2{\cdot}4\mathbf{j}) \times 12 + \tfrac{1}{2} \times 12^2 (-0{\cdot}8\mathbf{i} + 0{\cdot}1\mathbf{j}) + 13{\cdot}6\mathbf{i}$$

$$\mathbf{r} = 72\mathbf{i} + 28{\cdot}8\mathbf{j} - 57{\cdot}6\mathbf{i} + 7{\cdot}2\mathbf{j} + 13{\cdot}6\mathbf{i}$$

$$\mathbf{r} = 28\mathbf{i} + 36\mathbf{j}$$

$$\Rightarrow \text{distance} = \sqrt{28^2 + 36^2} = 4 \times \sqrt{130} = 45{\cdot}6\text{ m}$$

8 A golf ball is hit from a point on a horizontal surface, so that it has an initial velocity $V\,\text{m s}^{-1}$ at an angle α above the horizontal. The ball travels through the air and after 2.4 seconds hits a vertical wall at a height of 3 metres. The wall is at a horizontal distance of 38.4 metres from the point where the ball was hit. The path of the ball is shown in the diagram.



Assume that the weight of the ball is the only force that acts on it as it travels through the air.

(a) Find the horizontal component of the velocity of the ball. *(2 marks)*

(b) Find V. *(5 marks)*

(c) Find α. *(3 marks)*

QUESTION PART REFERENCE

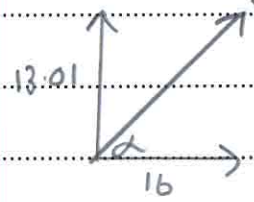
Answer space for question 8

(a) $V_H = \frac{\text{Distance}}{\text{time}} = \frac{38.4}{2.4} = 16\,\text{ms}^{-1}$

(b) Using suvat $s = 3$ $u = V_V$ $v = ?$ $a = -9.8$ $t = 2.4$

$\Rightarrow s = ut + \frac{1}{2}at^2$ $\quad 3 = V_V \times 2.4 - \frac{1}{2} \times 9.8 \times 2.4^2$

$\Rightarrow V_V = \frac{3 + 28.224}{2.4} = 13.01\,\text{ms}^{-1}$



$\Rightarrow V = \sqrt{16^2 + 13.01^2} = 20.621 = 20.6$

(c) $\tan\alpha = \frac{13.01}{16} \Rightarrow \alpha = \tan^{-1}\left(\frac{13.01}{16}\right) = 39.1^\circ$