Jan 2012 M1 Written Solutions

(1) Before

$u = (3\underline{i} + 8\underline{j})$ A (7kg)

$u = (6\underline{i} - 5\underline{j})$ B (3kg)

After

$v = ?$ C (10kg)

mv before = mv after.

$$7(3\underline{i} + 8\underline{j}) + 3(6\underline{i} - 5\underline{j}) = 10(x\underline{i} + y\underline{j})$$

$$(21 + 18)\underline{i} + (56 - 15)\underline{j} = 10x\underline{i} + 10y\underline{j}$$

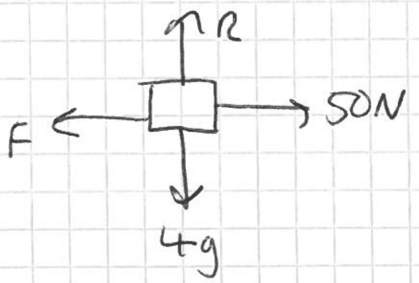
$$39\underline{i} + 41\underline{j} = 10x\underline{i} + 10y\underline{j}$$

so $39 = 10x \Rightarrow x = 3{\cdot}9$

$41 = 10y \Rightarrow y = 4{\cdot}1$

$$v = 3{\cdot}9\underline{i} + 4{\cdot}1\underline{j}$$

(2) (a)



(b) vertical forces balanced

so $R = 4g = 39{\cdot}2N$

(c) Use NII ($F = ma$).

$50 - F = 4 \times 3$

$50 - 12 = F$

So $F = 38N$

(d) Using $F = \mu R$

$$\mu = \frac{F}{R} = \frac{38}{39{\cdot}2} = 0{\cdot}969$$

(e) friction would be less (a is the same so net force is the same) so μ would be smaller

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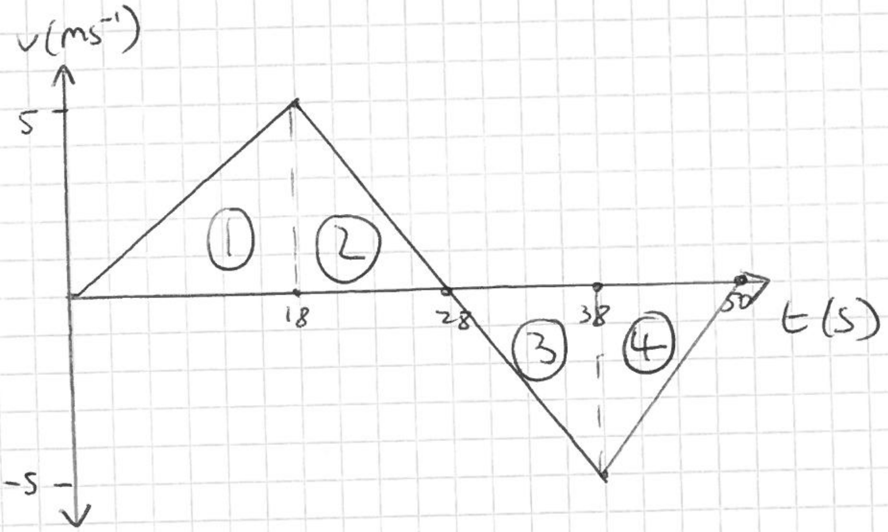


(a) Area under so ① + ②

$① = \frac{18\times5}{2} = 45m \qquad ② = \frac{10\times5}{2} = 25m$

$45+25 = \underline{\underline{70m}}$

(b) ① + ② + ③ + ④ (Notice → distance not displacement)

$70 + \frac{10\times5}{2} + \frac{12\times5}{2}$

$70 + 25 + 30 = \underline{\underline{125m}}$

(c) average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{125}{50} = \underline{\underline{2.5ms^{-1}}}$

(d) displacement

① and ② are in same direction so

70m in this direction

③ + ④ = 55 is in opposite direction

so $70-55 = \underline{\underline{15\ m}}$

(e) average velocity = $\frac{\text{total displacement}}{\text{total time}} = \frac{15}{50} = 0.3ms^{-1}$

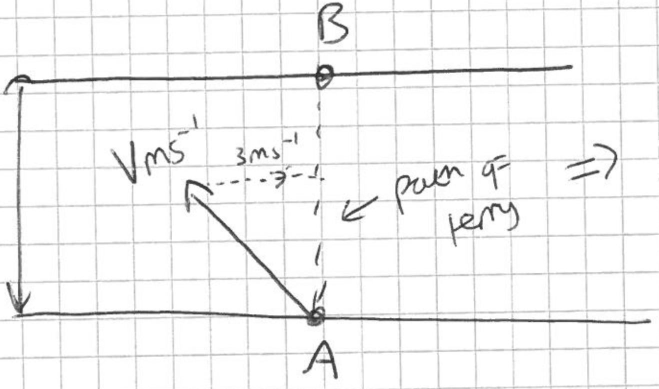
(f) acceleration is gradient i.e. $a = \frac{\Delta v}{t} = \frac{5}{18}$

$= 0.2\dot{7}$

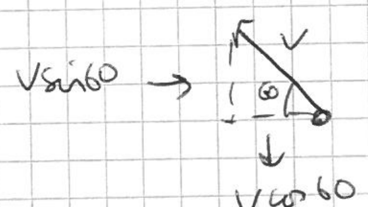
$= 0.28\,ms^{-2}$

(2sf)

(4)



$\Rightarrow$ ferry has no horizontal displacement so resultant velocity horizontally must be $0\,ms^{-1}$



so $V\cos 60 = 3$

$V = \frac{3}{\cos 60} = 6\,ms^{-1}$

(b) Only need vertical velocity & displacement

so velocity $= V\sin 60 = 6\sin 60$

displacement $= 200$

so $t = \frac{200}{6\sin 60} = 38.49$

so $\underline{\underline{38\,s}}$

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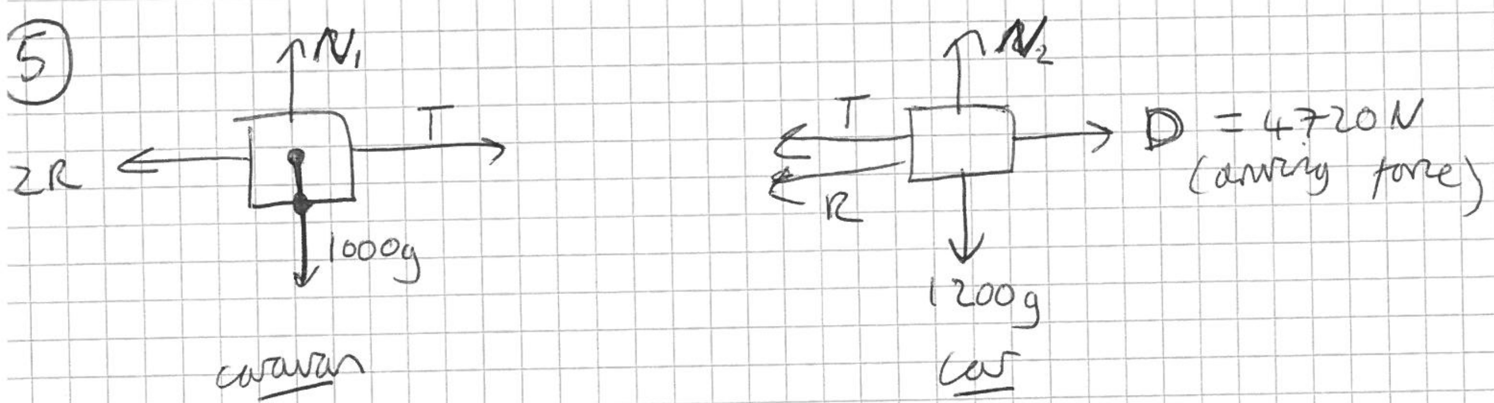


$$a = 1.6\ ms^{-2}$$

(a) Use NII $F=ma$ on each:

car: $D - T - R = 1200 \times 1.6$

$4720 - T - R = 1920$ ①

caravan. $T - 2R = 1000 \times 1.6$

$T - 2R = 1600$ ②

① + ② to cancel T gives

$4720 - 3R = 3520$

$3R = 1200$

$\Rightarrow R = \underline{\underline{400N}}$

(b) Use ② $T - 2(400) = 1600$

$T = 1600 + 800 = \underline{\underline{2400}}N$

⑥ a(i) $V^2 = u^2 + 2as \Rightarrow a = \dfrac{V^2 - u^2}{2s}$

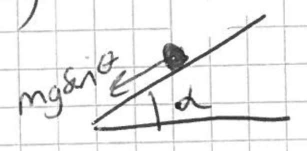
$a = \dfrac{10^2 - 4^2}{2 \times 50} = \dfrac{100 - 16}{100} = 0.84\,ms^{-2}$

(ii) $V = u + at \Rightarrow t = \dfrac{V - u}{a} = \dfrac{10 - 4}{0.84} = \dfrac{6}{0.84}$

$= 7.14\,s$

(b) $F = ma = 70 \times 0.84 = 58.8\,N$

c(i)



free-wheeling, no resistive forces

so $mg\sin\theta$ = net force

↓

only forces parallel to slope considered

$70g\sin\theta = 58.8$

$\sin\theta = 0.0\dot{8}57142\dot{}$

$\theta = 4.9171$

$= \underline{\underline{4.92^\circ}}$

(ii) Now net force is $mg\sin\theta - 30$

so $70g\sin\theta - 30 = 58.8$

$\sin\theta = \dfrac{58.8 + 30}{70g} = 0.129446$

$\theta = 7.43758$

$\theta = \underline{\underline{7.44^\circ}}$

(d) the cyclist is accelerating and resistive forces would vary (increase) with the speed so a constant resistive force is unrealistic.

7 (a) $a = 4.2\underline{i} + 2.5\underline{j}$ $\quad t = 20s$

consider $\underline{j}$ only for (a) (finding height)

vertically

$s = ?$
$u = 0$
v
$a = 2.5$
$t = 20$

$s = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 2.5 \times 20^2$
$= 500m$

(b) $V = u + at$
$= 0 + (4.2\underline{i} + 2.5\underline{j}) \times 20 \quad = (84\underline{i} + 50\underline{j})\ ms^{-1}$

(c) Find time when $\underline{j}$ component of $\underline{s}$ is 180

$S_y = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 2.5 \times t^2$ ← vertical component only so $a = 2.5\underline{j}$
$= 1.25t^2$

$180 = 1.25t^2 \Rightarrow t^2 = 144 \quad t = +12s$

(-12s makes no sense)

So $V = u + at$
$= (4.2\underline{i} + 2.5\underline{j}) \times 12 \quad = 50.4\underline{i} + 30\underline{j}$

so speed $= \sqrt{50.4^2 + 30^2} \quad = 58.6529$

$= 58.7\ ms^{-1}$ (3SF)

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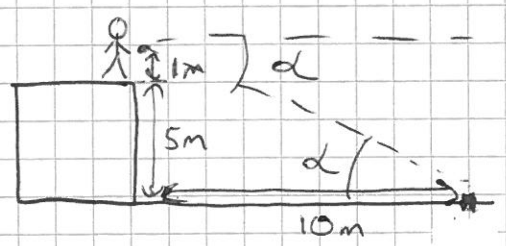
(a)



$$\tan\alpha = \left(\frac{5+1}{10}\right) \qquad \alpha = \tan^{-1}\left(\frac{6}{10}\right) = 30.9638$$

$$\alpha = 30.1 \quad (3SF)$$

(b) $S_y = -6$

$S_y = ut\sin\theta - \frac{1}{2}gt^2$

use $-\alpha$ as angle is below horizontal

$-6 = 8\sin(-30.9638)\,t - 4.9t^2$

$4.9t^2 + 4.11597t - 6 = 0$

use quadratic formula

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$t = 0.76359$

or $t = -1.60359$

so $\underline{t = 0.764\,s}$

(c) Find S_x when $t = 0.76359$

$S_x = ut\cos\theta = 8 \times 0.76359 \times \cos(-30.9638)$

$= 5.23818$

so $10 - 5.23818 = 4.76182$

$= \underline{4.76\,m}$ (3SF)