June 11 M1 written solutions

1(a)

$s = 0.9$
$u = 0$ (from rest)
$v = 0.6$
$a = ?$
t

$$v^2 = u^2 + 2as$$

$$\frac{v^2 - u^2}{2s} = a$$

$$a = \frac{0.6^2 - 0}{2 \times 0.9} = \frac{0.36}{1.8} = 0.2\,ms^{-2}$$

as required

(a)(ii) $s = ut + \frac{1}{2}at^2$

$$0.9 = 0 + \frac{1}{2} \times (0.2) \times t^2$$

$$0.9 = 0.1t^2$$

$$t = \sqrt{\frac{0.9}{0.1}} = \sqrt{9} = 3\,s$$

(b)

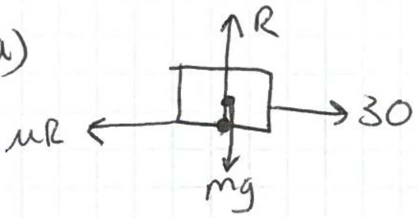
$T - mg$ ← Net force

$T - mg = ma$ ← NII ($F = ma$)

$$T - 800 \times 9.8 = 800 \times 0.2$$

$$T = 800 \times 0.2 + 800 \times 9.8 = 8000\,N$$

2(a)



(b) Vertical forces balanced

$\therefore$ $R = mg$
$= 4 \times 9.8 = 39.2\,N$

(c) $F = \mu R = 0.3 \times 39.2 = 11.76 \Rightarrow 11.8\,N$ (3SF)

(d) Net force $= 30 - \mu R = 30 - 11.76 = 18.24\,N$

NII: $\vec{F} = ma$

Net force

$18.24 = 4a \Rightarrow a = \frac{18.24}{4} = 4.56\,ms^{-2}$

(3) (a)

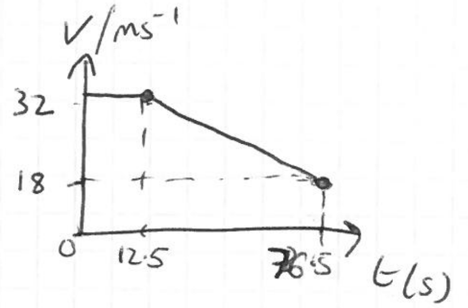
$s = ?$ ← not 2000 in part (a)
$\left.\begin{matrix} u \\ v \end{matrix}\right| = 32ms^{-1}$
$a = 0$ → travelling at $32ms^{-1}$ for the 12.5s
$t = 12.5$

$s = vt = 32 \times 12.5 = 400m$

(b) $s = 1600$ [← $2000 - 400$ (total distance − distance travelling at constant speed)]
$u = 32$
$v = 18$
$a =$
$t = ?$

$s = \frac{1}{2}(u+v)t$ $\qquad t = \frac{2s}{u+v} = \frac{2 \times 1600}{32+18} = \frac{3200}{50} = 64s$

(c)



$64 + 12.5 = 76.5s$

(d) average speed $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{2000}{76.5} = 26.144$

$= 26.1 ms^{-1}$

(4) A 6kg $\qquad$ B m kg
$u = 5\underline{i} + 18\underline{j}$ $\qquad u = 2i - 5\underline{j}$
$v = 8\underline{i}$ $\qquad v = V\underline{j}$

(a) mv before = mv after : $\quad 6(5\underline{i} + 18\underline{j}) + m(2i - 5j) = 6(8\underline{i}) + m(V\underline{j})$

$\therefore (30 + 2m)\underline{i} + (108 - 5m)\underline{j} = 48\underline{i} + mV\underline{j}$

equating $\underline{i}$ components gives $\quad 30 + 2m = 48 \rightarrow 2m = 18$
$\underline{\underline{m = 9}}$

(b) equating $\underline{j}$ components: $\quad 108 - 5m = mV$
$\Rightarrow 108 - 5(9) = 9V \quad \Rightarrow 63 = 9V \quad \Rightarrow \underline{\underline{V = 7}}$

(5)

P moves down

(a) $P: \quad 5g - T = 5a \quad (1) \quad$ (Net force $F = ma$)

$Q: \quad T - 3g = 3a \quad (2)$

$(1) + (2) \qquad 2g = 8a$

$a = \frac{1}{4}g = 2.45 \text{ ms}^{-2}$

(b) (2) $T - 3g = 3a$

$T = 3(a+g) = 3(2.45 + 9.8) = 36.75$

$= \underline{36.8 \text{ N}}$

(c) light and inextensible

(d) (i) $\downarrow$ = positive

$s = 0.196$

$u = 0$ ← "released from rest"

$v =$

$a = 2.45$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$0.196 = 0 + \frac{1}{2} \times 2.45 \times t^2$

$t^2 = \frac{0.196}{\frac{1}{2} \times 2.45} = \frac{4}{25} \Rightarrow t = \frac{2}{5} = 0.4\underline{5}$

(ii) $v^2 = u^2 + 2as$

$= 0 + 2 \times 2.45 \times 0.196 = 0.9604$

$v = \sqrt{0.9604} = 0.98 \text{ ms}^{-1}$

6 (a) Horizontally $\underline{\underline{a=0}}$

$$V = \frac{s}{t} = \frac{1000}{4} = 250\ ms^{-1}$$

(b) $\underline{\underline{\text{vertically}}}$

$$S_y = ut\sin\theta - \frac{1}{2}gt^2$$

$\downarrow$

$\sin\theta = 0$ (fired horizontally) $\theta = 0^\circ$

$$S_y = -\frac{1}{2}gt^2$$

$$= -\frac{1}{2}g \times 4^2 = -78 \cdot 4\ m$$

$-$ means: $(\downarrow)$

so $h = 78 \cdot 4\ m$

(c) $V_x = 250\ ms^{-1}$

$V_y = 0 - gt$ (this is just $v = u + at$ vertically)

$\downarrow$

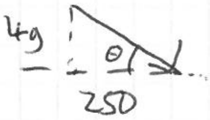
$\sin\theta = 0$

$$V_y = -4g$$

$$\text{Speed} = \sqrt{250^2 + (-4g)^2} = 253 \cdot 05$$

$\Rightarrow$ speed $= 253\ ms^{-1}$ (3SF)

(d) $V = 250\underline{i} - 4g\underline{j}$ when it hits the sea



$$\theta = \tan^{-1}\left(\frac{4g}{250}\right) = 8 \cdot 911$$

$$\theta = 8 \cdot 91^\circ \quad (3SF)$$

(7) (a) $\underline{\underline{u=0}}$ ("hovering over a lighthouse")

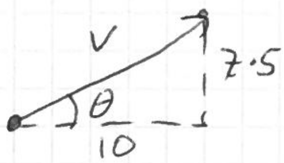
$$a = 0.5\underline{i} + 0.375\underline{j}$$

$$v = u + at = at$$

$$v = (0.5\underline{i} + 0.375\underline{j})\,20 = 10\underline{i} + 7.5\underline{j}$$

$\underline{\underline{\text{Speed}}}$ $\quad v = \sqrt{10^2 + 7.5^2} = 12.5\ \text{ms}^{-1}$

(b) $\quad v = 10\underline{i} + 7.5\underline{j}$



$$\theta = \tan^{-1}\left(\frac{7.5}{10}\right) = 36.87°$$

$$\phi = 90 - \theta \qquad = 90 - 36.87 = 53.13$$

$\Rightarrow$ Bearing is $\underline{\underline{053°}}$

bearings are 3 $\underline{\underline{\text{figures}}}$ → need the zero!

(c) Need expression for displacement first (in terms of t).

$$s = ut + \tfrac{1}{2}at^2$$
$$= 0 + \tfrac{1}{2}t^2(0.5\underline{i} + 0.375\underline{j})$$
$$= 0.25t^2\underline{i} + 0.1875t^2\underline{j}$$

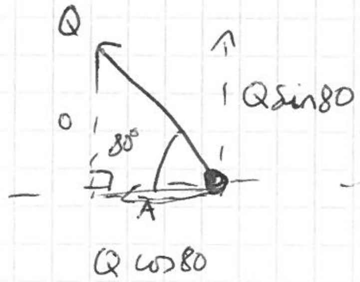
To find $\underline{\underline{\text{distance}}}$ we need Pythagoras.

$$500 = \sqrt{(0.25t^2)^2 + (0.1875t^2)^2} = \sqrt{\frac{25}{256}t^4}$$
$$= \frac{5}{16}t^2 \qquad = 0.3125t^2$$

$500 = 0.3125t^2 \qquad t^2 = 1600$

$$t = \underline{\underline{40\,s}}$$

(8)



(a) Vertically forces are balanced so

$$250g = P\sin 80 + Q\sin 80 \quad (1)$$

Horizontally: $P\cos 80 - Q\cos 80 = 250a$ (2) ($N\,II$ $F = ma$) net force

(1) $250g = \sin 80 (P+Q)$

$\Rightarrow P + Q = \dfrac{250g}{\sin 80}$ (1)'

(2) $250a = \cos 80 (P - Q)$

$\Rightarrow P - Q = \dfrac{250a}{\cos 80}$ (2)'

(1)' + (2)' (so that Q will cancel)

$2P = \dfrac{250g}{\sin 80} + \dfrac{250a}{\cos 80} \Rightarrow P = \dfrac{125g}{\sin 80} + \dfrac{125a}{\cos 80}$

$\Rightarrow P = 125\left(\dfrac{a}{\cos 80} + \dfrac{g}{\sin 80}\right)$

(b) $Q = 0$

use (1) and (2) (1)'' $250g = P\sin 80$ and (2)'' $P\cos 80 = 250a$

$\dfrac{(1)''}{(2)''} \Rightarrow \dfrac{P\sin 80}{P\cos 80} = \dfrac{250g}{250a} \Rightarrow \tan 80 = \dfrac{g}{a}$

$a = \dfrac{9.8}{\tan 80} = 1.728$

$a = 1.73\,ms^{-2}$