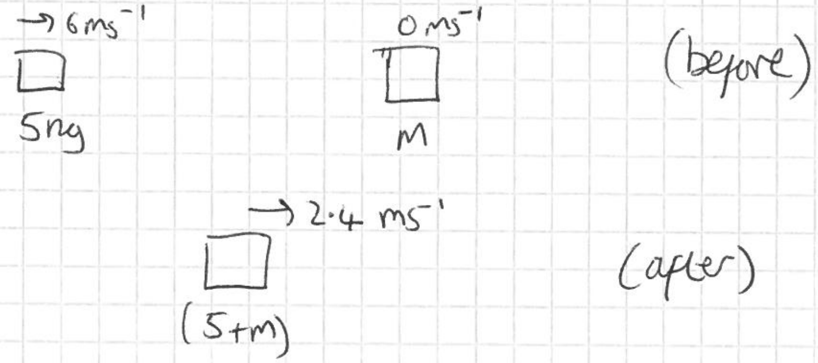
Jan 11 M1 Written Solutions

(1)



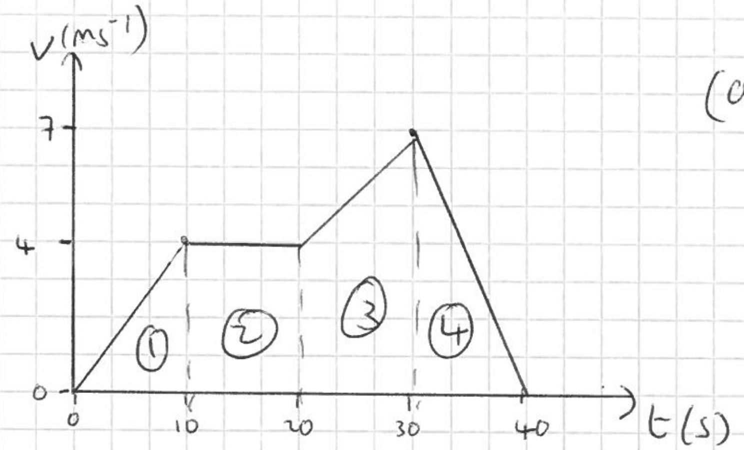
mv before = mv after.

$$5 \times 6 + 0 = (5+m)\,2.4$$

$$\frac{30}{2.4} - 5 = m$$

$$m = 7.5\text{ kg}$$

(2)



(a) distance = area under

(1) $\frac{10 \times 4}{2} = 20\text{ m}$

(2) $10 \times 4 = 40\text{ m}$

(3) $\frac{1}{2}(4+7) \times 10$
$= 55\text{ m}$

(4) $\frac{10 \times 7}{2} = 35$

$$20 + 40 + 55 + 35 = \underline{\underline{150\text{m}}}$$

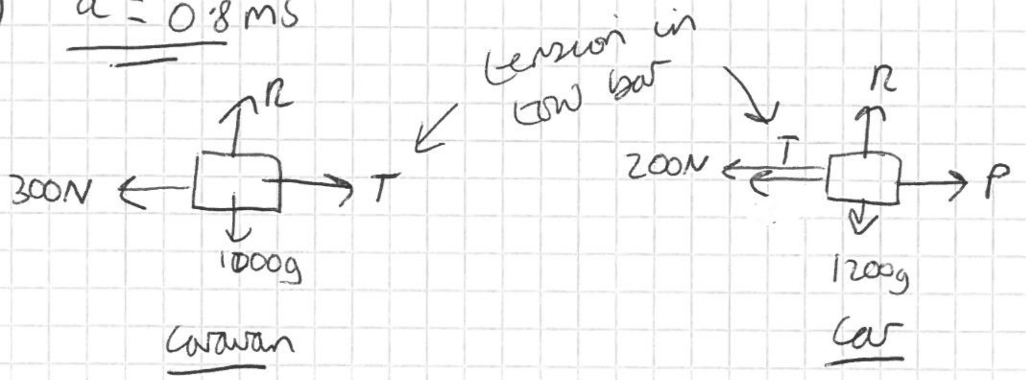
(b) average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{150}{40} = 3.75\text{ ms}^{-1}$

(c) $\underline{\text{gradient}}$ ie $\frac{\Delta v}{t} = \frac{4}{10} = 0.4\text{ms}^{-2}$

(d) $F = ma = 200000 \times 0.4 = 80000\text{ N}$

↓ 200 tonnes = 200 000 kg

(3) $\underline{a = 0.8 \text{ ms}^{-2}}$



(a) P acts on the car only.

using NII ($F=ma$) on the car:

$$\text{(1)} \quad P - 200 - T = 1200 \times 0.8$$

Need to know T so use the caravan. (NII again)

$$\text{(2)} \quad T - 300 = 1000 \times 0.8$$

$$T = 800 + 300 = 1100$$

so (1) $P - 200 - 1100 = 1200 \times 0.8$

$$\Rightarrow \quad P = \underline{\underline{2260 \text{ N}}}$$

(ii) This is the force I have called T

$T = 1100$ (see (2) above)

b) (i) $v = u + at$

$v = 15$

$u = 7$

$a = 0.8$

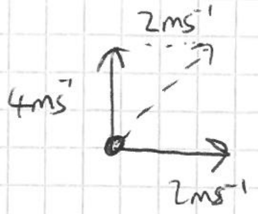
$$t = \frac{v-u}{a} = \frac{15-7}{0.8} = \frac{8}{0.8} = \underline{\underline{10 \text{s}}}$$

(ii) $s = ut + \frac{1}{2}at^2 = 7 \times 10 + \frac{1}{2} \times 0.8 \times 10^2 = 70 + 40 = 110 \text{m}$

(c) because resistive forces will vary as speed changes.

4

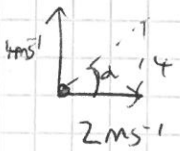
(a)



$$2^2 + 4^2 = 4 + 16 = 20$$

$$\Rightarrow \sqrt{20} = 2\sqrt{5} = 4.47\ ms^{-1}$$

(b)



$$\alpha = \tan^{-1}\left(\frac{O}{A}\right) = \tan^{-1}\left(\frac{4}{2}\right) = \tan^{-1}(2)$$

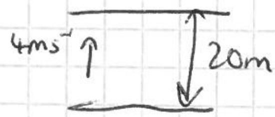
$$= 63.4349$$

$$\alpha = 63.4^\circ$$

(c) Easiest method:

"vertical" and "horizontal" motion are independent just like with projectiles

Consider "vertical" motion



$$t = \frac{s}{v} = \frac{20}{4} = \underline{\underline{5s}}$$

5

s

$\underline{u} = 4\underline{i} + 0.5\underline{j}$

v

$\underline{a} = -0.4\underline{i} + 0.2\underline{j}$

t

(a) use $v = u + at$

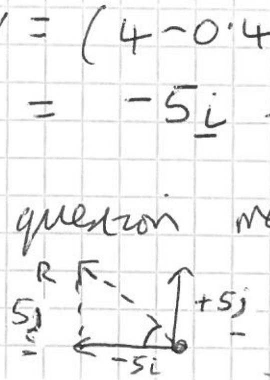
$$v = (4\underline{i} + 0.5\underline{j}) + t(-0.4\underline{i} + 0.2\underline{j})$$

$$\Rightarrow v = (4 - 0.4t)\underline{i} + (0.5 + 0.2t)\underline{j}$$

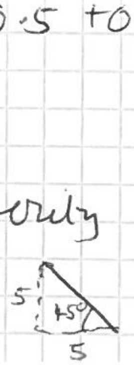
(b)(i) when $t = 22.5$ $\quad v = (4 - 0.4 \times 22.5)\underline{i} + (0.5 + 0.2 \times 22.5)\underline{j}$

$$= -5\underline{i} + 5\underline{j}$$

(ii) word "travelling" in question means use velocity vector.



$\Leftarrow$ isosceles $\therefore 45^\circ$



$\Leftarrow$ $\underline{\underline{NW}}$

(c) $V = (4 - 0.4t)\underline{i} + (0.5 + 0.2t)\underline{j}$

To find the <u>speed</u> we need Pythagoras.

$$\text{speed} = \sqrt{(4-0.4t)^2 + (0.5+0.2t)^2}$$

$$(16 - 3.2t + 0.16t^2) + (0.25 + 0.2t + 0.04t^2)$$

$$\text{Speed} = \sqrt{16.25 - 3t + 0.2t^2} = 5$$ ← this is speed we are looking for.

so $5^2 = 0.2t^2 - 3t + 16.25$

$(\times 5)$ $125 = t^2 - 15t + 81.25$

$0 = t^2 - 15t - 43.75$

use formula with $a = 1$ $b = -15$ $c = -43.75$

$$\frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(-43.75)}}{2 \times (1)} = \frac{15 \pm \sqrt{225 + 175}}{2}$$

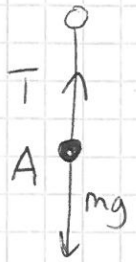
$$= \frac{15 \pm \sqrt{400}}{2} = \frac{15 \pm 20}{2}$$

$t = \frac{35}{2} = 17.5$ and $t = \frac{-5}{2} = -2.5$

reject negative solution

answer $\underline{\underline{t = 17.5\ s}}$

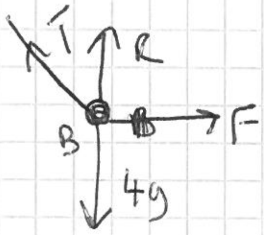
6 (a)



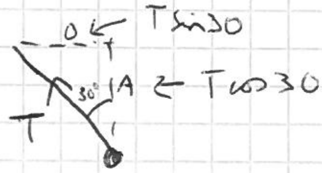
$T = mg$ (in equilibrium)

$T = 2 \times 9.8 = 19.6\,N$

(b)



(c) vertically: forces must be balanced. Don't forget that T has a vertical component.



vertically

$T\cos 30 + R = 4g$

$R = 4g - T\cos 30$

$= 4 \times 9.8 - 19.6 \times \cos 30$

$= 22.2259$

$= 22.2\,N$ (3SF)

(d) Horizontal forces also balanced

$T\sin 30 = F \leq \mu R$

$T\sin 30 \leq \mu R \Rightarrow \mu \geq \dfrac{T\sin 30}{R}$

$\mu \geq \dfrac{19.6 \times 0.5}{22.2259}$

≥ 0.4404

$\therefore$ least possible value is $\mu = 0.44$ $\quad \mu \geq 0.44$

(7) (a) vertical displacement S_y is $1 - 1.5 = -0.5$

$$S_y = ut\sin\theta - \frac{1}{2}gt^2$$

$$-0.5 = 12t\sin 30 - \frac{1}{2}\times 9.8 \times t^2$$

$$0 = -4.9t^2 + 6t + 0.5$$

use quadratic formula with $a = -4.9$
$b = +6$
$c = +0.5$

$$t = \frac{-6 \pm \sqrt{6^2 - 4(-4.9)(-0.5)}}{2\times(-4.9)}$$

$t = -0.0783$ and $t = 1.30281$

$\downarrow$ reject -ve.

$\downarrow$ so $t = 1.30$ (3sf).

(b) Horizontally

$$S_x = ut\cos\theta = 12\times 1.30281 \times \cos 30$$
$$= 13.5392$$
$$= 13.5\text{ m}$$

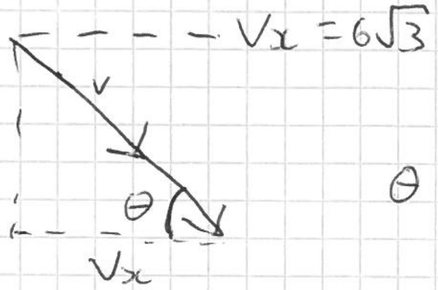
(c) Need V_x and V_y at $t = 1.30281$

$$V_x = u\cos\theta = 12\cos 30 = 6\sqrt{3}$$

$$V_y = u\sin\theta - gt = 12\sin 30 - 9.8\times 1.30281 = -6.76754$$

$$\text{Speed} = \sqrt{V_x^2 + V_y^2} = 12.401595$$
$$= 12.4\text{ ms}^{-1}$$

(d)



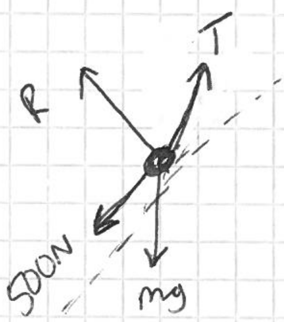
$\theta = \tan^{-1}\left(\frac{+V_y}{V_x}\right) = 33.07245$

$= 33.1°$

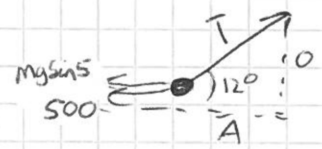
(e) No air resistance.

8

(a)



(b) $a = 0.6\ ms^{-2}$ use $F = ma$



↓
component of T parallel to the slope is $T\cos(12°)$

So Net force $F = T\cos 12 - 500 - mg\sin 5$

↓
weight component

$T\cos 12 - 500 - 2000g\sin 5 = 2000 \times 0.6$

$T = (1200 + 500 + 2000 \times 9.8 \times \sin 5) / \cos 12$

$= \frac{3408.252558}{\cos 12} = 3484.3949$

$= 3480\ N$ (3SF).