

Further Part 1 - January 2012

① a) $\alpha + \beta = -7/2$ $\alpha\beta = 4$

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
 $= (-7/2)^2 - 2(4)$
 $= 49/4 - 8 = 17/4$

c) Sum $1/\alpha^2 + 1/\beta^2 = \frac{\beta^2}{\alpha^2\beta^2} + \frac{\alpha^2}{\alpha^2\beta^2}$
 $= \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{17/4}{4^2} = 17/64$

Product $1/\alpha^3 \times 1/\beta^3 = \frac{1}{\alpha^2\beta^2} = \frac{1}{(\alpha\beta)^2} = \frac{1}{4^2} = 1/16$

$x^2 - \text{Sum} \cdot x + \text{Product} = 0$
 $\rightarrow x^2 - 17/64 x + 1/16 = 0$
 $64x^2 - 17x + 4 = 0$

② a) $\int_8^{\infty} x^{-2/3} dx = \int_8^p x^{-2/3} dx = \left[3x^{1/3} \right]_8^p$
 $= 3\sqrt[3]{p} - 3\sqrt[3]{8}$

As $p \rightarrow \infty$, $3\sqrt[3]{p}$ does not converge to a limit

$\therefore \int$ has no finite value

b) $\int_8^{\infty} x^{-4/3} dx = \int_8^p x^{-4/3} dx = \left[-3x^{-1/3} \right]_8^p$
 $= \frac{-3}{\sqrt[3]{p}} - \frac{-3}{\sqrt[3]{8}}$
 $= \frac{3}{2} - \frac{3}{\sqrt[3]{p}}$

As $p \rightarrow \infty$, $\frac{3}{\sqrt[3]{p}} \rightarrow 0$, so $\int \rightarrow 3/2$

$$\textcircled{3} \text{ a) i) } x^2 + 9 = 0 \rightarrow x^2 = -9 \rightarrow x = \pm \sqrt{-9} \rightarrow x = \pm 3i$$

$$\text{ii) } (x+2)^2 + 9 = 0 \rightarrow x+2 = \pm 3i$$

$$\rightarrow x = -2 \pm 3i$$

$$\text{b) i) } (1+x)^3 = 1 + 3x + 3x^2 + x^3$$

$$\text{ii) } (1+2i)^3 = 1 + 3(2i) + 3(2i)^2 + (2i)^3$$

$$= 1 + 6i + 12i^2 + 8i^3$$

$$= 1 + 6i - 12 - 8i = -11 - 2i$$

$$\text{iii) } z^* - z^3 = (1-2i) - (-11-2i)$$

$$= 1 - 2i + 11 + 2i$$

$$= 12$$

$$\textcircled{4} \text{ a) } \sum r^2 (4r-3) = \sum 4r^3 - 3 \sum r^2$$

$$= n^2(n+1)^2 - \frac{1}{2}n(n+1)(2n+1)$$

$$= \frac{1}{2}n(n+1) [2n(n+1) - (2n+1)]$$

$$= \frac{1}{2}n(n+1) [2n^2 + 2n - 2n - 1]$$

$$= \frac{1}{2}n(n+1)(2n^2 - 1)$$

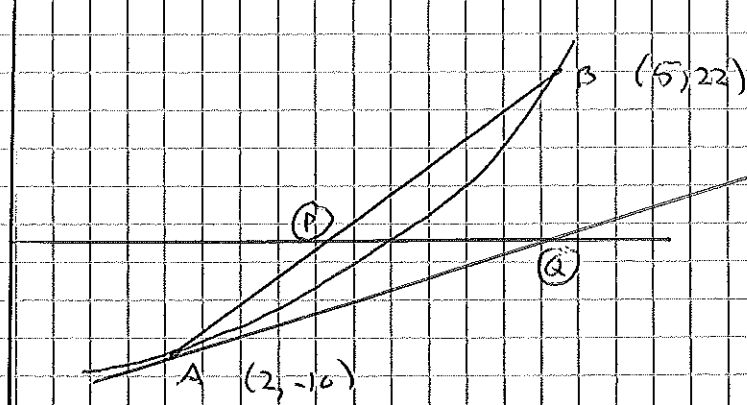
$$\text{b) } \sum_{20}^{40} = \sum_1^{40} - \sum_1^{19}$$

$$= \frac{1}{2}(40)(41)(2(40^2) - 1) - \frac{1}{2}(19)(20)(2(19^2) - 1)$$

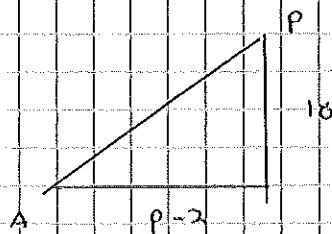
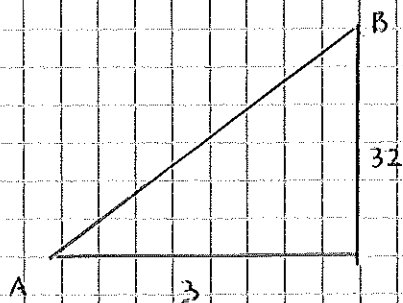
$$= 2623180 - 136996$$

$$= 2,486,184$$

$\textcircled{5} \text{ a)}$



ii)



$$\frac{p-2}{10} = \frac{3}{32}$$

$$\rightarrow p-2 = \frac{30}{32} \quad \rightarrow p = \frac{30}{32} + 2 = 47/16$$

b) iii) $x_1 = 2$

$$y_1 = -10$$

$$m = 8$$

$$y - y_1 = m(x - x_1)$$

$$y + 10 = 8(x - 2)$$

$$y + 10 = 8x - 16$$

$$\text{At } Q, y = 0 \quad \rightarrow \quad 10 = 8x - 16$$

$$8x = 26 \quad \rightarrow \quad x = 26/8 \text{ or } 3.25$$

6) a) $\theta = n\pi + \alpha$

$$\text{Key angle} = \tan^{-1}(1/\sqrt{3}) = \pi/6 = \alpha$$

$$\rightarrow x/2 - \pi/4 = n\pi + \pi/6$$

$$\rightarrow x/2 = n\pi + 5\pi/12$$

$$\rightarrow x = 2n\pi + 5\pi/6$$

b) $\tan^2(x/2 - \pi/4) = 1/3$

$\sqrt{}$ $\rightarrow \tan(x/2 - \pi/4) = \pm 1/\sqrt{3}$

$$\therefore \tan(x/2 - \pi/4) = 1/\sqrt{3}$$

$\boxed{\text{OR}}$ $\tan(x/2 - \pi/4) = -1/\sqrt{3}$

$$\rightarrow 2n\pi + 5\pi/6$$

(from a))

$$\text{Key angle: } \tan^{-1}(-1/\sqrt{3}) = -\pi/6$$

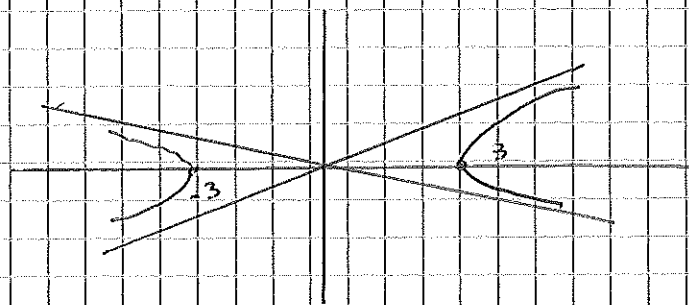
$$\rightarrow x/2 - \pi/4 = n\pi - \pi/6$$

$$\rightarrow x/2 = n\pi + \pi/12$$

$$\rightarrow x = 2n\pi + \pi/6$$

① a) $\frac{x}{3} = \pm y$ (from formula book) $\rightarrow y = \pm \frac{1}{3}x$

b)



Intersection = $(3, 0)$

$(-3, 0)$

a) i) $\frac{(x+3)^2}{9} - y^2 = 1$

ii) $y = x \rightarrow \frac{(x+3)^2}{9} - x^2 = 1$

$\rightarrow (x+3)^2 - 9x^2 = 9$

$\rightarrow x^2 + 6x + 9 - 9x^2 = 9$

$\rightarrow 8x^2 - 6x = 0$

$\rightarrow 4x^2 - 3x = 0$

$\rightarrow x(4x - 3) = 0$

$(0, 0)$

$x = 0$

$y = 0$

$x = \frac{3}{4}$

$(\frac{3}{4}, \frac{3}{4})$

$y = \frac{3}{4}$

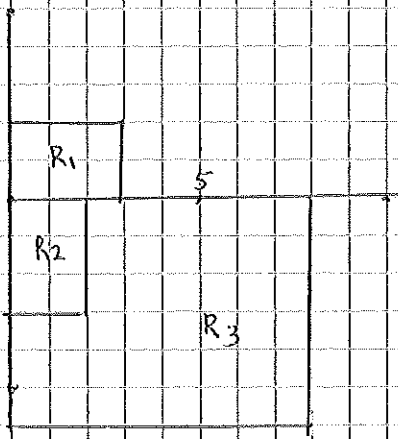
d) The curve shifts 3 to the right

$y = x - 3 \rightarrow y + 3 = x \therefore$ line also shifts 3 to right

\therefore Intersections are 3 to right

$= (3, 0) \text{ or } (\frac{3}{4}, \frac{3}{4})$

8)

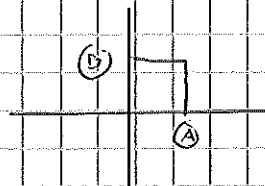


$$i) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$ii) \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$$

= stretch SF 4 in x
stretch SF 2 in y

$$b) i) R_1 \rightarrow R_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$$ii) R_1 \rightarrow R_3 = R_3 \times R_2$$

$$\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 4 \\ -2 & 0 \end{pmatrix}$$

9) a) Denominator $\rightarrow x = 1$

Numerator $\rightarrow y = 1/1 \rightarrow y = 1$

$$b) y = -4x + c \rightarrow -4x + c = x \quad (x=1)$$

$$\rightarrow (-4x + c)(x - 1) = x$$

$$\rightarrow -4x^2 + 4x + cx - c = x$$

$$\rightarrow 4x^2 - 3x - cx + c = 0$$

$$\rightarrow 4x^2 - (c + 3)x + c = 0$$

c) At tangent, $b^2 - 4ac = 0$

$$\rightarrow (-c - 3)^2 - 4(4)(c) = 0$$

$$\rightarrow c^2 + 6c + 9 - 16c = 0$$

$$\rightarrow c^2 - 10c + 9 = 0$$

$$(c - 9)(c - 1) = 0 \rightarrow c = 9, c = 1$$

ii)

$$C=1$$

$$4x^2 - 4x + 1 = 0$$

$$(2x - 1)(2x - 1) = 0$$

$$\rightarrow x = 1/2$$

$$\rightarrow y = -4(1/2) + 1 = -1 \quad \rightarrow (1/2, -1)$$

$$C=9$$

$$4x^2 - 12x + 9 = 0$$

$$(2x - 3)(2x - 3) = 0$$

$$\rightarrow x = 3/2$$

$$\rightarrow y = -4(3/2) + 9 = 3 \quad \rightarrow (3/2, 3)$$