FP1 - May/June 2010

(1) $y_{n+1} = y_n + h f(x_n)$

$y_1 = 3$

$x_1 = 1$

$h = 0.1$

$f(x) = 1 + 1^3$

$y_2 = 3 + 0.1[1 + 1^3]$

$= 3.2$

$y_2 = 3.2$

$x_2 = 1.1$

$f(x) = 1 + 1.1^3$

$y_3 = 3.2 + 0.1[1 + 1.1^3]$

$= 3.4331$

$y_3 = 3.4331$

$x_3 = 1.2$

$f(x) = 1 + 1.2^3$

$y_4 = 3.4331 + 0.1[1 + 1.2^3]$

$= \boxed{3.7059}$

(2) a) $(1 - 2i)(x + iy) - (x - iy)$

$\rightarrow x + iy - 2xi + 2y - x + iy$

$\rightarrow 2y + 2iy - 2xi$

REAL $2y$

IMAG $2iy - 2xi$ or $2y - 2x$

b) REAL $2y = 20 \rightarrow y = 10$

IMAG $2y - 2x = 10 \rightarrow x = 5$

$\rightarrow z = 5 + 10i$

③ $\theta = 360n \pm a$

Key angle $= \cos^{-1}(\cos 40) = 40$

$\rightarrow 5x - 20 = 360n \pm 40$

$5x = 360n + 20 \pm 40$

$x = 72n + 4 \pm 8$

or $72n + 12$, $72n - 4$

④ a)

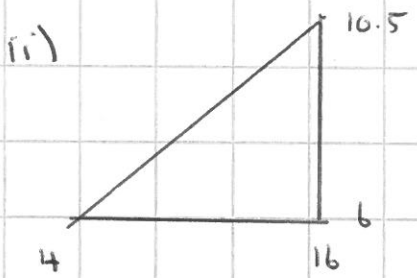
x	2	4	6	8
X	4	16	36	64

b) see mark scheme

c) i) $y = 15$, $X = 28$ $\rightarrow x^2 = 28$ $\rightarrow x = \sqrt{28}$

$= 5.3$

ii)

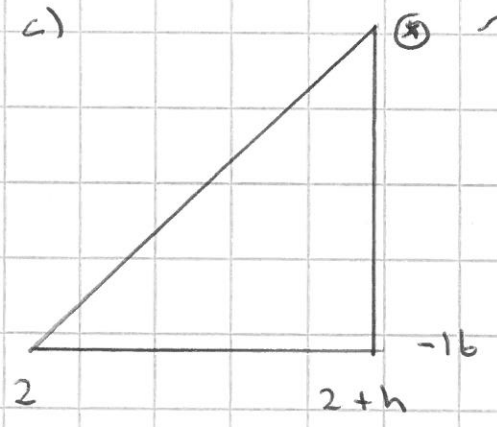


Gradient $= 4.5/12 = 0.375$

y intercept ≈ 4.5

$\rightarrow y = 0.38x^2 + 4.5$

⑤ c)



$\rightsquigarrow$ * $(2+h)^3 - 12(2+h)$

$= 2^3 + 3 \times 2^2 \times h + 3 \times 2 \times h^2 + h^3$

$-(24 + 12h)$

$= 8 + 12h + 6h^2 + h^3 - 24 - 12h$

$= h^3 + 6h^2 - 16$

$$\text{Gradient} = \frac{h^3 + 6h^2 - 16 - (-16)}{2 + h - 2}$$

$$= h^2 + 6h$$

b) As $h \to 0$, gradient $\to 0$

$\therefore$ Gradient at $A = 0$, which means it's a stationary point.

(6) a) $\cos\theta = 1/\sqrt{2} \to \theta = 45^\circ$

Rotation, 45°, Anti-clockwise

b) $\cos 2\theta = 1/\sqrt{2} \to \theta = 22.5^\circ$

Reflection in line $y = (\tan 22.5)x$

c)
$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\cos\theta = 0$

$\to \theta = 90$

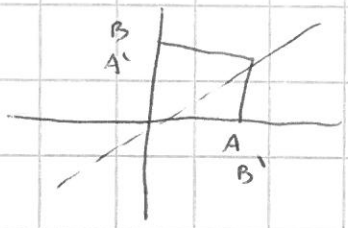
Rotation 90° anti-clockwise.

[NOTE: I should have known this by looking at A!]

d) B^2 must be a reflection followed by same reflection

$\to I$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

e)
$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

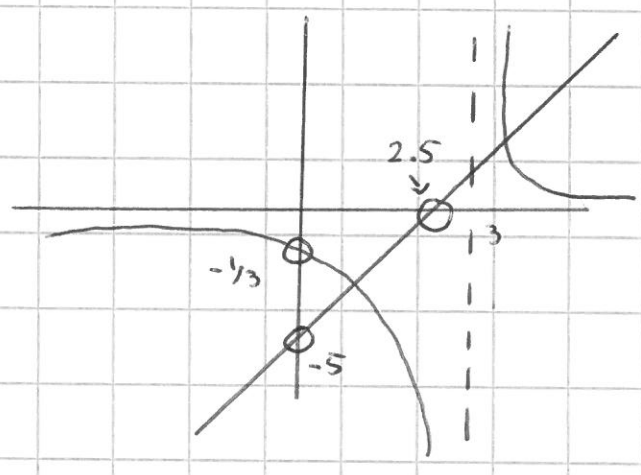


= Reflection in line $y = x$

(7) a) i) $x = 3$ As $x \to \infty$, $y \to 1/\infty = 0$

$\to y = 0$

ii)
iii)



b) i) $\dfrac{1}{x-3} = 2x - 5$

$\to 1 = (2x-5)(x-3)$

$1 = 2x^2 - 6x - 5x + 15$

$1 = 2x^2 - 11x + 15$

$2x^2 - 11x + 14 = 0$

$(2x-7)(x-2) = 0$

$\downarrow \qquad \downarrow$

$x = 7/2 \qquad x = 2$

ii) Curve lies below line: $2 < x < 3$ and $x > 7/2$

(8) a) $\alpha + \beta = 4$, $\alpha\beta = 10$

b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= 4/10 = 2/5$

c) SUM $\alpha + 2/\beta + \beta + 2/\alpha = \alpha + \beta + 2\alpha/\alpha\beta + 2\beta/\alpha\beta$

$= \alpha + \beta + \frac{2(\alpha + \beta)}{\alpha\beta}$

$= 4 + 8/10$

$= 24/5$

PRODUCT $(\alpha + 2/\beta)(\beta + 2/\alpha)$

$= \alpha\beta + 2 + 2 + 4/\alpha\beta$

$= 10 + 4 + 4/10 = 72/5$

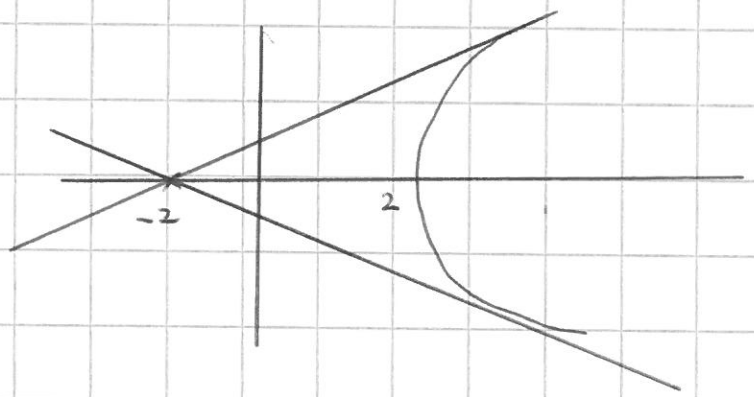
$\rightarrow x^2 - [\text{SUM}]x + [\text{PRODUCT}] = 0$

$\rightarrow x^2 - 24/5 x + 72/5 = 0$

$\rightarrow 5x^2 - 24x + 72 = 0$

(9) i)

ii)



b) i) $y = m(x+2)$

At intersection:

$m^2(x+2)^2 = x - 2$

$m^2[x^2 + 4x + 4] = x - 2$

$$m^2x^2 + 4m^2x + 4m^2 - x + 2 = 0$$
$$(m^2)x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$

ii) For equal roots, $b^2 - 4ac = 0$

$$\rightarrow (4m^2 - 1)^2 - 4 \times m^2 \times (4m^2 + 2) = 0$$
$$\rightarrow 16m^4 - 8m^2 + 1 - 16m^4 - 8m^2 = 0$$
$$\rightarrow -16m^2 + 1 = 0$$
$$\rightarrow 16m^2 = 1$$

iii) $m^2 = 1/16$

use: $(m^2)x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$

$$\rightarrow 1/16\,x^2 + (4/16 - 1)x + 4/16 + 2 = 0$$
$$\rightarrow 1/16\,x^2 - 3/4\,x + 9/4 = 0$$
$$\rightarrow x^2 - 12x + 36 = 0$$
$$(x - 6)(x - 6) = 0$$

$$\rightarrow x = 6$$

Find y:

$$y^2 = x - 2$$
$$y^2 = 6 - 2$$
$$y^2 = 4$$
$$\rightarrow y = \pm 2$$

$\therefore$ (6, -2) and (6, 2)