Further Pure 1 - June 2008

① $x^2 + x + 5 = 0$

a) $\alpha + \beta = -1$, $\alpha\beta = 5$

b) $\boxed{\text{SUM}}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2\times 5 = -9$

c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2}{\alpha\beta} + \frac{\beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{-9}{5}$

d) $\boxed{\text{SUM}} = -9/5$

$\boxed{\text{PRODUCT}}$ $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = \frac{\alpha\beta}{\alpha\beta} = 1$

$x^2 - \boxed{\text{SUM}}\, x + \boxed{\text{PRODUCT}} = 0$

$\rightarrow x^2 + 9/5\, x + 1 = 0$

$\rightarrow 5x^2 + 9x + 5 = 0$

② a) $z = x + iy$ $\quad z^* = x - iy$

$\rightarrow 3i(x + iy) + 2(x - iy)$

$= 3ix - 3y + 2x - 2iy$

REAL: $2x - 3y$

IMAG: $3ix - 2iy$

b) REAL: $2x - 3y = 7$ ①

Simultaneous equations

IMAG: $3x - 2y = 8$ ②

$\boxed{① \times 3}$ $6x - 9y = 21$

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$\boxed{② \times 2}$ $6x - 4y = 16$

$-5y = 5 \rightarrow y = -1$

Sub into ① $2x - 3(-1) = 7$

$2x + 3 = 7$

$2x = 4 \rightarrow x = 2$

$\rightarrow z = 2 - i$ $\quad (z = x + iy)$

(3) a) $\displaystyle\int_9^\infty \frac{1}{\sqrt{x}}\,dx = \int_9^n x^{-1/2}\,dx$

$= \left[2x^{1/2}\right]_9^n = 2\sqrt{n} - 2\sqrt{9}$

$= 2\sqrt{n} - 6$

As $n \to \infty$, $2\sqrt{n} - 6$ does not approach a limit

$\therefore$ Integral has no value

b) $\displaystyle\int_9^\infty \frac{1}{x\sqrt{x}} = \int_9^n x^{-3/2}$

$= \left[-2x^{-1/2}\right]_9^n = \left[\frac{-2}{\sqrt{x}}\right]_9^n$

$= \frac{-2}{\sqrt{n}} + \frac{2}{\sqrt{9}} = \frac{-2}{\sqrt{n}} + \frac{2}{3}$

As $n \to \infty$, $\frac{-2}{\sqrt{n}} \to 0$

$\therefore$ Integral $\to 2/3$

(4) a) $y = ax + \frac{b}{x+2}$

$y(x+2) = a(x+2)x + b$

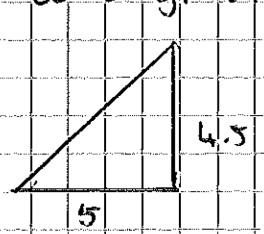
$\Rightarrow Y = aX + b$

b) i)

x	1	2	3	4
y	0.4	1.43	2.4	3.35
X	3	8	15	24
Y	1.2	5.72	12	20.1

ii) See Insert

iii) a = gradient



$\simeq \frac{4.5}{5}$

$\simeq 0.9$

b = intercept

≈ -2

(5) a) Radians: $\theta = 2n\pi \pm a$

Key angle $= \cos^{-1}(1/\sqrt{2}) = \pi/4 = a$

$\therefore \; x/2 + \pi/3 = 2n\pi \pm \pi/4$

$x/2 = 2n\pi - \pi/3 \pm \pi/4$

$\Rightarrow x = 4n\pi - 2\pi/3 \pm \pi/2$

b) Try values:

$\boxed{n=0}$ $\quad x = 0 - 2\pi/3 \pm \pi/2 = -\text{ve}!$ (both!)

$\boxed{n=1}$ $\quad x = 4\pi - 2\pi/3 - \pi/2 = 17\pi/6$

(6) a)

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \quad \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4I$$

c) $(AB)^2 =$

$$\begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix} \quad \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix} = -16I$$

$B^2 =$

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \quad \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

$A^2B^2 = \begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix}$

$\begin{pmatrix}4 & 0\\ 0 & 4\end{pmatrix} \begin{pmatrix}16 & 0\\ 0 & 16\end{pmatrix} = 16I$

$\therefore (AB)^2 \neq A^2B^2$

(7) a) Left 1, up 7 $= \begin{pmatrix}-1\\ 7\end{pmatrix}$

b) i) $x = -1$

As $x \to \infty$, $y \to 7$ $\therefore$ $y = 7$

ii) when $\boxed{x = 0}$, $y = 7 + 1/1 = 8 \Rightarrow (0, 8)$

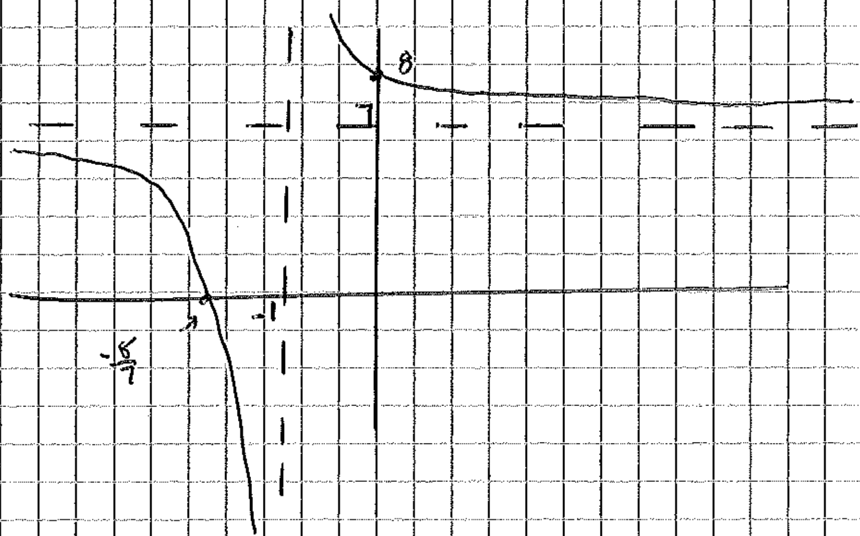
when $\boxed{y = 0}$ $0 = 7 + \frac{1}{x+1}$

$\Rightarrow 0 = 7(x+1) + 1$

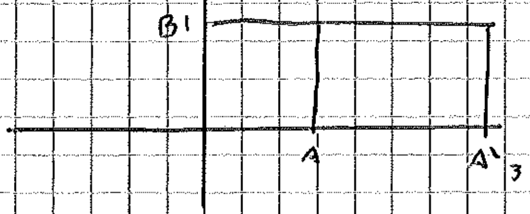
$\Rightarrow 0 = 7x + 7 + 1$

$\Rightarrow x = -8/7 \Rightarrow (-8/7, 0)$

c)



(8) a) From Diagram = Stretch, SF 3, parallel to x-axis



$= \begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}$

b) See Insert

c) $\boxed{1st}$ = stretch $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

$\boxed{2nd}$ = Reflection in $y = x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

order : $\boxed{2ND}$ $\boxed{1st}$ $\boxed{POINT}$

So, to find combined matrix :

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$$

(9) a) $x_1 = 3$

$y_1 = 4$

$m = m$

$y - 4 = m(x - 3)$

$y - 4 = mx - 3m$

b) Arrange eq of line to make $x =$

$\rightarrow \quad y - 4 = m(x - 3)$

$\frac{y - 4}{m} = x - 3$

$\rightarrow \quad x = \frac{y - 4}{m} + 3$

Sub into eq of parabola : $y^2 = 4\left[\frac{y - 4}{m} + 3\right]$

$y^2 = \frac{4y - 16}{m} + 12$

$my^2 = 4y - 16 + 12m$

$\rightarrow \quad my^2 - 4y + 16 - 12m = 0$

c) For tangents, $b^2 - 4ac = 0$

$\rightarrow \quad 16 - 4 \times m \times (16 - 12m) = 0$

$16 - 4m(16 - 12m) = 0$

$$16 - 64m + 48m^2 = 0$$

$\div 16$ $\quad 1 - 4m + 3m^2 = 0$

$$\rightarrow 3m^2 - 4m + 1 = 0$$

$$(3m-1)(m-1) = 0$$

$$m = 1/3 \qquad m = 1$$

From Part a) $\quad y - 4 = mx - 3m$

$\boxed{m = 1/3} \rightarrow y - 4 = \tfrac{1}{3}x - 1$

$$\rightarrow y = \tfrac{1}{3}x + 3$$

$\boxed{m = 1} \rightarrow y - 4 = x - 3$

$$\rightarrow y = x + 1$$

d) Tangents touch parabolas at the 2 m values

$$my^2 - 4y + 16 - 12m = 0$$

$\boxed{m = 1/3}$ $\quad \tfrac{1}{3}y^2 - 4y + 12 = 0$

$$\rightarrow y^2 - 12y + 36 = 0$$

$$(y-6)(y-6) = 0$$

$$y = 6 \qquad \boxed{y^2 = 4x}$$

$$x = 9$$

$$\rightarrow (9, 6)$$

$\boxed{m = 1} \rightarrow y^2 - 4y + 4 = 0$

$$(y-2)(y-2) = 0$$

$$y = 2 \qquad \boxed{y^2 = 4x}$$

$$x = 1$$

$$\rightarrow (1, 2)$$