Further Pure 1 - June 2007

(1) a) $M = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$

$= 3 \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

b) Enlargement SF 3, (so $p = 3$), reflection in line $y = -x$

c) $M^2 = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} = 9I$

(2) a) Let $f(x) = x^3 + x - 7$

$f(1.6) = 1.6^3 + 1.6 - 7 = -1.304$

$f(1.8) = 1.8^3 + 1.8 - 7 = 0.632$

Sign change, ∴ root between 1.6 and 1.8

b) $f(1.7) = 1.7^3 + 1.7 - 7 = -0.387$

∴ root lies between 1.7 and 1.8

$f(1.75) = 1.75^3 + 1.75 - 7 = 0.1093...$

∴ root lies between 1.7 and 1.75

∴ root = 1.7 (1dp)

(3) a) $z - 3iz^*$

$= (x + iy) - 3i(x - iy)$

$= x + iy - 3xi - 3y \qquad (i^2 = -1)$

$= x - 3y - 3xi + iy$

REAL $x - 3y$

IMAG $-3x + y$

b) $\boxed{\text{REAL}}$ $x - 3y = 16$ ①

$\boxed{\text{IMAG}}$ $-3x + y = 0$ ②

① × 3 → $3x - 9y = 48$

② $-3x + y = 0$ +

$$-8y = 48$$

$$\Rightarrow y = -6$$

use ① : $x - 3(-6) = 16 \Rightarrow x = -2$

$\therefore z = -2 - 6i$

④ a) $\alpha + \beta = -\left(-\frac{1}{2}\right) = \frac{1}{2}$ $\qquad \alpha\beta = \frac{4}{2} = 2$

b) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta}{\alpha\beta} + \frac{\alpha}{\alpha\beta}$

$$= \frac{\alpha + \beta}{\alpha\beta} = \frac{1/2}{2} = \frac{1}{4}$$

c) $\boxed{\text{SUM}}$ $\frac{4}{\alpha} + \frac{4}{\beta} = 4\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = 4\left(\frac{1}{4}\right) = 1$

$\boxed{\text{PRODUCT}}$ $\frac{4}{\alpha} \times \frac{4}{\beta} = \frac{16}{\alpha\beta} = \frac{16}{2} = 8$

$$\Rightarrow x^2 - \boxed{\text{SUM}}\, x + \boxed{\text{PRODUCT}} = 0$$

$$\Rightarrow x^2 - x + 8 = 0$$

⑤ a)

x	1	2	3	4
Y	0.584	0.788	0.992	1.196

b) $y = ab^x$

$\log_{10}(y) = \log_{10}(ab^x)$

$\Rightarrow \log_{10}(y) = x\log_{10}(b) + \log_{10}(a)$

$\Rightarrow Y = \underbrace{m}_{\log_{10}(b)}x + \underbrace{c}_{\log_{10}(a)}$

c) See sheet

d) From graph: gradient ≈ 0.2 $\qquad$ y intercept ≈ 0.38

$\therefore \log_{10}(b) \approx 0.2$ $\qquad$ $\therefore \log_{10}(a) \approx 0.38$

$\Rightarrow b \approx 10^{0.2} = 1.6$ $\qquad$ $\Rightarrow a = 10^{0.38} = 2.4$

(6) G.S $\theta = 2n\pi + \alpha$, $\theta = 2n\pi + (\pi - \alpha)$

Key angle (α) $= \sin^{-1}(\sqrt{3}/2) = \pi/3$

$\rightarrow 2x - \pi/2 = 2n\pi + \pi/3$, $2x - \pi/2 = 2n\pi + 2\pi/3$

$\rightarrow 2x = 2n\pi + 5\pi/6$, $2x = 2n\pi + 7\pi/6$

$\rightarrow x = n\pi + 5\pi/12$, $x = n\pi + 7\pi/12$

(7) a) Denominator = 0 when $x = -2$ $\rightarrow$ $x = -2$

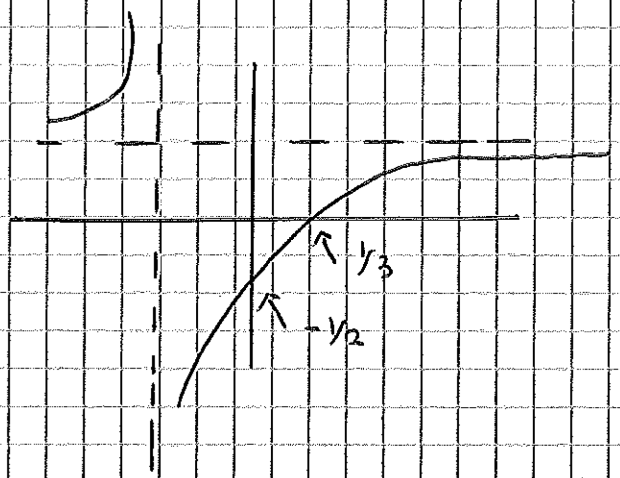
As $x \rightarrow \infty$, $y \rightarrow 3/1$ $\rightarrow$ $y = 3$

b) crosses x axis when $y = 0$

$\rightarrow 0 = \frac{3x-1}{x+2}$ $\rightarrow$ $0 = 3x - 1$ $\rightarrow$ $x = 1/3$

crosses y axis when $x = 0$

$\rightarrow y = \frac{3(0)-1}{0+2}$ $\rightarrow$ $y = -1/2$



c) From diagram, graph is between 0 and 3 when $x > 1/3$

(8) a) $\int_0^1 x^{1/3} + x^{-1/3}$ $\rightarrow$ $\int_p^1 x^{1/3} + x^{-1/3}$

$= \left[\frac{3}{4} x^{4/3} + \frac{3}{2} x^{2/3} \right]_p^1$

$= 3/4 + 3/2 - \left[\frac{3}{4} p^{4/3} + \frac{3}{2} p^{2/3} \right]$

$= 9/4 - \left[\frac{3}{4} \sqrt[3]{p^4} + \frac{3}{2} \sqrt[3]{p^2} \right]$

As $p \rightarrow 0$, $\sqrt[3]{p^4}$ and $\sqrt[3]{p^2} \rightarrow 0$

$\rightarrow \int \rightarrow 9/4$

b) $\int_0^1 \frac{x^{1/3} + x^{-1/3}}{x} = \int_p^1 x^{-2/3} + x^{-4/3}$

$= \left[3x^{1/3} - 3x^{-1/3} \right]_p^1$

$= 3(1) - 3(1) - \left[3\sqrt[3]{p} - \frac{3}{\sqrt[3]{p}} \right]$

As $p \to 0$, $3/\sqrt[3]{p}$ is undefined

$\therefore \int$ has no value.

(9) a) Intersects x axis when $y = 0$

$\to x^2/2 = 1 \to x^2 = 2 \to x = \pm\sqrt{2}$

$(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$

Intersects y axis when $x = 0$

$\to y^2 = 1 \to y = \pm 1 \to (0, 1)$ and $(0, -1)$

b) $\frac{(x - k)^2}{2} + y^2 = 1$

c) $x + y = 2 \to y = 2 - x$

sub into translated curve

$\to \frac{(x - k)^2}{2} + (2 - x)^2 = 1$

$\to (x - k)^2 + 2(2 - x)^2 = 2$

$\to x^2 - 2kx + k^2 + 2[4 - 4x + x^2] = 2$

$\to x^2 - 2kx + k^2 + 8 - 8x + 2x^2 = 2$

$\to 3x^2 - 2kx - 8x + k^2 + 6 = 0$

$\to 3x^2 - 2(k + 4)x + (k^2 + 6) = 0$

d) For tangent, $b^2 - 4ac = 0$

$\to [-2(k + 4)]^2 - 4(3)(k^2 + 6) = 0$

$\to (-2k - 8)^2 - 12k^2 - 72 = 0$

$\to 4k^2 + 32k + 64 - 12k^2 - 72 = 0$

$\rightarrow \; -8k^2 + 32k - 8 = 0$

$\div -8 \; \rightarrow \; k^2 - 4k + 1 = 0$

$\rightarrow \; (k - 2)^2 - 4 + 1 = 0$

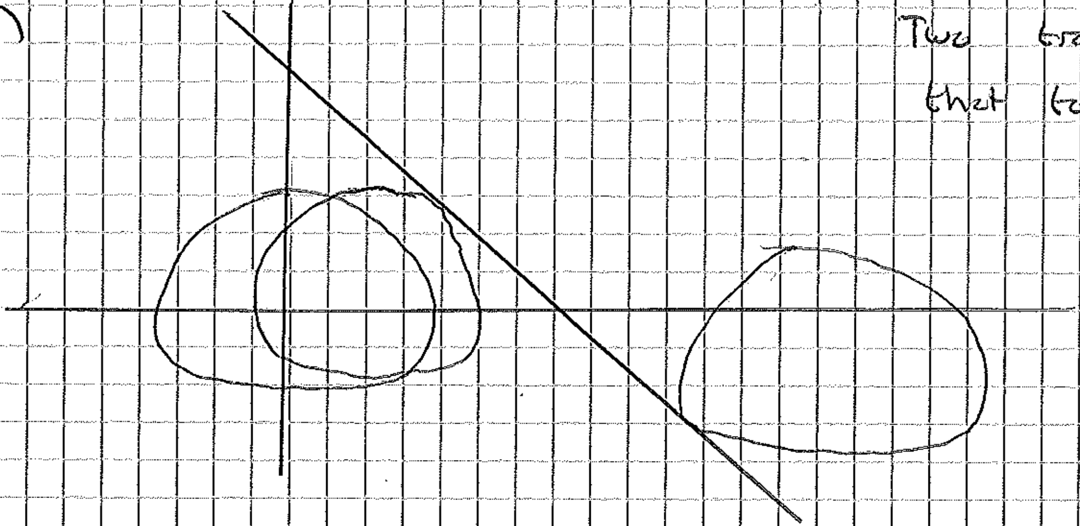
$\rightarrow \; (k - 2)^2 - 3 = 0$

$(k - 2)^2 = 3$

$k - 2 = \pm\sqrt{3}$

$\rightarrow \; k = 2 \pm \sqrt{3}$

e)



Two translated curves that touch line once!