Further Pure 1 - January 2007

① a) i) $x^2 + 16 = 0 \rightarrow x^2 = -16$

$x = \pm\sqrt{16} \rightarrow x = \pm 4i$

ii) $x^2 - 2x + 17 = 0$

$\rightarrow (x-1)^2 - 1 + 17 = 0 \rightarrow (x-1)^2 = -16$

$\rightarrow x - 1 = \pm 4i$

$\rightarrow x = 1 \pm 4i$

b) i) $(1+x)^3 = 1 + 3x + 3x^2 + x^3$

i) $= 1 + 3i + 3i^2 + i^3$

$= 1 + 3i - 3 - i \rightarrow -2 + 2i$

ii) $(1+i)^3 + 2(1+i) - 4i$

$= -2 + 2i + 2 + 2i - 4i$

$= 4i - 4i = 0$

② a) i) $A + B = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} + \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix}$

$= \begin{bmatrix} \sqrt{3} & 0 \\ 1 & 0 \end{bmatrix}$

ii) $BA \quad \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/2 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

b) i) $\theta = 30^\circ \rightarrow$ rotation 30° anti-cw about $(0,0)$

ii) $2\theta = 30^\circ \rightarrow$ reflection in line $y = (\tan 15)x$

iii) Reflection in x-axis

(3) a) $\alpha + \beta = -4/2 = -2 \qquad \alpha\beta = 3/2$

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-2)^2 - 2(3/2) = 4 - 3 = 1$

c) $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$= (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$

$= 1 - 2(3/2)^2 = 1 - 9/2 = -7/2$

(4) a) $y = ax^b$

$\log_{10} y = \log_{10}(ax^b)$

$\rightarrow \log_{10} y = \log_{10}(a) + b\log_{10}(x)$

b) Gradient $= -1/2 = b$

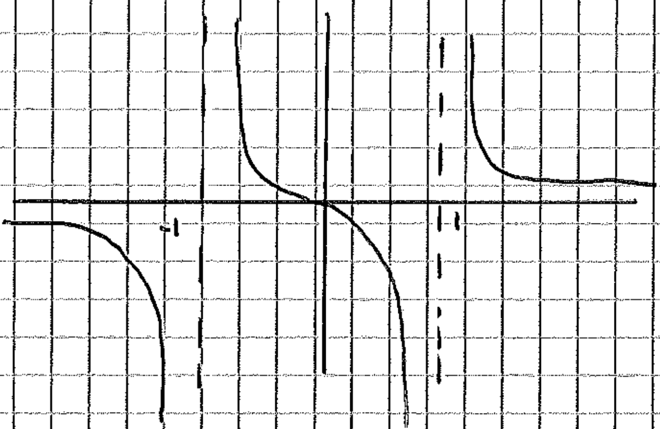
y intercept $= 1 \rightarrow \log_{10}(a) = 1$

$\rightarrow a = 10^1 = 10$

(5) a) Denominator $= 0$ when $x = 1$ or $-1 \rightarrow x = 1,\ x = -1$

As $x \rightarrow \infty$, $y \rightarrow 0/1 \rightarrow y = 0$

b) when $x = 0$, $y = 0$



c) From graph, curve lies above the x-axis when:

$-1 < x < 0$ and $x > 1$

(6) a) i) $(2r-1)^2 = 4r^2 - 4r + 1$

ii) $\sum(2r-1)^2 = 4\sum r^2 - 4\sum r + \sum 1$ $\quad \boxed{\sum 1 = n}$

$= \cancel{4\cdot\tfrac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n}$

$= \cancel{n[\ldots]}$

$= \cancel{1/3\, n(n+1)[2(2n+1) - 6] + 1}$

$= \frac{2}{3} n(n+1)(2n+1) - 2n(n+1) + n$

$= \frac{1}{3} n \left[2(n+1)(2n+1) - 6(n+1) + 3 \right]$

$= \frac{1}{3} n \left[2(2n^2 + 3n + 1) - 6n - 6 + 3 \right]$

$= \frac{1}{3} n \left[4n^2 + 6n + 2 - 6n - 6 + 3 \right]$

$= \frac{1}{3} n \left[4n^2 - 1 \right]$

b) Odd numbers are written in form $2n - 1$

if $101 = 2n - 1 \rightarrow n = 51$

if $199 = 2n - 1 \rightarrow n = 100$

$\therefore$ we need $\sum_{1}^{100} (2r-1)^2 - \sum_{1}^{50} (2r-1)^2$

$= \frac{1}{3}(100)(4 \times 100^2 - 1) - \frac{1}{3}(50)(4 \times 50^2 - 1)$

$= 1333300 - 166650$

$= 1,166,650$

(7) a) G.S. $\theta = 2n\pi + a, \quad \theta = 2n\pi + (\pi - a)$

Key angle $(a) = \sin^{-1}(0) = 0$

$\rightarrow x + \pi/6 = 2n\pi, \quad x + \pi/6 = 2n\pi + \pi$

$\rightarrow x = 2n\pi - \pi/6, \quad x = 2n\pi + 5\pi/6$

b) i) $g(0.05) = \frac{1}{2} + \frac{\sqrt{3}}{2}(0.05) - \frac{1}{4}(0.05)^2$

$= 0.54268... = 0.5427$ (4dp)

$f(0.05) = \sin(0.05 + \pi/6)$

$= 0.54266... = 0.5427$ (4dp)

ii) $g(h) = \frac{1}{2} + \frac{\sqrt{3}}{2}(h) - \frac{1}{4}(h)^2, \quad g(0) = \frac{1}{2}$

gradient $= \frac{g(h) - g(0)}{h} = \frac{\frac{1}{2} + \frac{\sqrt{3}}{2}(h) - \frac{1}{4}(h)^2 - \frac{1}{2}}{h}$

$= \frac{\sqrt{3}}{2} - \frac{1}{4}h$

iii) As $h \to 0$, gradient $\to \sqrt{3}/2$

(8) a) $x = 10 \to \frac{100}{25} - \frac{y^2}{9} = 1$

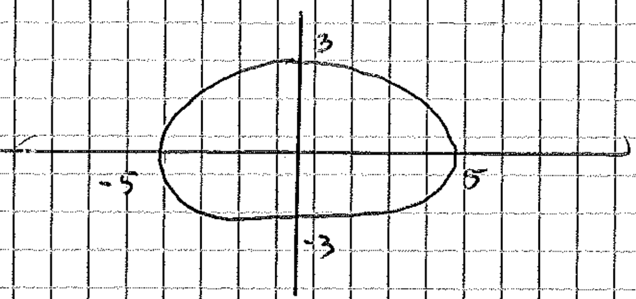
$= 4 - y^2/9 = 1$

$\to 36 - y^2 = 9$

$\to y^2 = 27 \to y = \pm\sqrt{27} = \pm 3\sqrt{3}$

b) when $x = 0$, $y^2/9 = 1 \to y = \pm 3$

when $y = 0$, $x^2/25 = 1 \to x = \pm 5$



c) tangent $x = 5$

d) i) $y = x - 4 \to \frac{x^2}{25} - \frac{(x-4)^2}{9} = 1$

$x^2 - \frac{25(x-4)^2}{9} = 25$

$\to 9x^2 - 25(x-4)^2 = 225$

$\to 9x^2 - 25[x^2 - 8x + 16] = 225$

$\to 9x^2 - 25x^2 + 200x - 400 = 225$

$\to -16x^2 + 200x - 625 = 0$

$\to 16x^2 - 200x + 625 = 0$

ii) $(4x - 25)(4x - 25) = 0$

$x = 25/4$

Equal roots, $\therefore$ the line is a tangent to the curve