

## Further Pure 1 - January 2006

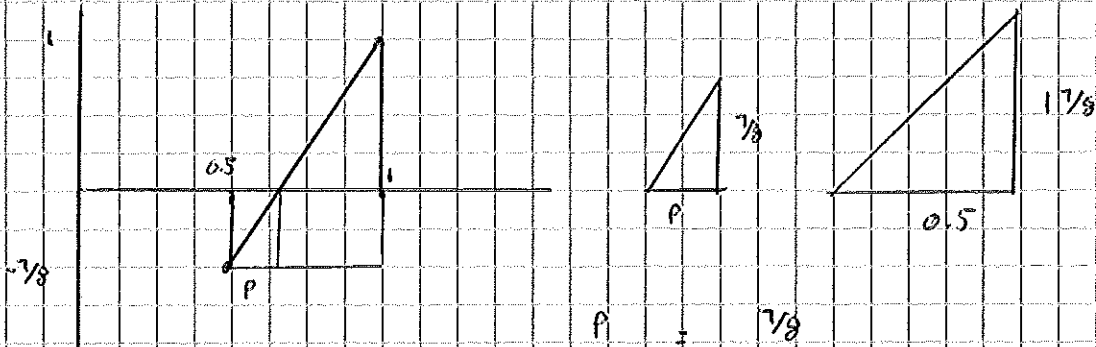
(1) a) Let  $f(x) = x^3 + 2x - 2$

$$f(0.5) = 0.5^3 + 2(0.5) - 2 = -\frac{7}{8}$$

$$f(1) = 1^3 + 2(1) - 2 = 1$$

Sign change,  $\therefore$  root lies between 0.5 and 1

b)



$$\frac{p}{0.5} = \frac{7/8}{17/8}$$

$$\Rightarrow p = \frac{0.5 \cdot 7/8}{17/8} = \frac{7}{30}$$

$$\therefore \text{root approximation} = 0.5 + p = 0.5 + \frac{7}{30} = \frac{11}{15} \text{ or } 0.7\bar{3}$$

(2) a)  $\int_0^9 \frac{1}{\sqrt{x}} = \int_0^9 x^{-1/2} = \left[ 2x^{1/2} \right]_0^9$

$$= 2\sqrt{9} - 2\sqrt{0}$$

$$= 6 - 0$$

As  $n \rightarrow 0$ ,  $2\sqrt{n} \rightarrow 0$ , so  $\int \rightarrow 6$

b)  $\int_0^9 \frac{1}{x\sqrt{x}} = \int_0^9 x^{-3/2} = \left[ -2x^{-1/2} \right]_0^9$

$$= -\frac{2}{\sqrt{9}} - \frac{-2}{\sqrt{n}}$$

$$= -\frac{2}{3} + \frac{2}{\sqrt{n}}$$

as  $n \rightarrow 0$ ,  $\frac{2}{\sqrt{n}}$  does not converge,  $\therefore \int$  has no value

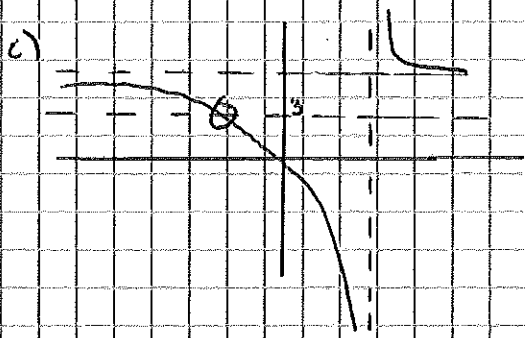
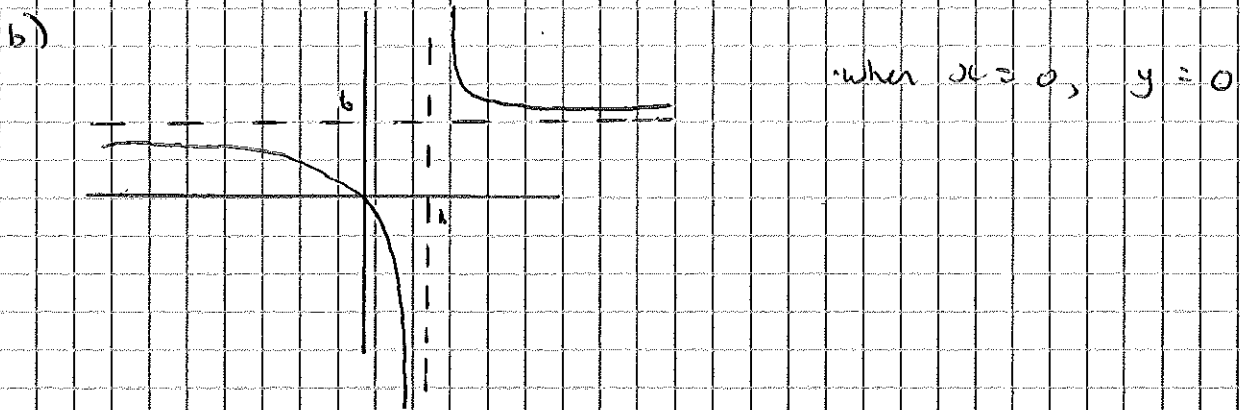
③ General solution for sine:  $\theta = 360n + a$ ,  $\theta = 360n + (180 - a)$

Key angle:  $\sin^{-1}(\sin(50)) = 50^\circ$

General solution:

$$\left. \begin{aligned} 40x + 10 &= 360n + 50 \\ \rightarrow 40x &= 360n + 40 \\ \rightarrow \boxed{xc} &= \boxed{90n + 10} \end{aligned} \right\} \begin{aligned} 40x + 10 &= 360n + (180 - 50) \\ \rightarrow 40x + 10 &= 360n + 130 \\ \rightarrow 40x &= 360n + 120 \\ \rightarrow \boxed{xc} &= \boxed{90n + 30} \end{aligned}$$

- ④ a) As  $xc \rightarrow \infty$ ,  $y \rightarrow 6/1 \therefore y = 6$  is asymptote  
 If  $xc = 1$ , denominator = 0  $\therefore xc = 1$  is asymptote



Find intersection:

$$\frac{6xc}{xc-1} = 3$$

$$\rightarrow 6xc = 3xc - 3$$

$$\rightarrow 4 = 3xc \rightarrow xc = \frac{4}{3}$$

$$\rightarrow 3x = -3 \rightarrow x = -1$$

From diagram, curve  $< 3$  when:  $-1 < xc < 1$

⑤ a) i)  $(2 + i\sqrt{5})(\sqrt{5} - i) = 2\sqrt{5} - 2i + 5i + \sqrt{5}$   $-i^2\sqrt{5} = \sqrt{5}$

$$= 3\sqrt{5} + 3i$$

ii) Let  $z = \sqrt{5} - i$  and  $z^* = \sqrt{5} + i$

$$(2 + i\sqrt{5})(\sqrt{5} - i) = 3\sqrt{5} + 3i$$

$$= 3(\sqrt{5} + i) = 3z^*$$

$\therefore \sqrt{5} - i$  is a root

b) i)  $\sqrt{5} + i$  (complex conjugate)

ii)  $\boxed{\text{Sum}}$   $\sqrt{5} - i + \sqrt{5} + i = 2\sqrt{5}$

$\boxed{\text{Product}}$   $(\sqrt{5} - i)(\sqrt{5} + i) = 5 - -1 = 6$

iii)  $x^2 - \boxed{\text{Sum}}x + \boxed{\text{Product}} = 0$

$\rightarrow p = -2\sqrt{5}, \quad q = 6$

6) a) see sheet

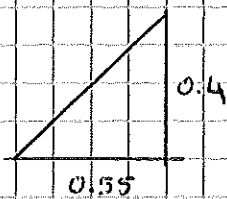
b)  $y = kx^n$

$\log_{10}(y) = \log_{10}(k) + n \log_{10}(x)$

$\rightarrow Y = a x + b$   
 $\uparrow \qquad \qquad \qquad \uparrow$   
 $n \qquad \qquad \qquad \log_{10}(k)$

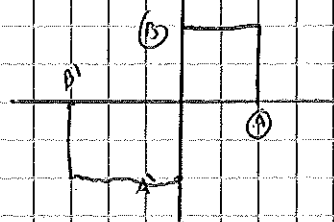
c) see sheet

d)  $n = \text{gradient}$



$= 0.4/0.55 = 0.72$

7) a) ii)



Reflection in  $y = -x$

ii)  $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

iii) A reflection in  $y = -x$  followed by another

reflection in  $y = -x$  returns the object to the starting position.

b)

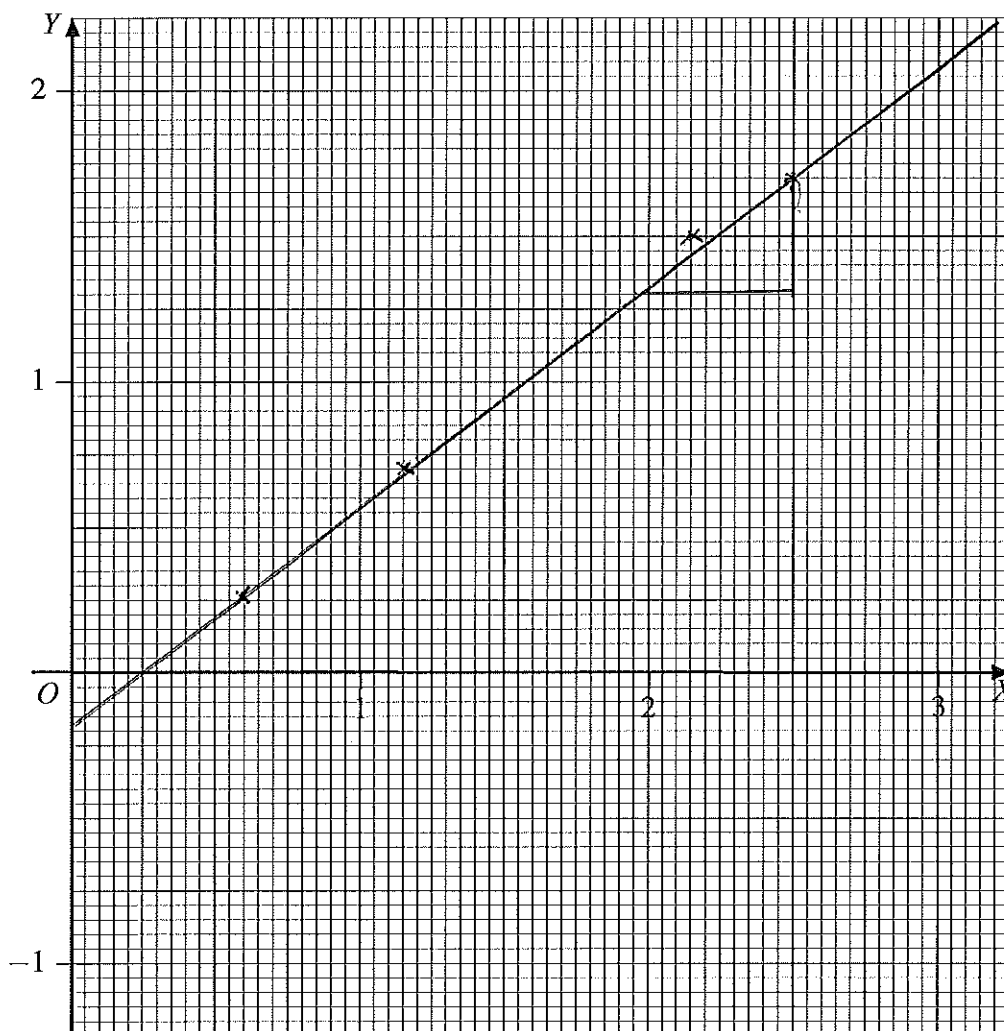
$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Figure 1 (for use in Question 6)

$X$	0.60	1.23	2.18	2.48
$Y$	0.26	0.70	1.48	1.70

Figure 2 (for use in Question 6)



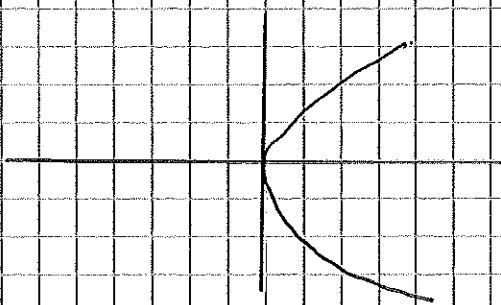
$$B^2 - A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\text{ii) } (B+A) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$(B-A) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$(B+A)(B-A) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

8) a)  $y^2 = 12x$



b) i)  $(y-2)^2 = 12x$

ii)  $x$  and  $y$  swap

$$\rightarrow x^2 = 12y$$

c) i)  $y = x + c \rightarrow (x+c)^2 = 12x$

$$\rightarrow x^2 + 2xc + c^2 = 12x$$

$$\rightarrow x^2 + 2xc - 12x + c^2 = 0$$

$$\rightarrow x^2 + (2c-12)x + c^2 = 0$$

ii) For tangent,  $b^2 - 4ac = 0$

$$\rightarrow (2c-12)^2 - 4 \times 1 \times c^2 = 0$$

$$\rightarrow 4c^2 - 48c + 144 - 4c^2 = 0$$

$$\rightarrow -48c + 144 = 0$$

$$\rightarrow c = \frac{144}{48} = 3$$

$$\text{iii)} \quad x^2 + (2c - 12)x + c^2 = 0$$

$$c = 3$$

$$\rightarrow x^2 + (6 - 12)x + 3^2 = 0$$

$$\rightarrow x^2 - 6x + 9 = 0$$

$$\rightarrow (x - 3)(x - 3) = 0$$

$$\hookrightarrow x = 3$$

$$y = x + c \rightarrow y = 3 + 3 = 6$$

$$\therefore \text{co-ordinates} = (3, 6)$$

$$\text{iv)} \quad \text{if } c = 4$$

$$\rightarrow x^2 + (8 - 12)x + 4^2 = 0$$

$$\rightarrow x^2 - 4x + 16 = 0$$

check discriminant:

$$b^2 - 4ac$$

$$\rightarrow (-4)^2 - 4 \times 1 \times 16 = -48$$

Discriminant is negative

$\therefore$  No real solutions

$\therefore$  Line does NOT intersect the curve