FP1 - June 2005

① a) i) $\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$

ii)

$$\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 8 & 6 \\ 6 & 8 \end{bmatrix}$$

b) $\begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix} = -5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

② $y_{n+1} \approx y_n + h f(x_n)$

$x_1 = 0.5$

$y_1 = 1$

$f(x_1) = \sin(2 \times 0.5) = 0.8414\ldots$

$h = 0.1$

$y_2 \approx 1 + 0.1 \times 0.8414$

$= 1.08414\ldots$

$x_2 = 0.6$

$y_2 = 1.0841\ldots$

$f(x_2) = \sin(2 \times 0.6) = 0.9320\ldots$

$y_3 \approx 1.0841 + 0.1 \times 0.932$

$= 1.1773\ldots$

$= 1.18$ (3sf)

③ a) $\sum r^2(r-1) = \sum r^3 - \sum r^2$

$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$

$= \frac{1}{12}n(n+1)\left[3(n)(n+1) - 2(2n+1)\right]$

$= \frac{1}{12}n(n+1)\left[3n^2 + 3n - 4n - 2\right]$

$= \frac{1}{12}n(n+1)\left[3n^2 - n - 2\right]$

$= \frac{1}{12}n(n+1)(3n+2)(n-1)$

$= \frac{1}{12}n(n+1)(n-1)(3n+2)$

$= \frac{1}{12}n(n^2-1)(3n+2)$

Diff of 2 squares

b) $\sum_{4}^{11} r^2(r-1) = \sum_{1}^{11} r^2(r-1) - \sum_{1}^{3} r^2(r-1)$

$= 1/12\,(11)(11^2-1)(33+2) - 1/12\,(3)(3^2-1)(9+2)$

$= 3850 - 22 = 3828$

(4) a) $f(x) = x^3 + x$

$f(2+h) = (2+h)^3 + (2+h)$

$= 8 + ~~4~~12h + 6h^2 + h^3 + 2 + h$

$= h^3 + 6h^2 + 13h + 10$

$f(2) = 2^3 + 2 = 10$

$\therefore f(2+h) - f(2) = h^3 + 6h^2 + 13h + 10 - 10$

$= h^3 + 6h^2 + 13h$

OR/ $13h + 6h^2 + h^3$

b) Gradient $= \dfrac{13h + 6h^2 + h^3}{h} = 13 + 6h + h^3$

As $h \to 0$, gradient $\to 13$

(5) a) Tan General Solution: $\theta = n\pi + a$

Key angle : $\tan^{-1}(\sqrt{3}) = \pi/3$

$\Rightarrow 3x = n\pi + \pi/3$

$\Rightarrow x = n\pi/3 + \pi/9$

b) Key angle : $\tan^{-1}(-\sqrt{3}) = -\pi/3$

$\Rightarrow 3x - \pi/3 = n\pi - \pi/3$

$\Rightarrow 3x = n\pi$

$\Rightarrow x = n\pi/3$

(6) a) i) $\alpha + \beta = 4$, $\alpha\beta = 13$

ii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 4^2 - 2(13) = -10$

iii) The square of a real number is ≥ 0

$\therefore$ The sum of squares of real numbers is ≥ 0

b) i) $(\alpha + i) + (\beta + i) = \alpha + \beta + 2i$

$= 4 + 2i$

ii) $(\alpha + i)(\beta + i) = \alpha\beta + \alpha i + \beta i + i^2$

$= \alpha\beta + i(\alpha + \beta) - 1$

$= 13 + 4i - 1 = 12 + 4i$

c) sum $= 4 + 2i$

Product $= 12 + 4i$

$x^2 - \boxed{\text{sum}}\, x + \boxed{\text{product}} = 0$

$\Rightarrow x^2 - (4+2i)x + (12 + 4i) = 0$

7 a) i)

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 4 & 8 & 10 \\ 0 & -4 & -2 \end{bmatrix}$$

D = (4, 0)

E = (8, -4)

F = (10, -2)

ii) See sheet

b) i)

$$\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$$

From diagram:

CB = 1

$FE = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\therefore$ Enlargement SF $= 2\sqrt{2}$

ii) $2\sqrt{2} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} \cos(45) & \sin(45) \\ -\sin(45) & \cos(45) \end{bmatrix}$

= Rotation 45° clockwise

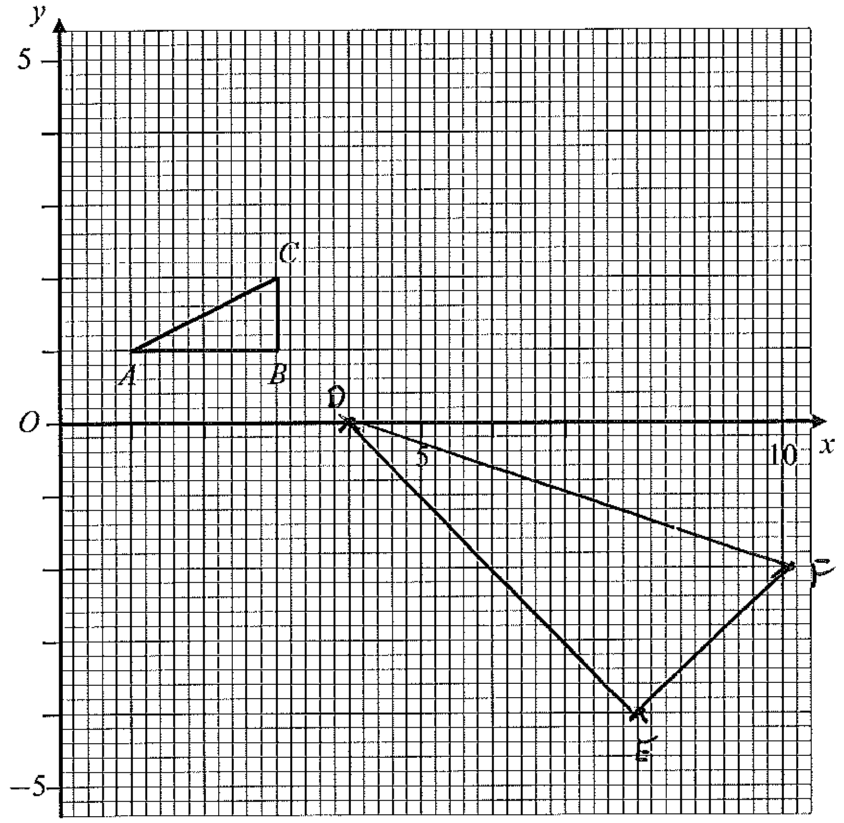

Figure 1 (for use in Question 7)

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(8) a) At P, $y = 0 \Rightarrow x = 2 \Rightarrow (2, 0)$

b) Find equation of chord:

$x_1 = 2$ $\qquad$ $y - y_1 = m(x - x_1)$

$y_1 = 0$ $\qquad$ $y = 2(x - 2)$

$m = 2$ $\qquad$ $\Rightarrow y = 2x - 4$

Use to find x:

$$\frac{x^2}{4} - \frac{(2x - 4)^2}{6} = 1$$

$(\times 12)$ $\quad 3x^2 - 2(2x - 4)^2 = 12$

$3x^2 - 2[4x^2 - 16x + 16] = 12$

$3x^2 - 8x^2 + 32x - 32 = 12$

$0 = 5x^2 - 32x + 44$

$(5x - 22)(x - 2) = 0$

$x = 2 \quad (P)$

$5x - 22 = 0$

$\Rightarrow x = 22/5 \quad (Q)$

$y = 2x - 4 \Rightarrow 2(22/5) - 4 = 24/5$

$\therefore Q = (4.4, 4.8)$

(9) a) i) As $x \to \infty$, $f(x) \to 1/1 \quad \therefore y = 1$ is asymptote

ii) Denominator needs to $= 0$

$x^2 + 9 = 0$ has no real solutions

b) Let $f(x) = k \Rightarrow k = \dfrac{x^2 + 4x}{x^2 + 9}$

$\Rightarrow k(x^2 + 9) = x^2 + 4x$

$\Rightarrow kx^2 + 9k = x^2 + 4x$

$\Rightarrow kx^2 - x^2 - 4x + 9k = 0$

$\Rightarrow (k - 1)x^2 - 4x + 9k = 0$

For equal roots, $b^2 - 4ac = 0$

$\rightarrow (-4)^2 - 4 \times (k-1) \times 9k = 0$

$\rightarrow 16 - 36k(k-1) = 0$

$\rightarrow 16 - 36k^2 + 36k = 0$

$\rightarrow 36k^2 - 36k - 16 = 0$

$\rightarrow 9k^2 - 9k - 4 = 0$

c) Solve: $9k^2 - 9k - 4 = 0$

$\rightarrow (3k+1)(3k-4) = 0$

$3k + 1 = 0$

$\rightarrow k = -1/3$

$\rightarrow y = -1/3$

$\boxed{\text{Find } x}$

$(k-1)x^2 - 4x + 9k = 0$

$-4/3\,x^2 - 4x - 3 = 0$

$-4x^2 - 12x - 9 = 0$

$4x^2 + 12x + 9 = 0$

$(2x+3)(2x+3) = 0$

$2x + 3 = 0$

$\rightarrow x = -3/2$

Co-ordinate of SP $= (-3/2, -1/3)$

$3k - 4 = 0$

$\rightarrow k = 4/3$

$\rightarrow y = 4/3$

$\boxed{\text{Find } x}$

$(k-1)x^2 - 4x + 9k = 0$

$1/3\,x^2 - 4x + 12 = 0$

$x^2 - 12x + 36 = 0$

$(x-6)(x-6) = 0$

$x = 6$

Co-ordinates of SP $= (6, 4/3)$