Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA
General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MD01

Unit Decision 1

Friday 25 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

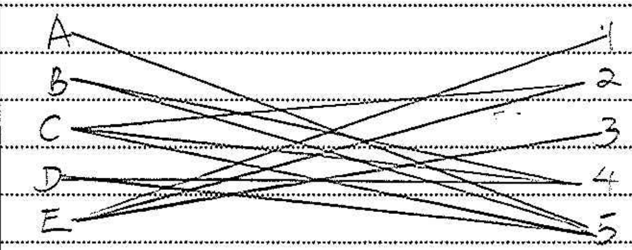
1 (a) Draw a bipartite graph to represent the following adjacency matrix.

	1	2	3	4	5
A	0	0	0	0	1
B	0	0	0	1	1
C	0	1	0	1	1
D	0	0	0	1	1
E	1	1	1	0	1

(2 marks)

(b) If A, B, C, D and E represent five people and 1, 2, 3, 4 and 5 represent five tasks to which they are to be assigned, explain why a complete matching is impossible. *(2 marks)*

Answer space for question 1

1.
(a)


(b) One person cannot do 2 tasks
example only E can do tasks
1 and 3 which is impossible

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QUESTION PART REFERENCE

Answer space for question 1

Turn over ▶

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2 (a) Use a Shell sort to arrange the following numbers into ascending order.

7 8 1 6 3 4 5 2 *(4 marks)*

(b) Write down the number of comparisons on the first pass. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

n=8. 4 sublists.

(a) 7 7 3 C1
8 8 4 C2
1 1 5 C3
6 6 2 C4.
3
4
5
2

This becomes
3 7
4 8
5
2 6

3 becomes 3 1
4 2 4 2
1 sublists 1 3
2 2 → 4
7 7 5
8 8 6
5 5 7
6 6 8

1 2 3 4 5 6 ~~7~~ 6 8
1 2 3 4 5 6 7 8

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QUESTION PART REFERENCE

Answer space for question 2

(b) 4 comparisons.

Turn over ▶

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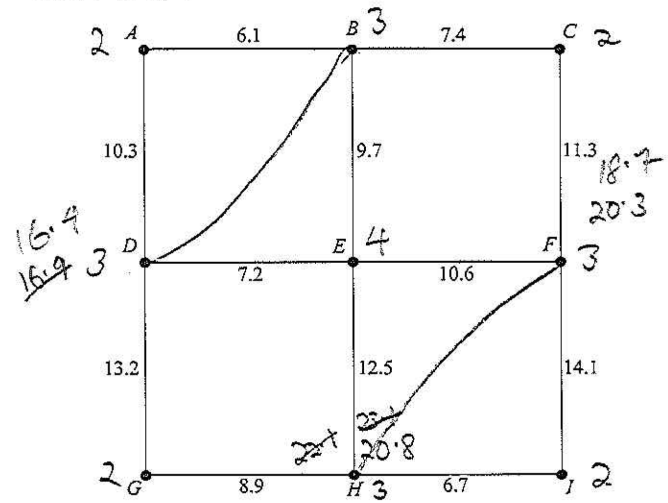
3 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.

A delivery man lives in village A and is to drive along all the roads at least once before returning to A.



Total length of all the roads is 118 miles

(a) Find the length of an optimal Chinese postman route around the nine villages, starting and finishing at A. *(5 marks)*

(b) For an optimal Chinese postman route corresponding to your answer in part (a), state:

(i) the number of times village E would be visited;

(ii) the number of times village I would be visited. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

(a) Odds → B, D, F, H

BD + FH = 16.4 + 20.8 = 37.2

BF + DH = 18.7 + 19.7 = 38.4

BH + DF = 22.2 + 17.8 = 40

Optimum length = 37.2.

Optimal Chinese route = 118 + 37.2

length = 155.4 miles

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

(b)

(i) E twice

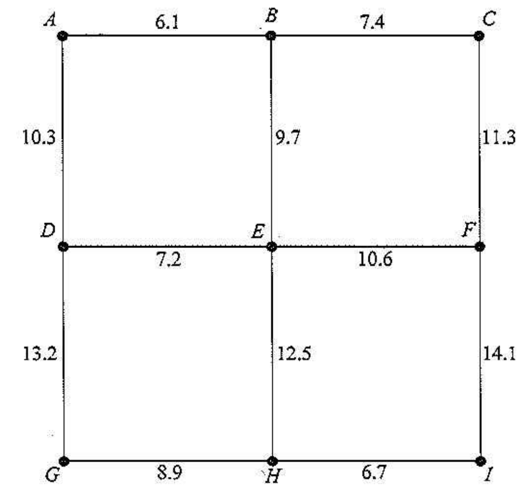
(ii) I twice

Turn over ▶

Do not write outside the box

4 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.

A programme of resurfacing some roads is undertaken to ensure that each village can access all other villages along a resurfaced road, while keeping the amount of road to be resurfaced to a minimum.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network.

(ii) State the length of your minimum spanning tree.

(iii) Draw your minimum spanning tree. *(7 marks)*

(b) Given that Prim's algorithm is used with different start vertices, state the final edge to be added to the minimum spanning tree if:

(i) the start vertex is E;

(ii) the start vertex is G. *(2 marks)*

(c) Given that Kruskal's algorithm is used to find the minimum spanning tree, state which edge would be:

(i) the first to be included in the tree;

(ii) the last to be included in the tree. *(2 marks)*

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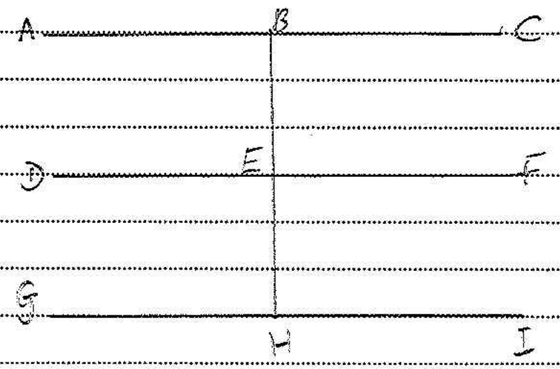
QUESTION PART REFERENCE | **Answer space for question 4**

(a)
(i)

AB	6.1
BC	7.4
BE	9.7
DE	7.2
EF	10.6
EH	12.5
HI	6.7
GH	8.9

(ii) (length) $= 6.4 + 7.4 + 9.7 + 7.2 + 10.6 + 12.5 + 6.7 + 8.9 = 69.1$

(iii)



Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE | **Answer space for question 4**

(b)
(i) GH
(ii) EF

(c)
(i) 1st AB
(ii) last EH

AB Kruskal's
HI
DE
BC
GH
BE
EF
EH

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

5 The feasible region of a linear programming problem is defined by

$$x + y \leqslant 60$$
$$2x + y \leqslant 80$$
$$y \geqslant 20$$
$$x \geqslant 15$$
$$y \geqslant x$$

(a) On the grid opposite, draw a suitable diagram to represent these inequalities and indicate the feasible region. *(5 marks)*

(b) In each of the following cases, use your diagram to find the maximum value of P on the feasible region. In each case, state the corresponding values of x and y.

(i) $P = x + 4y$ *(2 marks)*

(ii) $P = 4x + y$ *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

(a) $2x + y \leq 80$

x	0	10	40
y	80	60	0

$x + y \leq 60$

x	0	20	50
y	60	40	10

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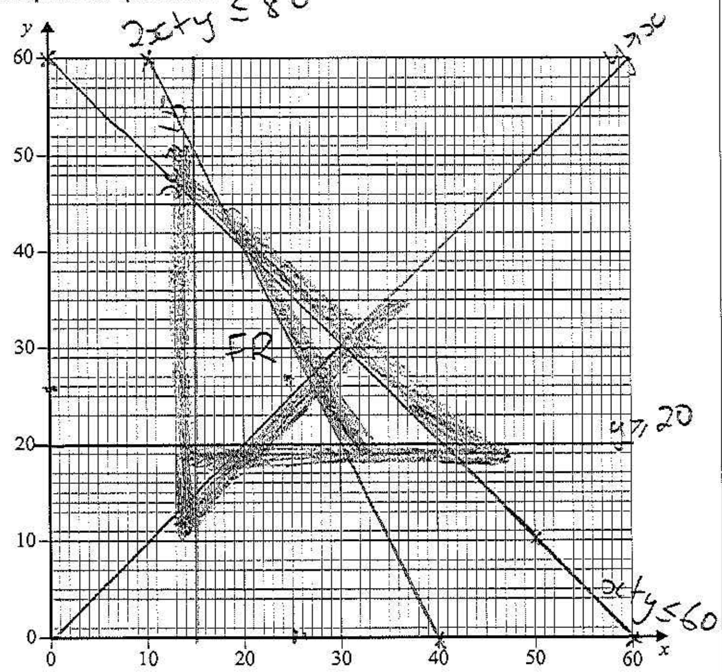
QUESTION PART REFERENCE

Answer space for question 5

(a)



(b)
(i) $P = x + 4y$
Maximum at $(15, 45)$
$P = 15 + (4 \times 45) = 195$

(ii) $P = 4x + y$ $(x = 26, y = 26)$
$P = (4 \times 26) + 26$
$P = 130$

Turn over ▶

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6 The network opposite shows some roads connecting towns. The number on each edge represents the length, in miles, of the road connecting a pair of towns.

(a) (i) Use Dijkstra's algorithm on the network to find the minimum distance from A to J.

(ii) Write down the corresponding route. *(7 marks)*

(b) The road AJ is a dual carriageway. Ken drives at 60 miles per hour on this road and drives at 50 miles per hour on all other roads.

Find the minimum time to travel from A to J. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

(a)
(ii) Route A B C F I J

(b)
(i) A - J = 30 using Dijkstra's

$\frac{30 \times 60}{50} = \frac{1800}{50} = 36$ minutes.

Direct A - J = $\frac{35 \times 60}{60}$ = 35 minutes

Minimum time is 35 minutes from A to J.

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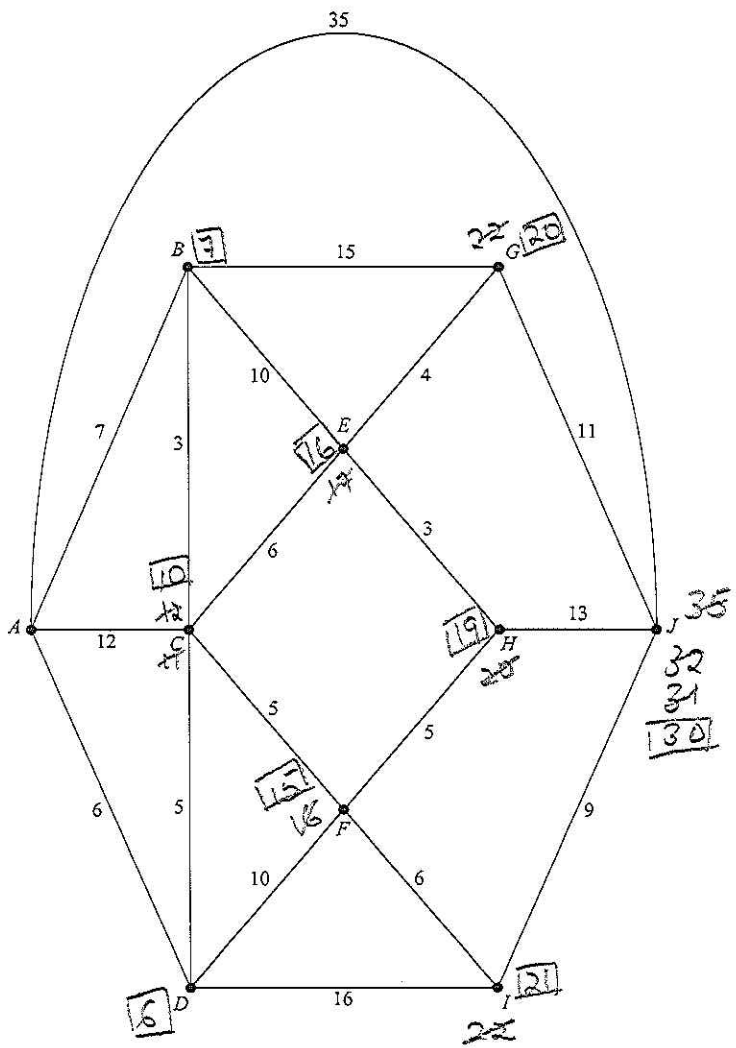
QUESTION PART REFERENCE

Answer space for question 6

(a)



Turn over ▶

Do not write outside the box

7 (a) A simple connected graph X has eight vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(b) A simple connected graph Y has n vertices.

(i) State the minimum number of edges of the graph.

(ii) Find the maximum number of edges of the graph. *(2 marks)*

(c) A simple graph Z has six vertices and each of the vertices has the same degree d.

(i) State the possible values of d.

(ii) If Z is connected, state the possible values of d.

(iii) If Z is Eulerian, state the possible values of d. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 7

7

(a)(i) edges = (vertices − 1) = 8 − 1 = 7

(ii) maximum edges = $\frac{n(n-1)}{2} = \frac{8 \times 7}{2}$

= 28

(b)

(i) $n-1$

(ii) $\frac{n(n-1)}{2}$ = maximum number of edges

c

(i) $d = 0, 1, 2, 3, 4, 5$

(ii) $d = 2, 3, 4, 5$

(iii) $d = 2, 4$ (even number of vertices)

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QUESTION PART REFERENCE | **Answer space for question 7**

Turn over ▶

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8 Tony delivers paper to five offices, A, B, C, D and E. Tony starts his deliveries at office E and travels to each of the other offices once, before returning to office E. Tony wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the offices.

	2	3	5	4	1
	A	B	C	D	E
A	–	10	16	20	8
B	10	–	21	15	9
C	16	21	–	10	23
D	20	15	10	–	17
E	8	9	23	17	–

(a) Find the travelling time of the tour $ACDBEA$. *(1 mark)*

(b) Hence write down a tour, starting at E, which has the same total travelling time as your answer to part **(a)**. *(1 mark)*

(c) Use the nearest neighbour algorithm, starting at E, to find an upper bound for the minimum travelling time for Tony's tour. *(4 marks)*

(d) By deleting E, find a lower bound for the minimum travelling time for Tony's tour. *(4 marks)*

(e) Sketch a network showing the edges that give the lower bound in part **(d)**, and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE | **Answer space for question 8**

(a) A C D B E A

16 + 10 + 15 + 9 + 8 = 58

(b) E A C D B E

(c) E A B D C E

8 + 10 + 15 + 10 + 23 = 66

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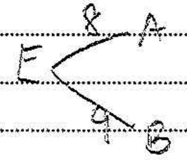
QUESTION PART REFERENCE

Answer space for question 8

19.



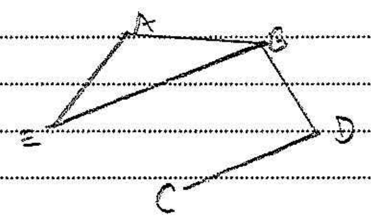
	A^1	B^2	C^4	D^3
A	–	10	16	20
B	10	–	21	15
C	16	21		10
D	20	15	10	

ABDC

$10 + 15 + 10 = 35$

lower bound $= 35 + 17$
$= 52$

(e)



doesn't give a tour (travels to all vertices and returns to the start

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Turn over ▶

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9 A factory can make three different kinds of balloon pack: gold, silver and bronze. Each pack contains three different types of balloon: A, B and C.

Each gold pack has 2 type A balloons, 3 type B balloons and 6 type C balloons.

Each silver pack has 3 type A balloons, 4 type B balloons and 2 type C balloons.

Each bronze pack has 5 type A balloons, 3 type B balloons and 2 type C balloons.

Every hour, the maximum number of each type of balloon available is 400 type A, 400 type B and 400 type C.

Every hour, the factory must pack at least 1000 balloons.

Every hour, the factory must pack more type A balloons than type B balloons.

Every hour, the factory must ensure that no more than 40% of the total balloons packed are type C balloons.

Every hour, the factory makes x gold, y silver and z bronze packs.

Formulate the above situation as 6 inequalities, in addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, simplifying your answers. *(8 marks)*

QUESTION PART REFERENCE

Answer space for question 9

$2x + 3y + 5z \leq 400$

$3x + 4y + 3z \leq 400$

$6x + 2y + 2z \leq 400$ same as

$3x + y + z \leq 200$ (halving).

$(2x + 3x + 6x) + (3y + 4y + 2y) + (5z + 3z + 2z) \geq 1000$

$11x + 9y + 10z \geq 1000$

more type A than type B is

$2x + 3y + 5z > 3x + 4y + 3z$

$(-2x \ -3y \ -3z) \quad (-2x \ -3y \ -3z)$

$2z > x + y$

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QUESTION PART REFERENCE

Answer space for question 9

$6x + 2y + 2z \le \frac{40}{100}(11x + 9y + 10z)$

$600x + 200y + 200z \le 440x + 360y + 400z$

$-440x \quad -200y \quad -200z \qquad -440x - 200y - 200z$

$160x \le 160y + 200z$

$(\div 40)$

$4x \le 4y + 5z$

Turn over ▶

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QUESTION PART REFERENCE

Answer space for question 9

END OF QUESTIONS