Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MD01

Unit Decision 1

Monday 24 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

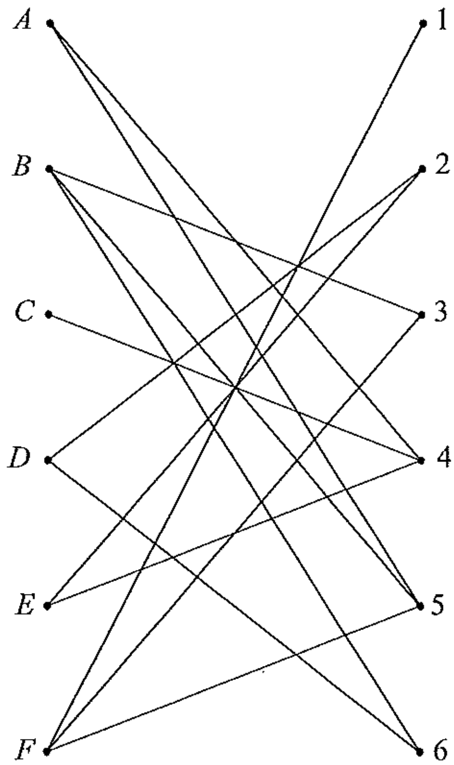
Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, B is assigned to task 5, D to task 2, E to task 4 and F to task 3.

Demonstrate, by using an algorithm from this initial matrix, how each person can be allocated to a task. *(5 marks)*

QUESTION PART REFERENCE

1(a)

	1	2	3	4	5	6
A	0	0	0	1	1	0
B	0	0	1	0	1	1
C	0	0	0	1	0	0
D	0	1	0	0	0	1
E	0	1	0	1	0	0
F	1	0	1	0	1	0

Do not write outside the box

QUESTION PART REFERENCE

(b) Initial Match:

B + 5

D + 2

E + 4

F + 3

Start with A:

A − 4 + E − 2 + D − 6

A + 4 E + 2 D + 6

New Match:

A + 4

B + 5 → unchanged from before

D + 6

E + 2

F + 3 → also unchanged

C − 4 + A − 5 + B − 3 + F − 1

C + 4 A + 5 B + 3 F + 1

Final Match: A5 B3 C4 D6 E2 F1

Do not write outside the box

2 A student is using a quicksort algorithm to rearrange a set of numbers into ascending order. She uses the first number in each list (or sublist) as the pivot.

Her correct solution for the first three passes is as follows.

Initial list	10	7	4	22	13	16	19	5
After 1st pass	7	4	5	10	22	13	16	19
After 2nd pass	4	5	7	10	13	16	19	22
After 3rd pass	4	5	7	10	13	16	19	22

(a) State the pivots used for the 2nd pass. *(2 marks)*

(b) Write down the number of comparisons on each of the three passes. *(3 marks)*

(c) Explain whether the student has completed the algorithm. *(1 mark)*

QUESTION PART REFERENCE

(a) 7 and 22

(b) 1st Pass → 10 is compared with all other numbers

10 So $\boxed{7}$

7

4

22

13

16

19

5

Do not write outside the box

QUESTION PART REFERENCE

2nd Pass = $\boxed{5}$ comparisons | 3rd Pass = $\boxed{3}$

7 4 and 5 compared with 7 = $\boxed{2}$
4
5

10

22 13, 16, 19 compared with 22 = $\boxed{3}$
13
16
19

4 4 and 5 compared = $\boxed{1}$
5
7
10
13 16, 19 compared with 13 = $\boxed{2}$
16
19
22

So : 1st = 7 2nd = 5 3rd = 3

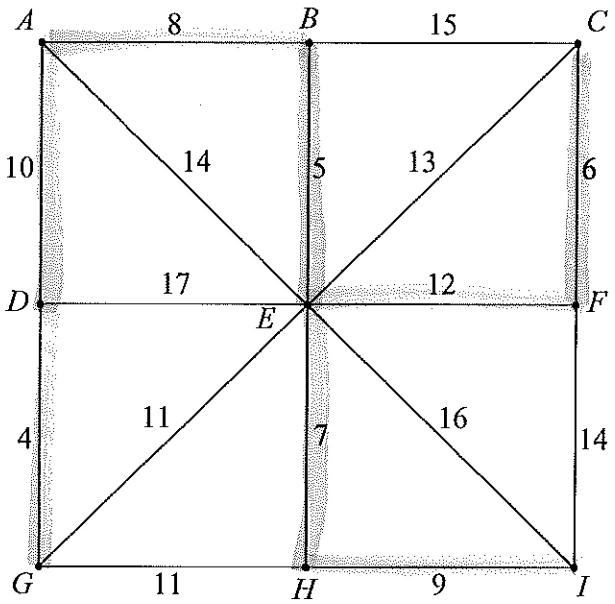
(c) After 3rd Pass :

4
5
7
10
13
16
19
22

← Not finished as 16 and 19 have not been compared yet.

Turn over ▶

3 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.



(a) (i) Use Prim's algorithm starting from E, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

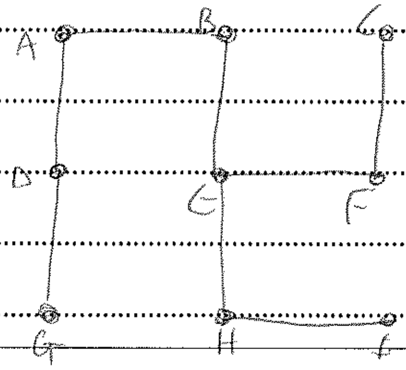
(b) On a particular day, village B is cut off, so its connecting roads cannot be used.

Find the length of a minimum spanning tree for the remaining eight villages. *(2 marks)*

QUESTION PART REFERENCE

a(i) EB (5), EH (7), AB (8), HI (9),
AD (10), DG (4), EF (12) CF (6)

(ii) Total = 61 (iii)



Do not write outside the box

QUESTION PART REFERENCE

(b) Obviously, you need to delete AB and BE (total 13)

The shortest edge which connects A, D, G to the others is 11 (EG or GH)

so 61 − 13 + 11 = 59

Do not write outside the box

4 The network below shows some paths on an estate. The number on each edge represents the time taken, in minutes, to walk along a path.

(a) (i) Use Dijkstra's algorithm on the network to find the minimum walking time from A to J. *(6 marks)*

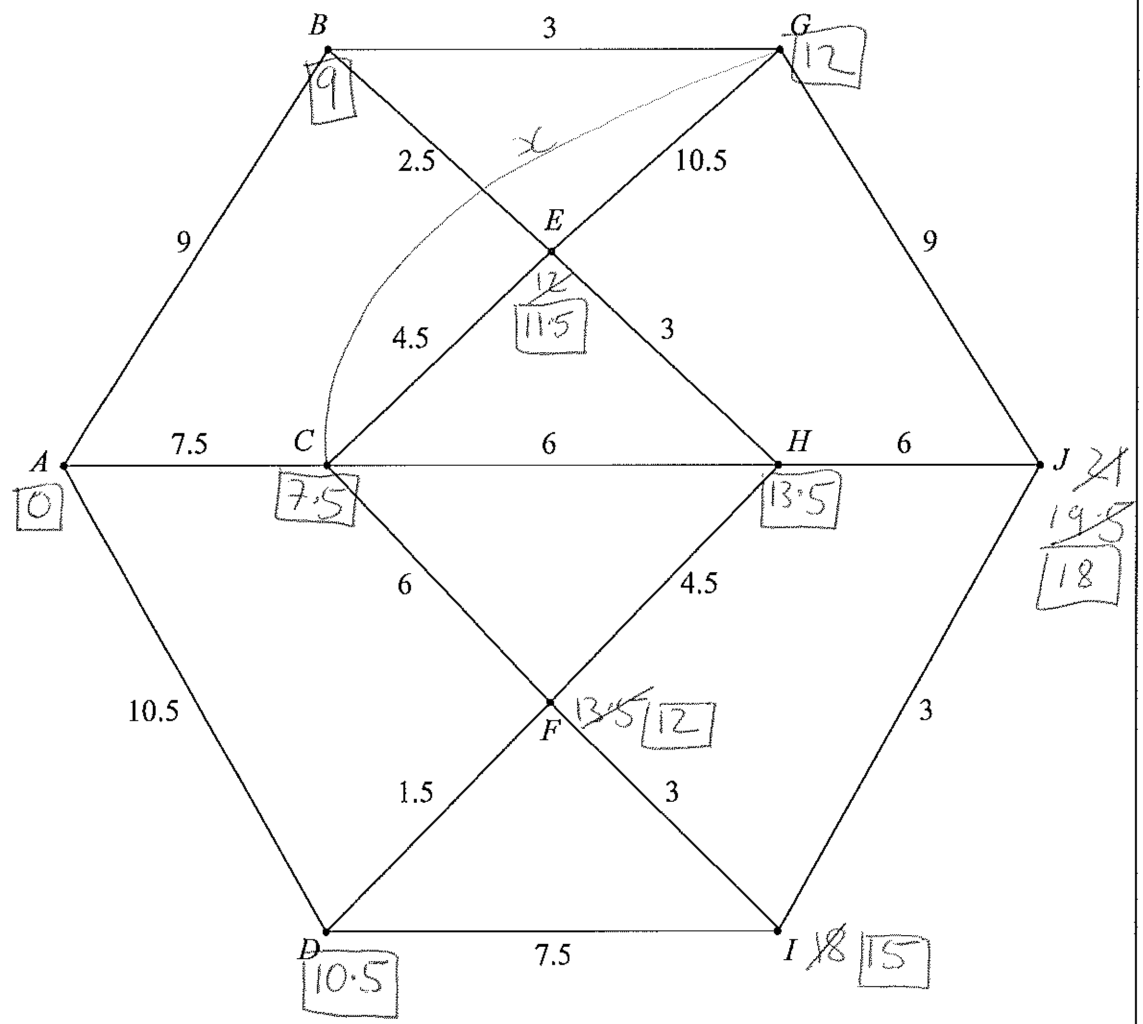
(ii) Write down the corresponding route. *(1 mark)*

(b) A new subway is constructed connecting C to G directly. The time taken to walk along this subway is x minutes. The minimum time taken to walk from A to G is now reduced, but the minimum time taken to walk from A to J is not reduced.

Find the range of possible values for x. *(3 marks)*

QUESTION PART REFERENCE

(a)(i)



Do not write outside the box

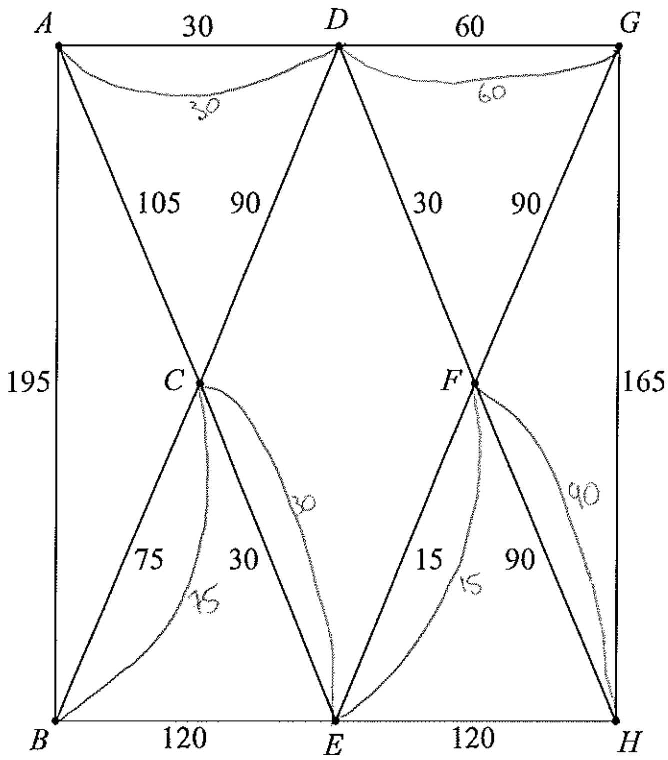
QUESTION PART REFERENCE	
(a)(ii)	ADFIJ
(b)	If CG makes A→G shorter then ACG must be <u>less</u> than 12
	so $7.5 + x < 12 \Rightarrow \boxed{x < 4.5}$
	Since CG does not shorten A→J, we know that ACGJ must be equal to or greater than 18.
	so $7.5 + x + 9 \geq 18$
	$16.5 + x \geq 18$
	$\boxed{x \geq 1.5}$
	so $1.5 \leq x < 4.5$

Turn over ▶

Do not write outside the box

5 Norris delivers newspapers to houses on an estate. The network shows the streets on the estate. The number on each edge shows the length of the street, in metres.

Norris starts from the newsagents located at vertex A, and he must walk along all the streets at least once before returning to the newsagents.



The total length of the streets is 1215 metres.

(a) Give a reason why it is not possible to start at A, walk along each street once only, and return to A. *(1 mark)*

(b) Find the length of an optimal Chinese postman route around the estate, starting and finishing at A. *(5 marks)*

(c) For an optimal Chinese postman route, state:

(i) the number of times that the vertex F would occur; *(1 mark)*

(ii) the number of times that the vertex H would occur. *(1 mark)*

Do not write outside the box

QUESTION PART REFERENCE

5

(a) There are some vertices with odd order (A B G H) so the graph is not Eulerian.

(b) Odd vertices: A B G H

AB = 180 (ACB)
AG = 90 (ADG)
AH = 150 (ADFH)
BG = 210 (ACEFG)
BH = 210 (ACEFH)
GH = 165

AB + GH = 180 + 165 = 345
AG + BH = 90 + 210 = 300
AH + BG = 150 + 210 = 360

Repeat AG and BH. Length = 1215 + 300
= 1515 metres

(c)(i) Repeated edges drawn on diagram
F: $\frac{6}{2} = 3$ (ii) $\frac{4}{2} = 2$

Number of edges divided by 2.

Turn over ▶

Do not write outside the box

6 (a) The complete graph K_n has every one of its n vertices connected to each of the other vertices by a single edge.

(i) Find the total number of edges in the graph K_5. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for the graph K_5. *(1 mark)*

(iii) State the number of edges in a Hamiltonian cycle for the graph K_5. *(1 mark)*

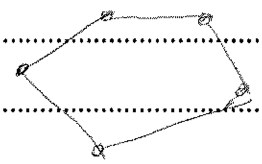
(b) A simple graph G has six vertices and nine edges, and G is Eulerian. Draw a sketch to show a possible graph G. *(2 marks)*

QUESTION PART REFERENCE

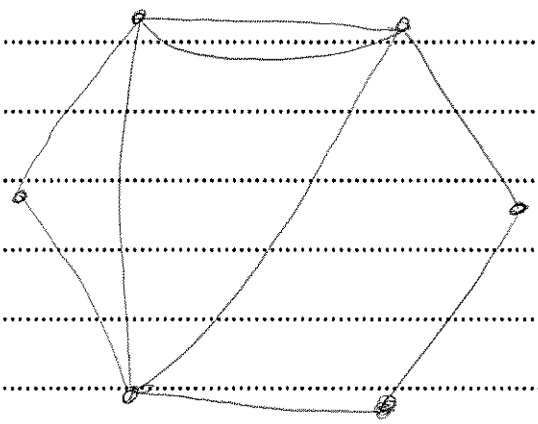
6(a)(i) $4+3+2+1 = 10$

(ii) 4 (1 less than no. of vertices)

(iii) 5.



(b) Eulerian → all orders have to be even



Do not write outside the box

7 Fred delivers bread to five shops, A, B, C, D and E. Fred starts his deliveries at shop B, and travels to each of the other shops once before returning to shop B. Fred wishes to keep his travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	1	2		4	3
	A	B	C	D	E
A	–	3	11	15	5
B	3	–	18	12	4
C	11	18	–	5	16
D	15	12	5	–	10
E	5	4	16	10	–

(a) Find the travelling time for the tour $BACDEB$. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at B, to find another upper bound for the travelling time for Fred's tour. *(3 marks)*

(c) By deleting C, find a lower bound for the travelling time for Fred's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

(a) $3 + 11 + 5 + 10 + 4 = 33$ minutes

(b) $B \to A \to E \to D \to C \to B$
3 5 10 5 18 $= \underline{41}$

(c) Cross out C in table after choosing smallest 2 : $5 + 11 = 16$.

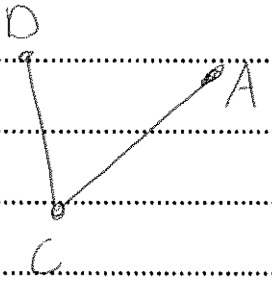
Then use Prims in table. I started from A

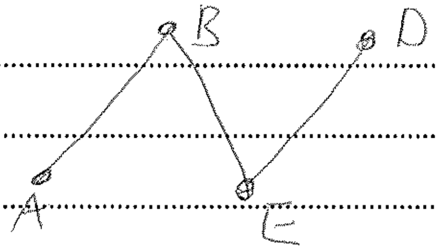
AB BE DE
(3) (4) (10) $= 17$.

so $17 + 16 = \underline{33}$

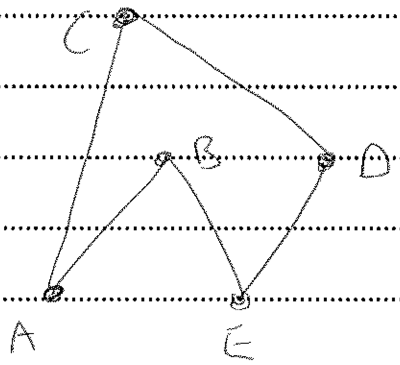
QUESTION PART REFERENCE

(d)





Putting these together:



This is a tour - so 33 is the optimal solution.

8 A student is tracing the following algorithm with positive integer values of A and B.

The function INT gives the integer part of a number, eg $\text{INT}(2.3) = 2$ and $\text{INT}(3.8) = 3$.

Line 10	Let $X = 0$
Line 20	Input A, B
Line 30	If $\text{INT}(A/2) = A/2$ then go to Line 50
Line 40	Let $X = X + B$
Line 50	If $A = 1$ then go to Line 90
Line 60	Let $A = \text{INT}(A/2)$
Line 70	Let $B = 2 \times B$
Line 80	Go to Line 30
Line 90	Print X
Line 100	End

(a) Trace the algorithm in the case where the input values are $A = 20$ and $B = 8$. *(4 marks)*

(b) State the purpose of the algorithm. *(1 mark)*

(c) Another student changed Line 50 to

Line 50 If $A = 1$ then go to Line 80

Explain what would happen if this algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

(ii)		X	A	B
	10	0		
	20		20	8
	30		Int(10) = 10 ✓	
	50		A is not 1	
	60		10	
	70			16
	30		Int(5) = 5 ✓	
	50		A is not 1	
	60		5	
	70			32
	30		Int(2.5) ≠ 2.5	
	40	32		
	50		A is not 1	
	60		2	
	70			64
	30		Int(1) = 1 ✓	
	50		A ≠ 1 ✗	
	60		1	
	70			128
	30		Int(0.5) ≠ 0.5	
	40	160		
	50		A = 1 ✓	
	90	160		

End

X = 160

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

(b) Multiplication

(c) You will end up in a continuous loop and would never reach line 90.

Turn over ▶

9 Herman is packing some hampers. Each day, he packs three types of hamper: basic, standard and luxury.

Each basic hamper has 6 tins, 9 packets and 6 bottles.

Each standard hamper has 9 tins, 6 packets and 12 bottles.

Each luxury hamper has 9 tins, 9 packets and 18 bottles.

Each day, Herman has 600 tins and 600 packets available, and he must use at least 480 bottles.

Each day, Herman packs x basic hampers, y standard hampers and z luxury hampers.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find three inequalities in x, y and z that model the above constraints, simplifying each inequality. *(4 marks)*

(b) On a particular day, Herman packs the same number of standard hampers as luxury hampers.

(i) Show that your answers in part **(a)** become

$$x + 3y \leqslant 100$$

$$3x + 5y \leqslant 200$$

$$x + 5y \geqslant 80$$

(2 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Herman's situation, indicating the feasible region. *(4 marks)*

(iii) Use your diagram to find the maximum total number of hampers that Herman can pack on that day. *(2 marks)*

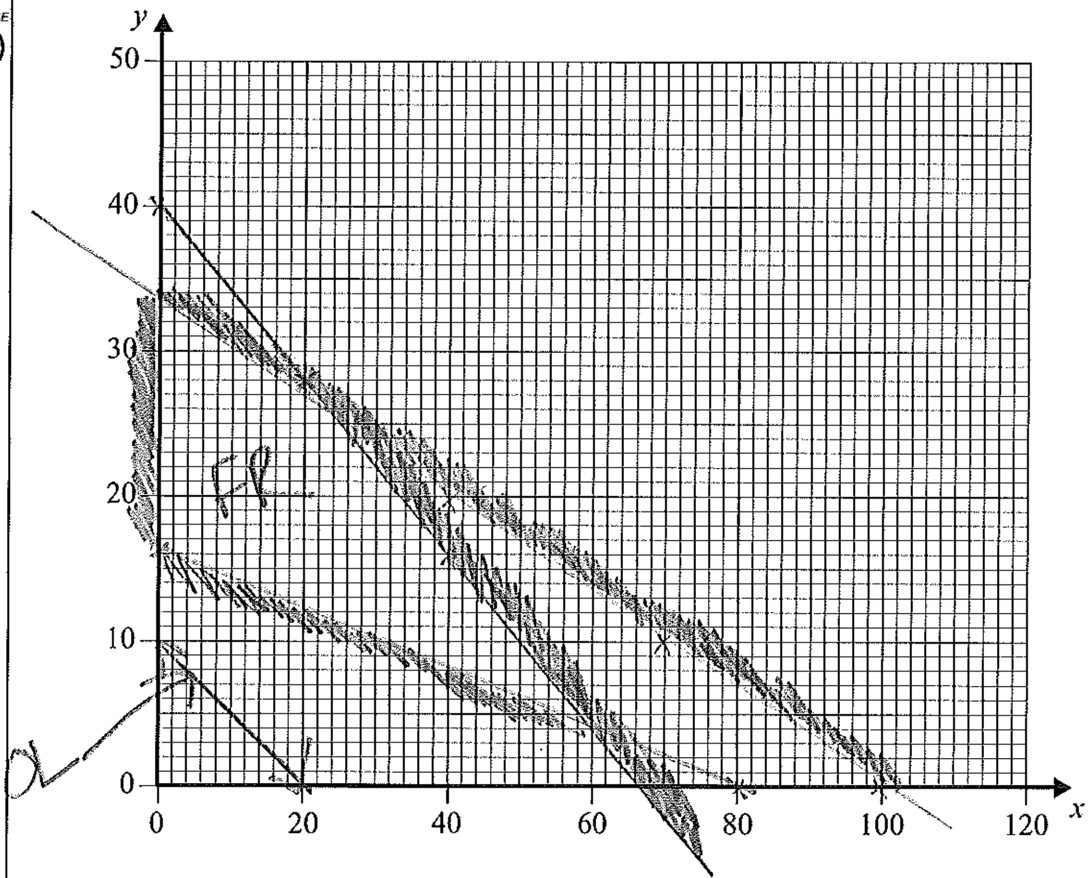
(iv) Find the number of each type of hamper that Herman packs that corresponds to your answer to part **(b)(iii)**. *(1 mark)*

QUESTION PART REFERENCE		
		Simplified:
(a)	$6x + 9y + 9z \leq 600$ (Tins)	① = $2x + 3y + 3z \leq 200$
	$9x + 6y + 9z \leq 600$ (packets)	② = $3x + 2y + 3z \leq 200$
	$6x + 12y + 18z \geq 480$ (bottles)	③ = $x + 2y + 3z \geq 80$

Do not write outside the box

QUESTION PART REFERENCE

(b)(ii)



(b)(i) $y = z$

① $2x + 3y + 3y \leq 200 \Rightarrow 2x + 6y \leq 200$
$\Rightarrow x + 3y \leq 100$

② $3x + 2y + 3y \leq 200 \Rightarrow 3x + 5y \leq 200$

③ $x + 2y + 3y \geq 80 \Rightarrow x + 5y \geq 80$

(ii) See diagram

(iii) Max number: $N = x + y + z = x + 2y$ →PTO

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Objective function $N = x + 2y$

gradient $= -\frac{1}{2}$

(iii) Solution at intercept of

$2x + 6y = 200$ ①

$3x + 5y = 200$ ②

$6x + 18y = 600$

$6x + 10y = 400$

$8y = 200$

$y = 25$

$\Rightarrow x = 25$

So Max number $= 25 + 25 + 25 = 75$

(iv) 25 of each type.

END OF QUESTIONS