Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2009

Mathematics MD01

Unit Decision 1

Specimen paper for examinations in June 2010 onwards

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the space provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 (a) Draw a bipartite graph representing the following adjacency matrix.

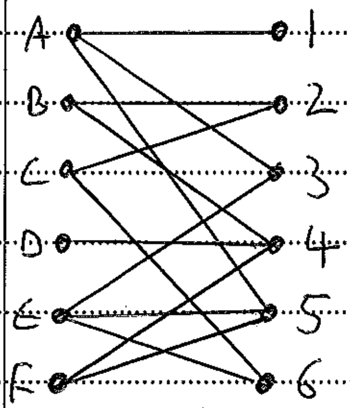
	1	2	3	4	5	6
A	1	0	1	0	1	0
B	0	1	0	1	0	0
C	0	1	0	0	0	1
D	0	0	0	1	0	0
E	0	0	1	0	1	1
F	0	0	0	1	1	0

(2 marks)

(b) Initially, *A* is matched to 3, *B* is matched to 4, *C* is matched to 2, and *E* is matched to 5. Use the maximum matching algorithm, from this initial matching, to find a complete matching. List your complete matching. *(5 marks)*

QUESTION PART REFERENCE

1a



1(b) Initial Match: A3 B4 C2 E5

D−4+B−2+C−6

D+4 B+2 C+6

→

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QUESTION PART REFERENCE

New match: A3 B2 C6 D4 E5

F − 5 + E − 3 + A − 1
F +5 E+3 A + 1

Complete match:

A1 B2 C6 D4 E3 F5

Note: Failed attempts with F can occur e.g.

F − 4 + D not worked

F − 5 + E − 6 + C − 2 + B − 4 + D not worked

No marks lost if you have shown these

Do not write outside the box

2 A student is using a shuttle sort to rearrange a set of numbers into ascending order.

Her correct solution is as follows.

Initial list	5	6	3	9	4	13	1
After 1st pass	5	6	3	9	4	13	1
After 2nd pass	3	5	6	9	4	13	1
After 3rd pass	3	5	6	9	4	13	1
After 4th pass	3	4	5	6	9	13	1
After 5th pass	3	4	5	6	9	13	1
After 6th pass	1	3	4	5	6	9	13

Write down the number of comparisons and swaps on each of the passes. *(6 marks)*

QUESTION PART REFERENCE

2 1st Pass:

Comparisons = 1 Swaps = 0

2nd: C = 2 S = 2

3rd: C = 1 S = 0

4th: C = 4 S = 3

5th: C = 1 S = 0

6th C = 6 S = 6

Note: On 3rd and 5th pass - you only need to do 1 comparison as you did not have to do a swap on those 2 numbers.

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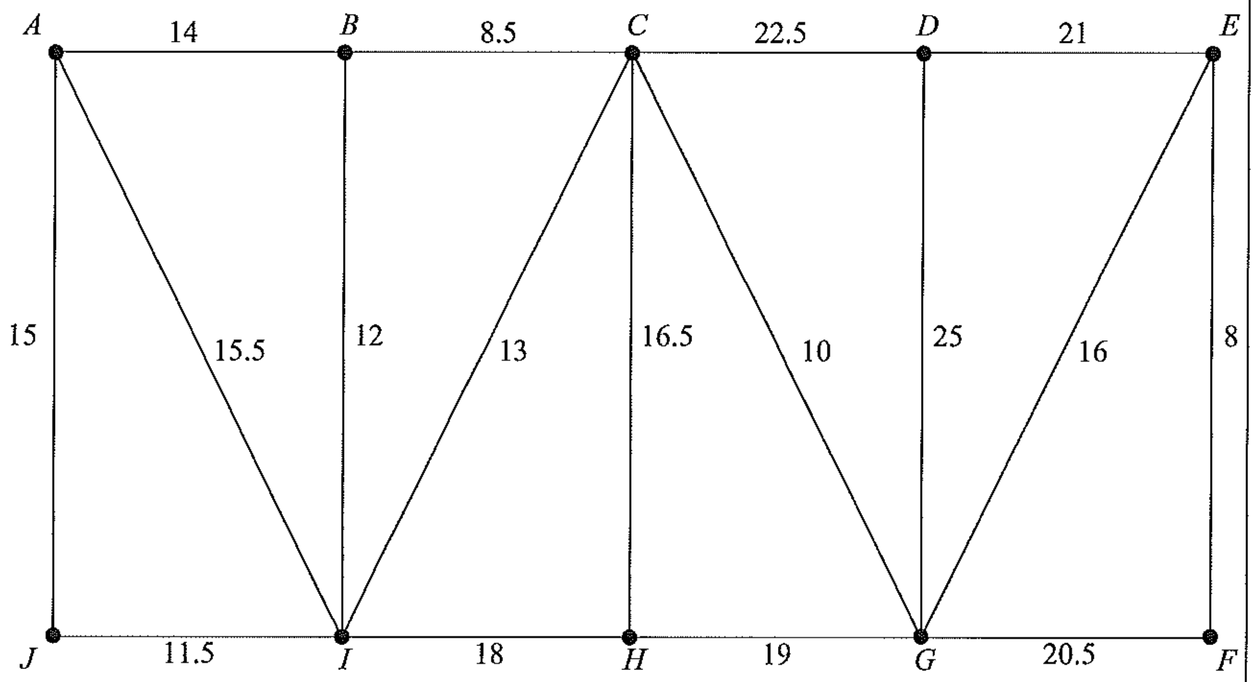
3 (a) (i) State the number of edges in a minimum spanning tree for a network with 10 vertices. *(1 mark)*

(ii) State the number of edges in a minimum spanning tree for a network with n vertices. *(1 mark)*

(b) The following network has 10 vertices: A, B, ..., J. The number on each edge represents the distance between a pair of adjacent vertices.



(i) Use Kruskal's algorithm to find the minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

QUESTION PART REFERENCE

3(a)

(i) 9

(ii) n−1

→

Do not write outside the box

QUESTION PART REFERENCE

b(i) EF (8)

BE (8·5)

CG (10)

IJ (11·5)

BI (12)

AB (14)

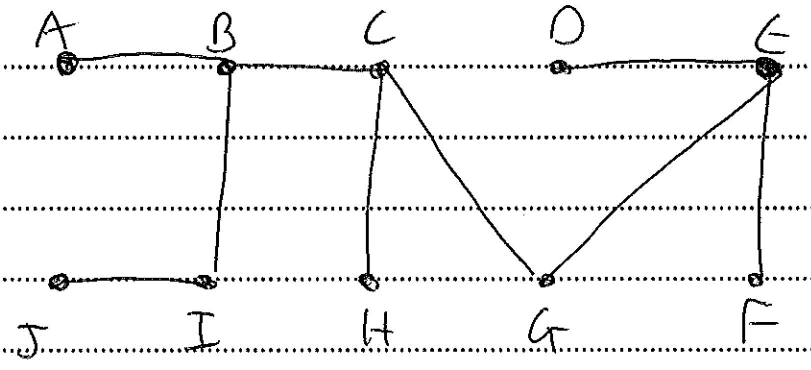
EG (16)

CH (16·5)

DE (21)

(ii) 117·5

(iii)



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4 The diagram opposite shows a network of roads on a housing estate. The number on each edge is the length, in metres, of the road.

Joe is starting a kitchen-fitting business.

(a) Joe delivers leaflets advertising his business. He walks along all of the roads at least once, starting and finishing at C. Find the length of an optimal Chinese postman route for Joe. *(6 marks)*

As C is an odd vertex - it may be easier to do (b) first. (ie. to help find shortest routes).

(b) Joe gets a job fitting a kitchen in a house at T. Joe starts from C and wishes to drive to T. Use Dijkstra's algorithm on the diagram opposite to find the minimum distance to drive from C to T. State the corresponding route. *(7 marks)*

QUESTION PART REFERENCE

4(a) Odd vertices : B C H F

BC = 160

BH = 280 (BINH)

BF = 360 (BINF)

CH = 210

CF = 520 (CABINF)

HF = 320

$\boxed{BC + HF = 160 + 320 = 480}$

BH + CF = 280 + 520 = 800

BF + CH = 360 + 210 = 570

Repeat BC and HF

Length = 2410 + 480 = 2890

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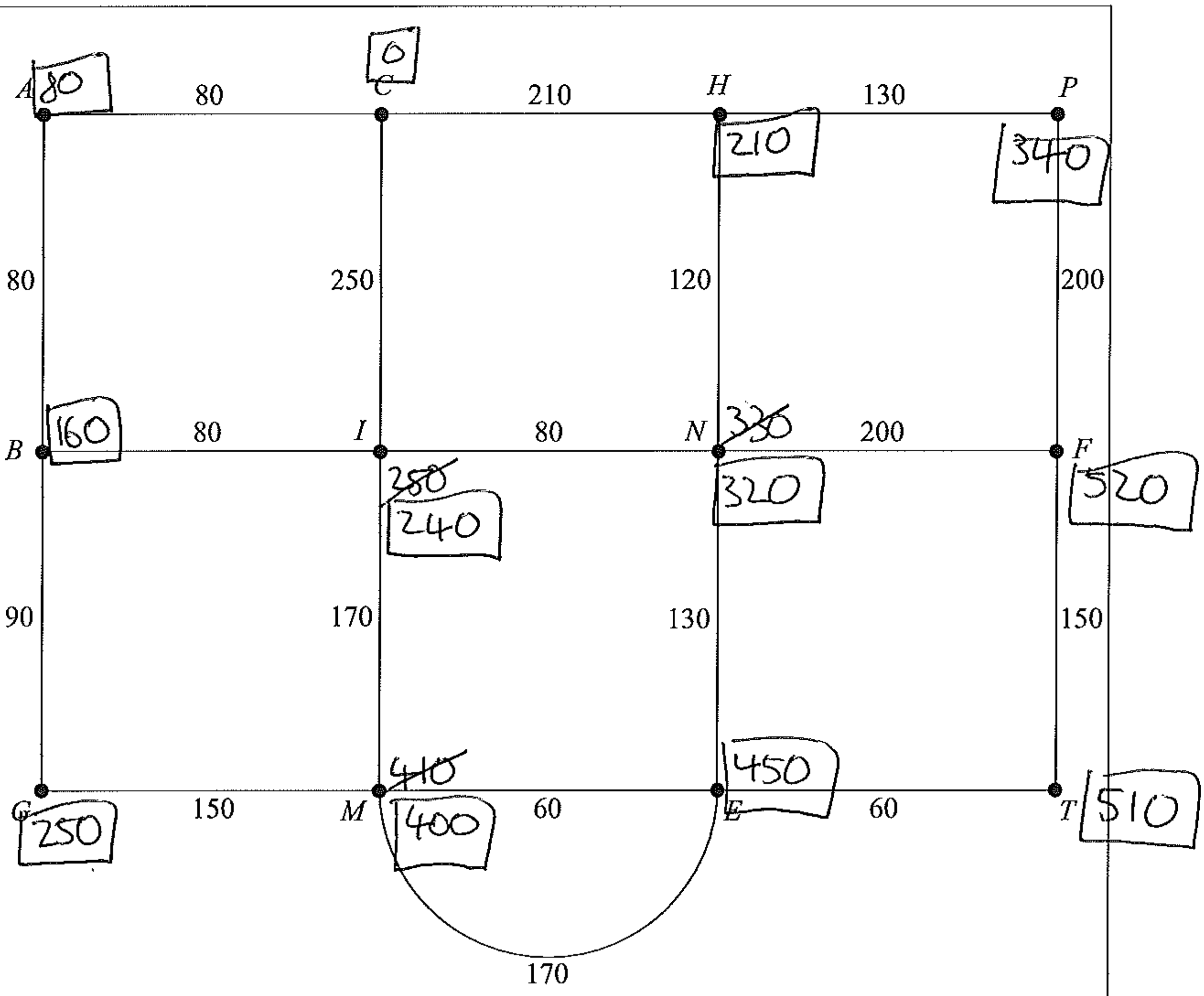
QUESTION PART REFERENCE

(b)



Total length of roads = 2410 metres

(b) See diagram

Route:

CABINET

0 9

Do not write outside the box

5 Angelo is visiting six famous places in Palermo: A, B, C, D, E and F. He intends to travel from one place to the next until he has visited all of the places before returning to his starting place. Due to the traffic system, the time taken to travel between two places may be different dependent on the direction travelled.

The table shows the times, in minutes, taken to travel between the six places.

From \ To	A	B	C	D	E	F
A	–	25	20	20	27	25
B	15	–	10	11	15	30
C	5	30	–	15	20	19
D	20	25	15	–	25	10
E	10	20	7	15	–	15
F	25	35	29	20	30	–

(a) Give an example of a Hamiltonian cycle in this context. *(2 marks)*

(b) (i) Show that, if the nearest neighbour algorithm starting from F is used, the total travelling time for Angelo would be 95 minutes. *(3 marks)*

(ii) Explain why your answer to part **(b)(i)** is an upper bound for the minimum travelling time for Angelo. *(2 marks)*

(c) Angelo starts from F and visits E next. He also visits B before he visits D. Find an improved upper bound for Angelo's total travelling time. *(3 marks)*

QUESTION PART REFERENCE

5(a) ABCDEFA

b(i) $F \to D \to C \to A \to B \to E \to F$

20 15 5 25 15 15 = 95

(ii) Nearest neighbour doesn't necessarily give the best answer, but it does give a Hamiltonian cycle.

Do not write outside the box

QUESTION PART REFERENCE

(c) $F \rightarrow E \rightarrow C \rightarrow A \rightarrow B \rightarrow D \rightarrow F$

30 7 5 25 11 10 $= 88$

B: 2
C: 4

Do not write outside the box

6 Each day, a factory makes three types of widget: basic, standard and luxury. The widgets produced need three different components: type A, type B and type C.

Basic widgets need 6 components of type A, 6 components of type B and 12 components of type C.
Standard widgets need 4 components of type A, 3 components of type B and 18 components of type C.
Luxury widgets need 2 components of type A, 9 components of type B and 6 components of type C.
Each day, there are 240 components of type A available, 300 of type B and 900 of type C.
Each day, the factory must use at least twice as many components of type C as type B.

Each day, the factory makes x basic widgets, y standard widgets and z luxury widgets.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find four inequalities in x, y and z that model the above constraints, simplifying each inequality. *(8 marks)*

(b) Each day, the factory makes the maximum possible number of widgets. On a particular day, the factory must make the same number of luxury widgets as basic widgets.

(i) Show that your answers in part **(a)** become

$$2x + y \leqslant 60, \quad 5x + y \leqslant 100, \quad x + y \leqslant 50, \quad y \geqslant x$$ *(3 marks)*

(ii) Using the axes opposite, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region. *(5 marks)*

(iii) Find the total number of widgets made on that day. *(2 marks)*

(iv) Find all possible combinations of the number of each type of widget made that correspond to this maximum number. *(3 marks)*

QUESTION PART REFERENCE

6 (a)
$x = 6A + 6B + 12C$
$y = 4A + 3B + 18C$
$z = \underbrace{2A}_{\leqslant 240} + \underbrace{9B}_{\leqslant 300} + \underbrace{6C}_{\leqslant 900}$

This step is not necessary, but can make it easier for you

A: $6x + 4y + 2z \leqslant 240$ B: $6x + 3y + 9z \leqslant 300$

C: $12x + 18y + 6z \leqslant 900$

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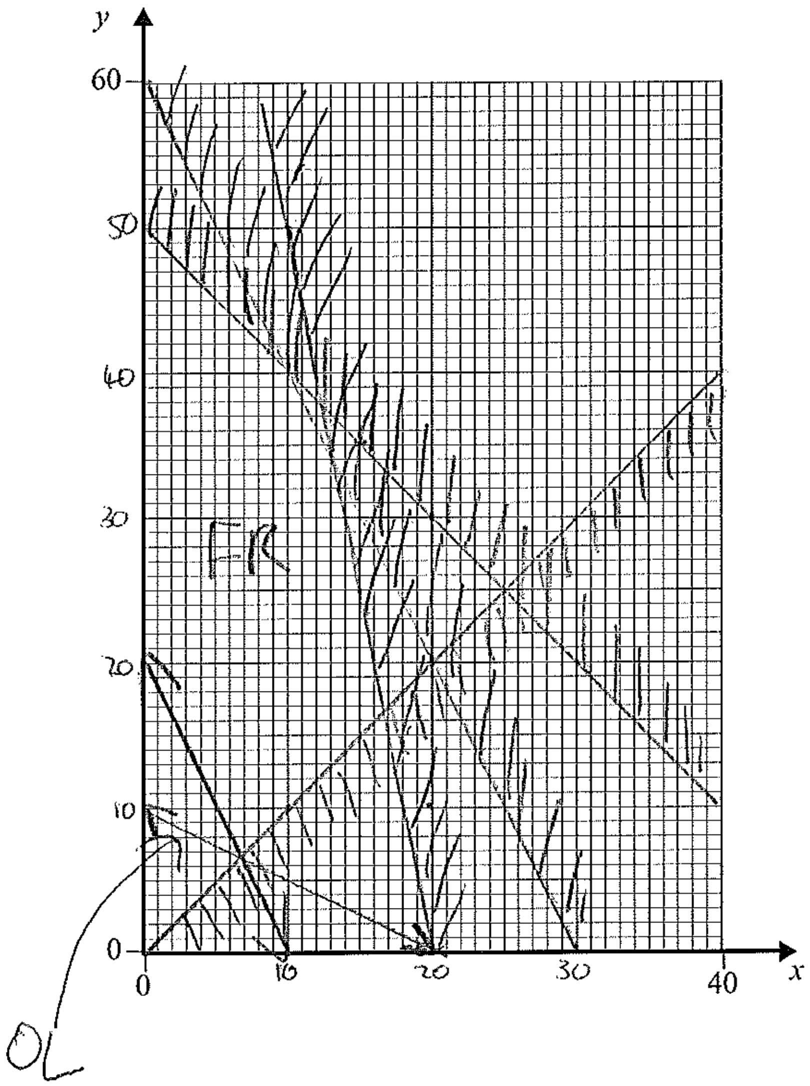
QUESTION PART REFERENCE

(b)



Simplified:

$3x + 2y + z \leq 120$ $2x + y + 3z \leq 100$

$2x + 3y + z \leq 150$

$2B \leq C$: $2(6x + 3y + 9z) \leq 12x + 18y + 6z$

$12x + 6y + 18z \leq 12x + 18y + 6z$

$0 \leq 12y - 12z$

$12z \leq 12y$

$\boxed{z \leq y}$

Do not write outside the box

QUESTION PART REFERENCE

(b)(i) $\boxed{x = z}$

$3x + 2y + x \leq 120$

$4x + 2y \leq 120$

$\boxed{2x + y \leq 60}$

$2x + y + 3x \leq 100$

$\boxed{5x + y \leq 100}$

$2x + 3y + x \leq 150$

$3x + 3y \leq 150$

$\boxed{x + y \leq 50}$

$z \leq y$

$\boxed{x \leq y}$

(ii) See graph on previous page.

$2x + y = 60$

$(30, 0)$ $(20, 20)$ $(0, 60)$

$5x + y \leq 100$

$(20, 0)$ $(10, 50)$ $(15, 25)$

(b)(iii) OL:

Maximise $N = x + y + z = 2x + y$

since $x = z$

$m = \frac{-2}{1} = \frac{-20}{10}$ (gradient)

Do not write outside the box

QUESTION PART REFERENCE

(iv) from objective line – it is not too clear if we should look at (10, 40) or (13, 34)

So: $N = 2x + y$

$y \downarrow$ $x \rightarrow$	10	11	12	13
34	54	56	58	60
36	56	58	60	/
38	58	60	/	/
40	60	/	/	/

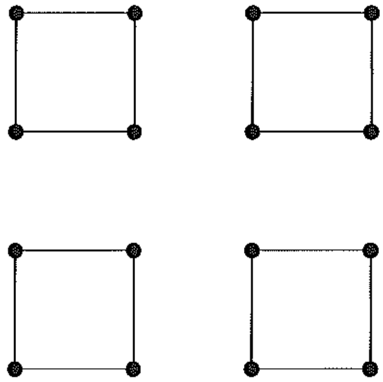
↑ out of FR

x	y	z
10	40	10
11	38	11
12	36	12
13	34	13

Max = 60

Do not write outside the box

7 (a) The diagram shows a graph with 16 vertices and 16 edges.



(i) On **Figure 1** below, add the minimum number of edges to make a connected graph. *(1 mark)*

(ii) On **Figure 2** opposite, add the minimum number of edges to make the graph Hamiltonian. *(2 marks)*

(iii) On **Figure 3** opposite, add the minimum number of edges to make the graph Eulerian. *(2 marks)*

(b) A complete graph has n vertices and is Eulerian.

(i) State the condition that n must satisfy. *(1 mark)*

(ii) The number of edges in a Hamiltonian cycle for the graph is the same as the number of edges in an Eulerian trail. State the value of n. *(2 marks)*

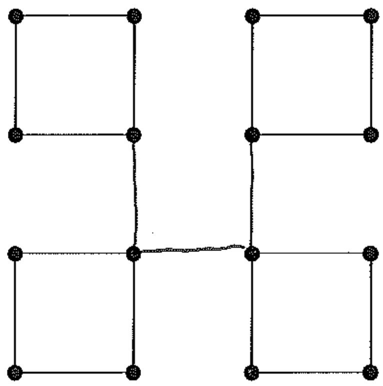
QUESTION PART REFERENCE

(a)(i)

Figure 1

Connected Graph

a(i)



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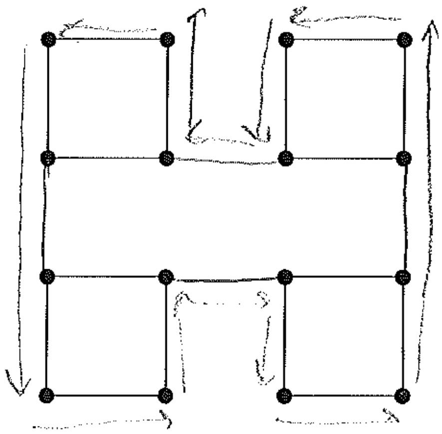
QUESTION PART REFERENCE

(a)(ii)

Figure 2

Hamiltonian Graph

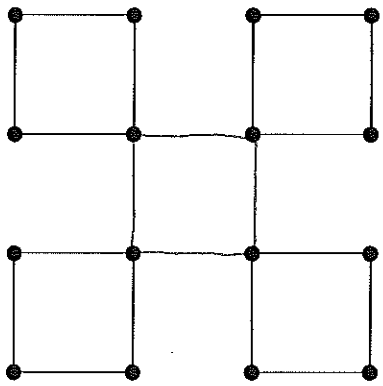


Must have a Hamiltonian cycle (only need to visit each vertex not travel along every edge)

(a)(iii)

Figure 3

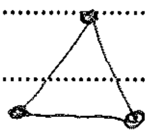
Eulerian Graph



connected and all even orders.

(b)(i) n must be odd

(ii)



$n=3$

END OF QUESTIONS