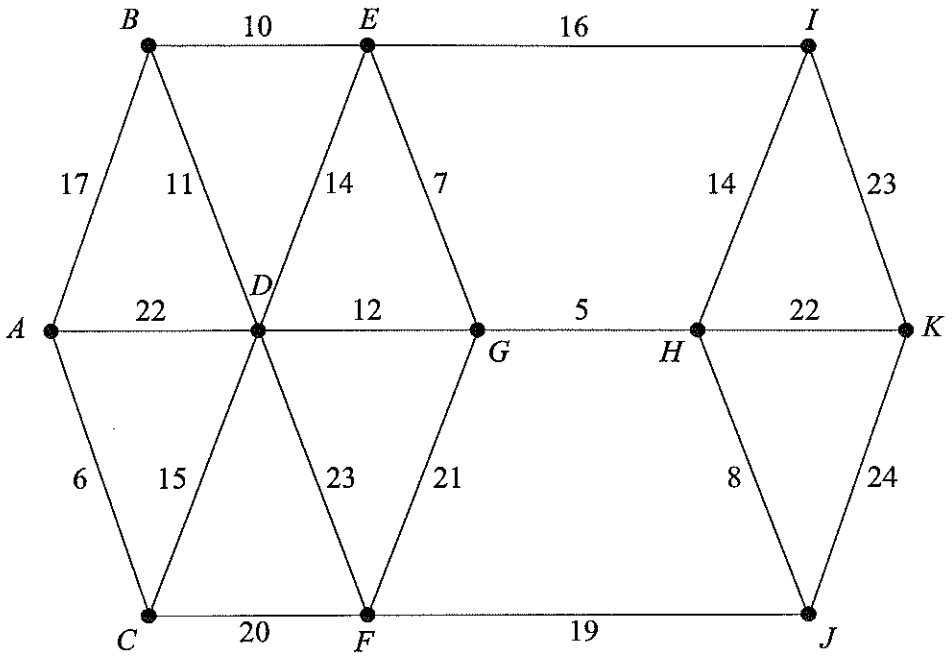
Jan 09

Answer **all** questions.

1 The following network shows the lengths, in miles, of roads connecting 11 villages, *A*, *B*, ..., *K*.



(a) Starting from *G* and showing your working at each stage, use Prim's algorithm to find a minimum spanning tree for the network. *(6 marks)*

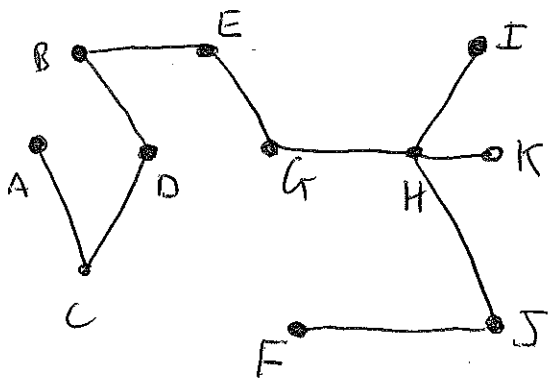
(b) State the length of your minimum spanning tree. *(1 mark)*

(c) Draw your minimum spanning tree. *(3 marks)*

(a) GH EG HJ BE BD HI CD AC FJ HK
5 7 8 10 11 14 15 6 19 22

(b) 117

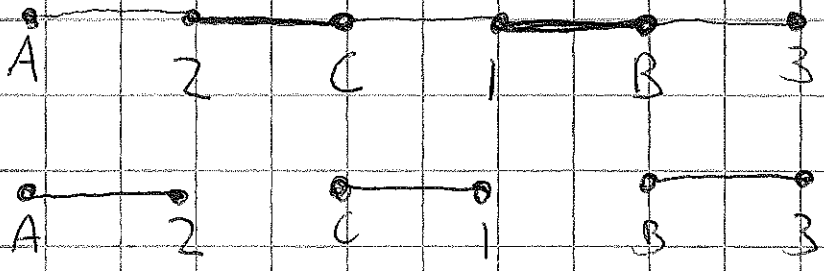
(c)



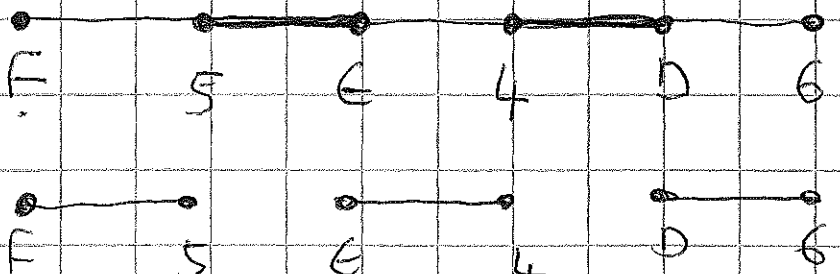
2(a)

	1	2	3	4	5	6
A	0	1	0	0	0	0
B	1	1	1	0	0	0
C	1	1	0	0	0	0
D	0	0	0	1	0	1
E	0	0	0	1	1	0
F	0	0	0	0	1	0

(b) Initial match: B-1 C-2 D-4 E-5



New match: A-2
B-3
C-1
D-4
E-5



Complete match:

A2 B3 C1 D6 E4 F5

3(a) (i) See diagram on next page.

(ii) DEFBAH

(b) (i) 24 (this is just the label boxed at A)

(ii) Odd vertices:

A C D G

AC = 19 (ABC)

AD = 24 (DEFBA)

AG = 19 (ABFG)

CD = 6

CG = 10 (CFG)

DG = 15 (DEFG)

AC + DG = 19 + 15 = 34

AD + CG = 24 + 10 = 34

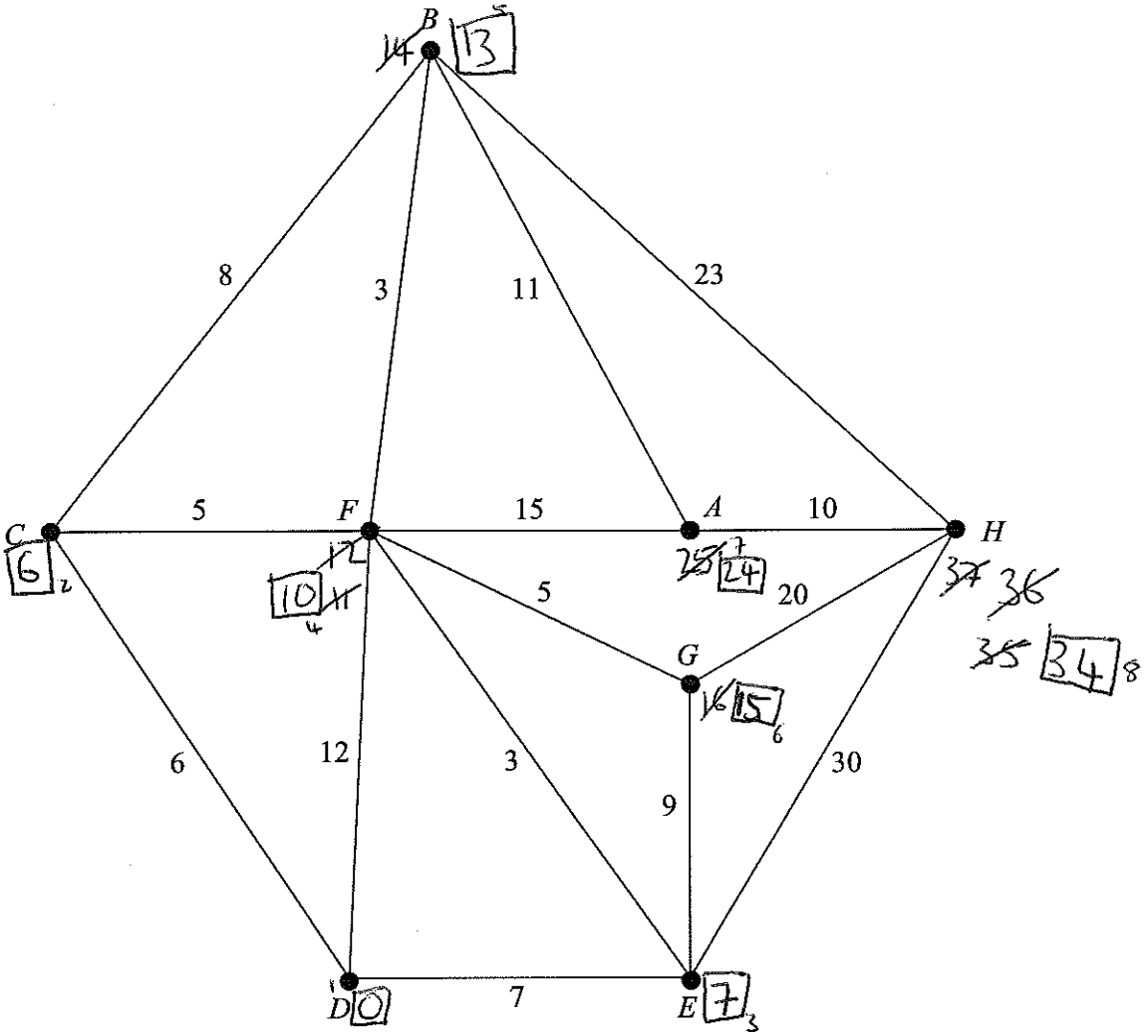
AG + CD = 19 + 6 = 25

so - repeat AG and CD

Length = 167 + 25 = 192

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows roads connecting some places of interest in Berlin. The numbers represent the times taken, in minutes, to walk along the roads.



The total of all walking times is 167 minutes.

(a) Mia is staying at D and is to visit H.

(i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from D to H. *(6 marks)*

(ii) Write down the corresponding route. *(1 mark)*

(b) Each day, Leon has to deliver leaflets along all of the roads. He must start and finish at A.

(i) Use your answer to part (a) to write down the shortest walking time from D to A. *(1 mark)*

(ii) Find the walking time of an optimum Chinese Postman route for Leon. *(6 marks)*

4(a)

$x + y + z \geq 110$

$x \leq y$

$y + z \leq 150$

$16x + 8y + 24z \leq 3120$

Maximise: $P = 70x + 30y + 50z$

(b) $z = 30$

$y \geq x$ as in (a)

$16x + 8y + 24(30) \leq 3120$

$16x + 8y \leq 2400$

$2x + y \leq 300$

$x + y + z \geq 110$

$x + y + 30 \geq 110 \Rightarrow x + y \geq 80$

$y + z \leq 150$

$y + 30 \leq 150$

$y \leq 120$

$P = 70x + 30y + 1500$

(b)(i) see over page.

OL: $P = 70x + 30y + 1500$

$m = \frac{-70}{30} = \frac{-7}{3} \left(= \frac{-70}{30} \right)$

⇓

more useful as this fraction for graph.

(ii) Consider points (90, 120) and (100, 100)

$P = 70(40) + 30(120) + 1500$

$= 11400$

$P = 70(100) + 30(100) + 1500$

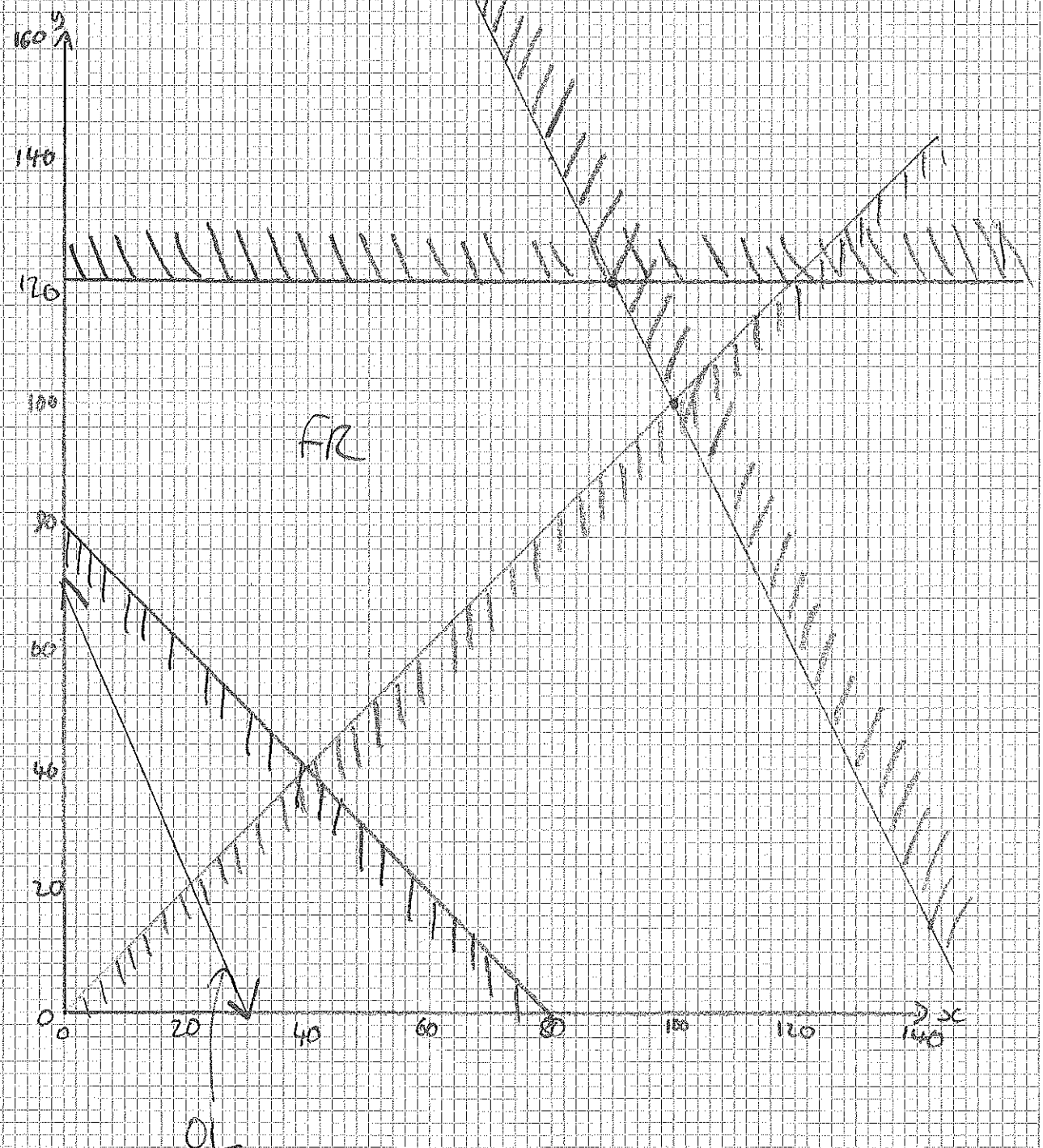
$= 11500$

So £11500 ⇒ 100 goats, 100 pigs and 30 sheep.

$2x + y = 300$

$(100, 100)$

$(80, 140)$



A	B	C	D	E	F	G	H
1	3	0					
			1	2	0		
						1·5	
							2·25
		7					
3							
	7						
					5		
			2				
				5			
						1·4	
							1·96
		17					
7							
	17						
					12		
			5				
				12			
						1·416̇	
							2·00694

$\sqrt{2} \approx 1.41\dot{6}$

$\sqrt{2} \approx 1.417$

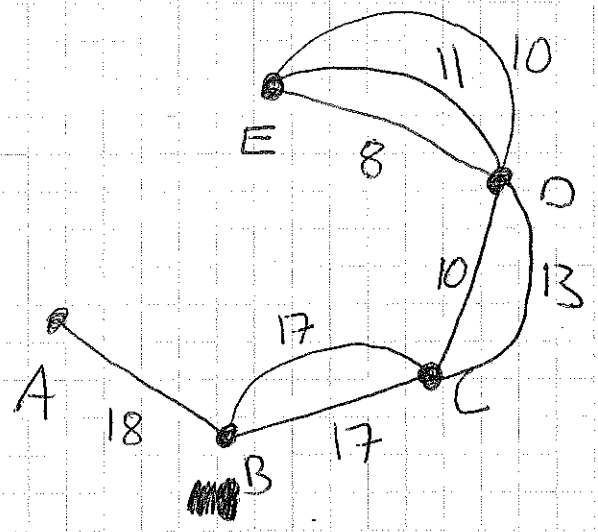
6(a) 5 vertices ⇒ 4 edges.

$8 + 10 + 10 + 11 = 39$

(b) $18 + 17 + 17 + 8 = 60$

Note: we are still looking for the minimum spanning tree. The layout of the graph will effect the MST length, but you would never have to use the four largest.
The worst case is the smallest with the 3 largest.

(c) [53] Could use 18, 17, 10, 8



5.

A	B	C	D	E	F	G	H
1	3	0					
			1	2	0		
						1·5	
							2·25
		7					
3	[illegible]						
	7						
					5		
			2				
				5			
						1·4	
							1·96
	[illegible]	17					
7							
	17						
					12		
			5				
				12			
						1·46	
							2·0069

$\sqrt{2} \approx 1.416$ $\sqrt{2} \approx 1.417$

7(a)

(i) $2x - 4 < x + 6$

$x - 4 < 6$

$x < 10$

(ii) $2x - 4 < 3x - 7$

$-4 < x - 7$

$3 < x$

so $\boxed{x > 3}$

$2x - 4 < 4x - 14$

$-4 < 2x - 14$

$10 < 2x$

$\boxed{x > 5}$

(b)(i) $2x - 1 < 3x - 7$

$-1 < x - 7$

$6 < x$

$\boxed{x > 6}$

$2x - 1 < x + 8$

$x - 1 < 8$

$\boxed{x < 9}$

(ii) $DE < DB$

$2x - 2 < 3x - 9$

$-2 < x - 9$

$7 < x$

$\boxed{x > 7}$

Since x is an integer and $x < 9$ $x > 7$

$\Rightarrow \boxed{x = 8}$

(iii)

A C D E B A

$2x - 4$ $2x - 1$ $2x - 2$ $x + 9$ $x + 6$

Total = $8x + 8$

$8(8) + 8$

$= \boxed{72}$