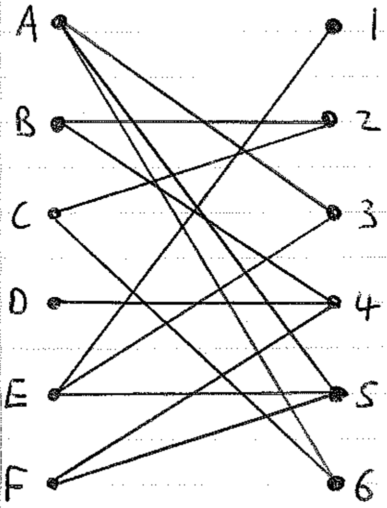
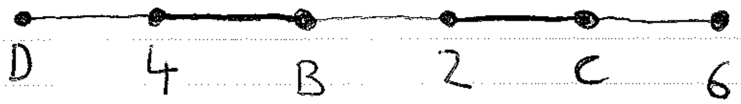
June 2008 Decision 1

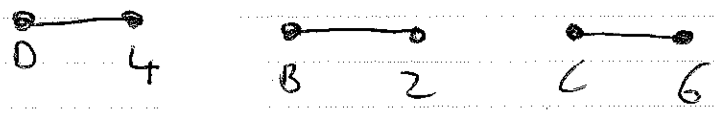
1(a)



(b) Initial Match: A-3 ~~B-4~~ ~~C-2~~ E-5

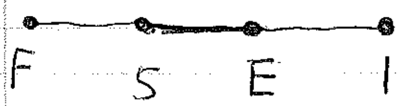
Alt path:

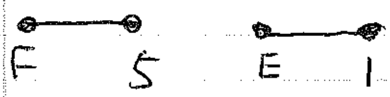




New match:
A-3
B-2
C-6
D-4
~~E-5~~

Alt. path:





Complete Match:
A3
B2
C6
D4
E1
F5

2(a)

$\underline{P} \quad B \quad M \quad N \quad J \quad K \quad R \quad D$

$\underline{B} \quad M \quad N \quad J \quad K \quad D \quad \boxed{P} \quad \underline{R}$

$\boxed{B} \quad \underline{M} \quad N \quad J \quad K \quad D \quad \boxed{P} \quad \boxed{R}$

$\boxed{B} \quad \underline{J} \quad K \quad D \quad \boxed{M} \quad \underline{N} \quad \boxed{P} \quad \boxed{R}$

$\boxed{B} \quad \underline{D} \quad \boxed{J} \quad \underline{K} \quad \boxed{M} \quad \boxed{N} \quad \boxed{P} \quad \boxed{R}$

(b)(i) $1 + 2 + 3 + 4 + 5 + 6 + 7 = \boxed{28}$

(ii) They must have been in reverse order (ie. descending).

3(a)

(i) 10

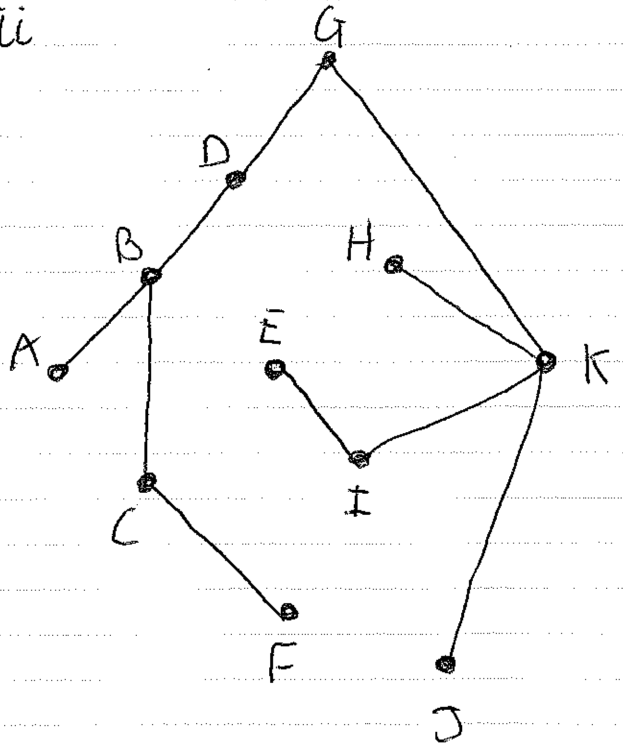
(ii) $n-1$

(b) (i)

AB = 18
BC = 13
BD = 15
CF = 16
DG = 17
GK = 12
JK = 16
KI = 17
EI = 14
HK = 17

(ii) 155

iii



4(a)(i) $8 + 48 + 18 + 43 + 13 = 130$

(ii)

$T \to P \to C \to B \to V \to T$

$\quad 8 \quad 18 \quad 43 \quad 18 \quad 51$

total $= 138$

(iii) Nearest Neighbour doesn't give the best solution. The answer above may be improved on.

(b)(i) Two shortest from B: $18 + 43 = \underline{61}$

(Use prims in table ignoring B row & column to get spanning tree)

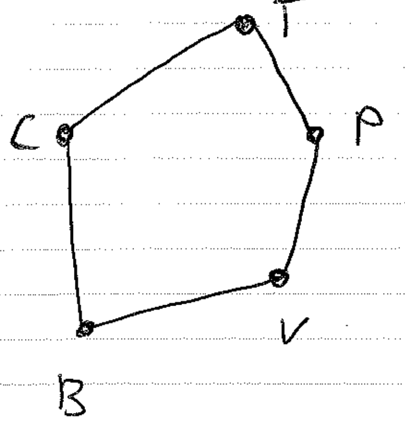
PT	CT	PV	
8	13	48	$= 69$

$61 + 69 = \boxed{130}$

(Lower bound)

(ii) The solution can't possibly be lower, but the above may not be a tour.

(c)



This is a tour.

~~[illegible]~~

Must be optimum as it is equal to tour found in a(i).

4 David, a tourist, wishes to visit five places in Rome: Basilica (B), Coliseum (C), Pantheon (P), Trevi Fountain (T) and Vatican (V). He is to start his tour at one of the places, visit each of the other places, before returning to his starting place.

The table shows the times, in minutes, to travel between these places. David wishes to keep his travelling time to a minimum.

b(i)

	B	C	P	T	V
B	–	43	57	52	18
C	43	–	18	13	56
P	57	18	–	8	48
T	52	13	8	–	51
V	18	56	48	51	–

(a) (i) Find the total travelling time for the tour $TPVBCT$. *(1 mark)*

(ii) Find the total travelling time for David's tour using the nearest neighbour algorithm starting from T. *(4 marks)*

(iii) Explain why your answer to part (a)(ii) is an upper bound for David's minimum total travelling time. *(2 marks)*

(b) (i) By deleting B, find a lower bound for the total travelling time for the minimum tour. *(5 marks)*

(ii) Explain why your answer to part (b)(i) is a lower bound for David's minimum total travelling time. *(2 marks)*

(c) Sketch a network showing the edges that give the lower bound found in part (b)(i) and comment on its significance. *(2 marks)*

5 (a) Odd vertices:

A B C D

AB = 270 (AEFB)

AC = 290 (AEHGC)

$\boxed{AD = 260 \ (AEHD)}$

$\boxed{BC = 270 \ (AFGC)}$

BD = 290 (BFEHD)

CD = 270 (CGHD)

AB + CD = 270 + 270 = 540

AC + BD = 290 + 290 = 580

$\boxed{AD + BC = 260 + 270 = 530}$

repeat AD
BC

1920 + 530 = <u>2450</u>

(b) No need to repeat AD. So 1920 + 270 = <u>2190</u>

(c) (i) Just need to make the graph semi-eulerian. So, repeat only AD = 260 (minimum pairing found in A).

1920 + 260 = <u>2180</u>

(ii) Would have to start from one of the two remaining odd vertices.

<u>B or C</u>

6(a)

$x \geq 0 \quad x \leq 100$

$y \geq 0 \quad y \leq 80$

$x < y$

$x + y \geq 60$

$2x + 8y \geq 320$

Minimise: $C = 1.5x + 3y$

(b) See figure 1

Note: to find gradient of OL:

$C = 1.5x + 3y$

$(\div 3) \quad \frac{C}{3} = \frac{1}{2}x + y$

$y = \frac{C}{3} \boxed{-\frac{1}{2}} x$

so $m = \frac{-1}{2}$

(c) Solution is at intersection of

$x + y = 60 \quad (1)$

$x + 4y = 160 \quad (2)$

$(2) - (1)$

$3y = 100 \quad \Rightarrow \quad x = 26\frac{2}{3}$

$y = 33\frac{1}{3}$

PTO →

Consider integers $26 \leqslant x \leqslant 28$

$32 \leqslant y \leqslant 34$

Values for C:

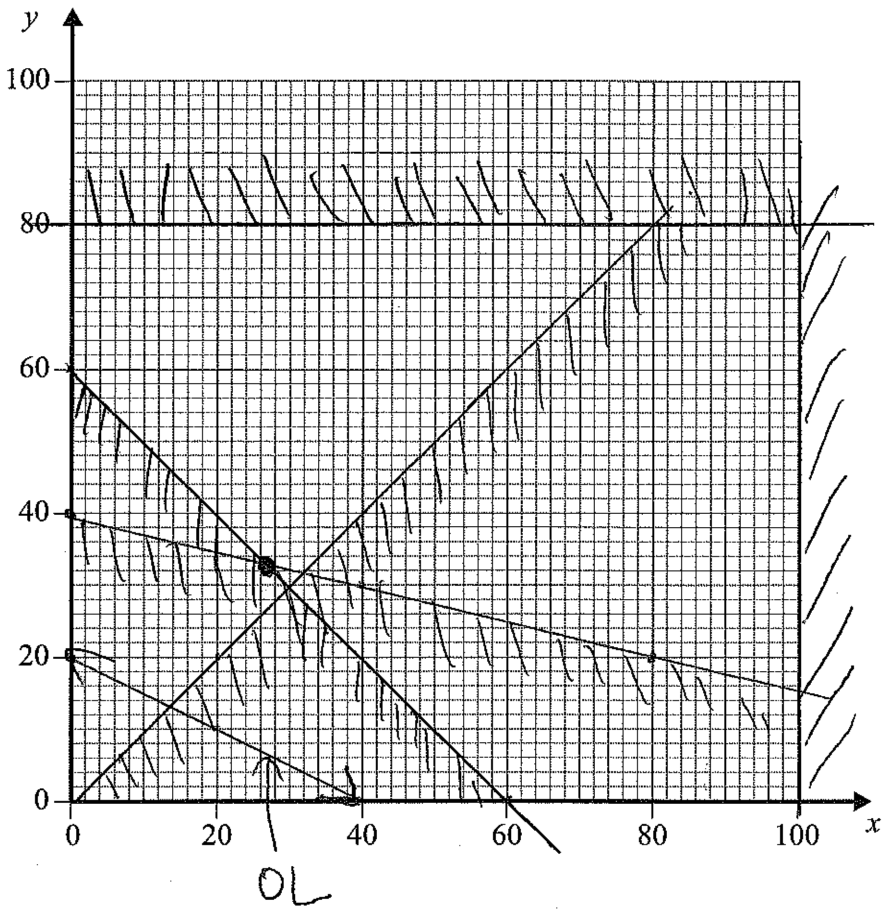
$y \downarrow$ $x \rightarrow$	26	27	28
32	/	/	/
33	/	/	141
34	141	142.5	144

/ = Outside feasible region

so $x = 26 \quad y = 34$

$\boxed{C = £141}$

Figure 1 (for use in Question 6)



$\boxed{2x + 8y \geq 320}$

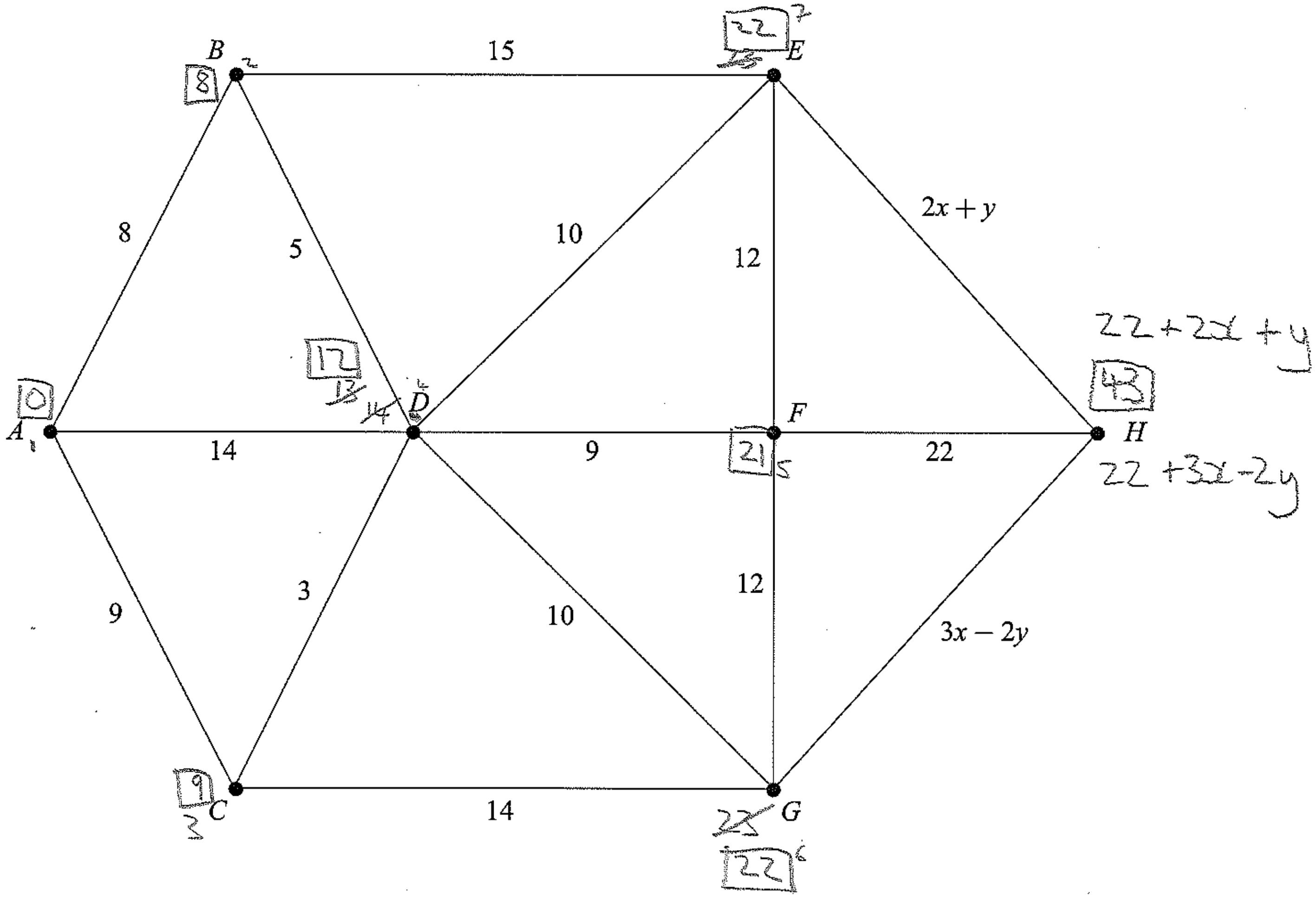
$x + 4y \geq 160$ → (consider $x + 4y = 160$

$y = 40 - \frac{1}{4}x$)

$(0, 40)$

$m = \frac{-1}{4} = \frac{-10}{40}$ → down 10 across 40.

Figure 2 (for use in Question 7)



$\boxed{7}$

7(a) 43

(b) $22 + 2x + y = 43$

$22 + 3x - 2y = 43$

$2x + y = 21$ (1)

$3x - 2y = 21$ (2)

(1) $\times 2$

$4x + 2y = 42$

$+\ 3x - 2y = 21$

$7x = 63$

$\boxed{x = 9}$

$3 \times 9 - 2y = 21$

$27 - 2y = 21$

$2y = 6$ $\boxed{y = 3}$