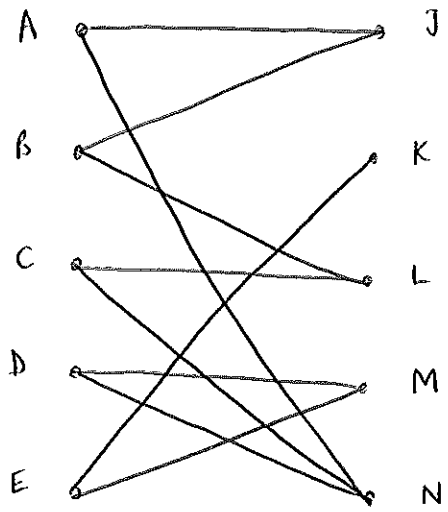
Jan '08

1a)



b) D - M * E - K

AN
BJ
CL
DM
EK

2. See sheet

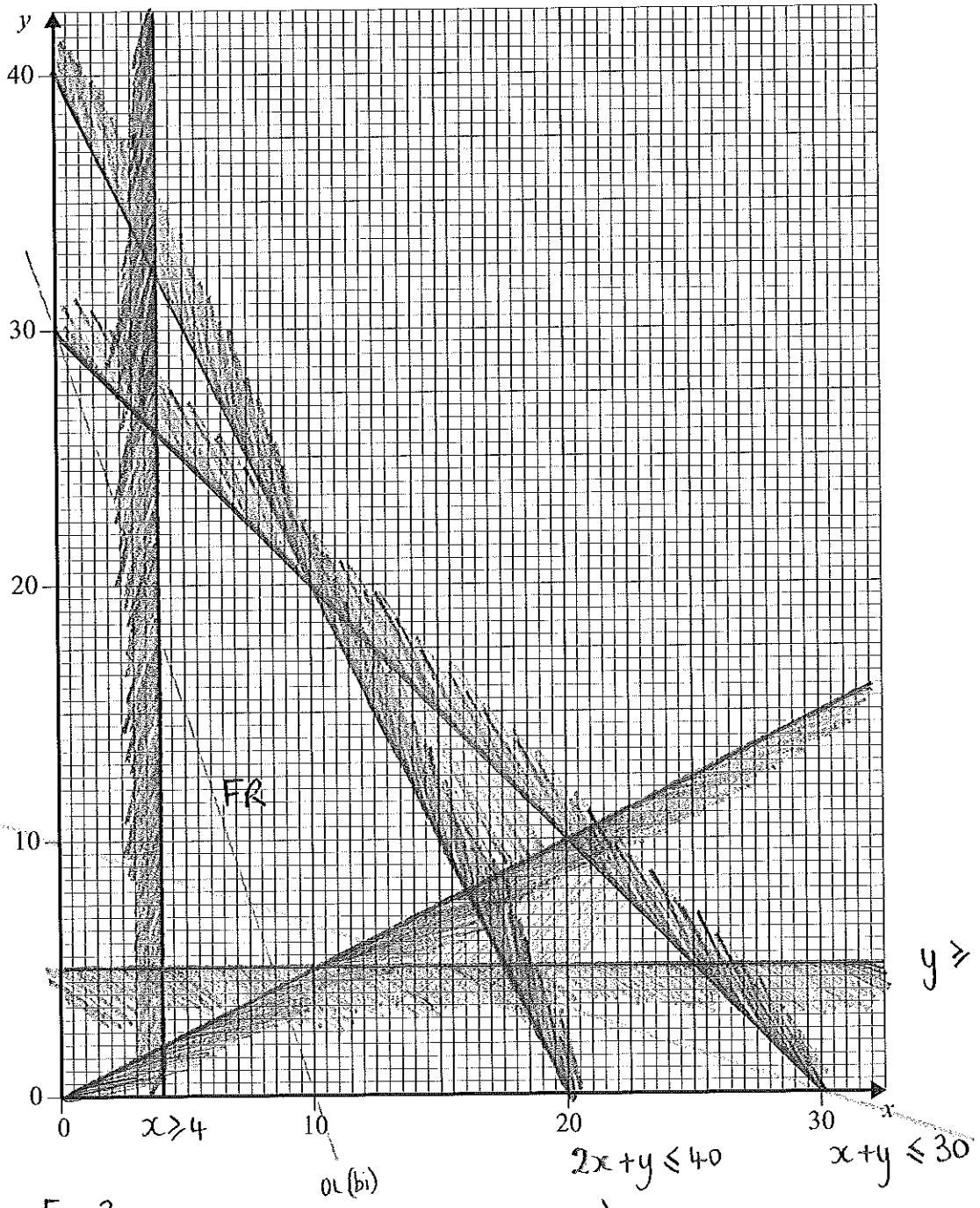
3a)

DF	1·2
IH	1·8
BC	2·1
AJ	2·2
EF	2·4
HG	2·6
FG	2·7
AB	2·8
IJ	2·9
	20·7

b) 20·7

Figure 1 (for use in Question 2)

Q2 a)



bi) $F = 3x + y$

$30 = 3x + y$

$(0, 30)$ $(10, 0)$

maximum at $(16, 8)$

$F = 3(16) + 8 = 56$

bii) $F = x + 3y$

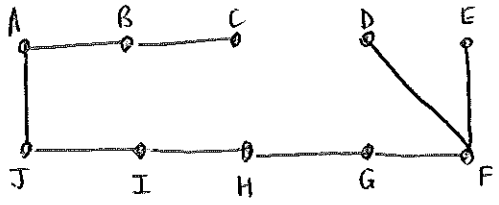
$30 = x + 3y$

$(0, 10)$ $(30, 0)$

maximum at $(4, 26)$

$F = 4 + 3(26) = 82$

c)



Prim's

AJ
AB
BC
IJ
IH
HG
GF
FD
EF

d) EF

4ai) See sheet

ii) A B E I K

b) odd vertices A, D, H, K

AD	27	(ABD)
AH	20	(ACD)
AK	46	(ABEIK)
DH	40	(from Q)
DK	20	(DIK)
HK	30	(DJK)

AD + HK = 27 + 30 = 57

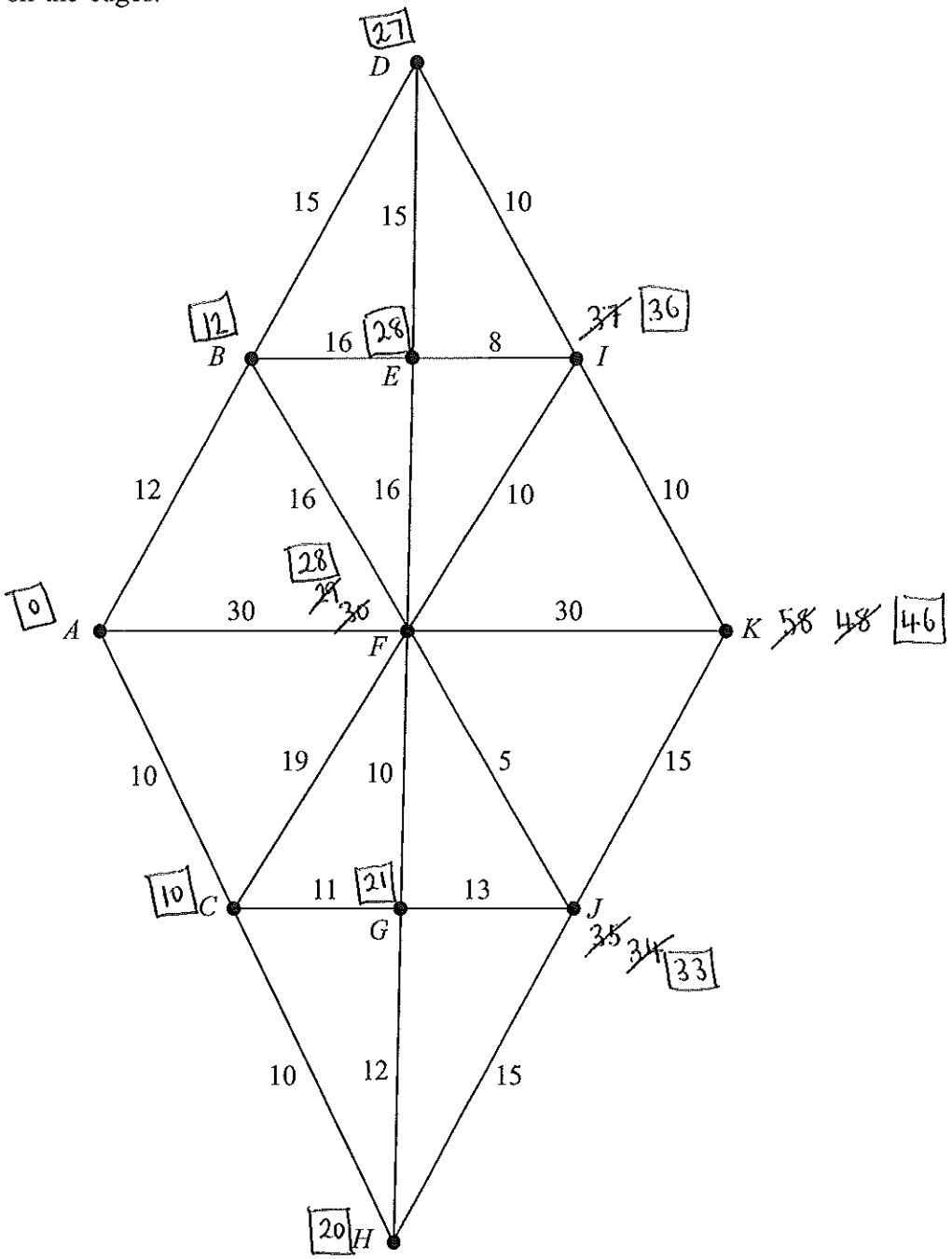
AH + DK = 20 + 20 = 40

AK + DH = 46 + 40 = 86

308 + 40 = 348 mins

4 [**Figure 2**, printed on the insert, is provided for use in this question.]

The network shows 11 towns. The times, in minutes, to travel between pairs of towns are indicated on the edges.



The total of all of the times is 308 minutes.

(a) (i) Use Dijkstra's algorithm on **Figure 2** to find the minimum time to travel from A to K. *(6 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) Find the length of an optimum Chinese postman route around the network, starting and finishing at A. (The minimum time to travel from D to H is 40 minutes.) *(5 marks)*

5ai) 40

ii) 40

b) The optimal solution must lie between 45 and 55

$45 \leqslant T \leqslant 55$

ci)

	A	B	C	D
A	–	20	38	35
B	20	–	18	15
C	38	18	–	33
D	35	15	33	–

ii) A → B → D → C → A

20 15 33 38 = 106

iii) A B D B C B A

6ai)

A	B	C	D	K	N	X	Y
1	-6	11	-6				
				1			
					0		
						1	
							0
					1		
				2			
						2	
							0
					2		
				3			
						3	
							0
					3		

ii)

A	B	C	D	K	N	X	Y
1	-10	29	-20				
				1			
					0		
						1	
							0
					1		
				2			
						2	
							6
				3			
						3	
							4
				4			
						4	
							0
					2		
				5			
						5	
							0
					3		

b) If line 90 wasn't there the algorithm would never end

7 The numbers 17, 3, 16 and 4 are to be sorted into ascending order.

The following four methods are to be compared: bubble sort, shuttle sort, Shell sort and quick sort (with the first number used as the pivot).

A student uses each of the four methods and produces the correct solutions below. Each solution shows the order of the numbers after each pass.

						Comparisons	Swaps
Solution 1	17	3	16	4	Shuttle	1	1
	3	17	16	4			
	3	16	17	4			
	3	4	16	17			
Solution 2	17	3	16	4	Shell	2	1
	16	3	17	4			
	3	4	16	17			
Solution 3	17	3	16	4	Quick	3	3
	3	16	4	17			
	3	16	4	17			
	3	4	16	17			
Solution 4	17	3	16	4	Bubble	3	3
	3	16	4	17			
	3	4	16	17			
	3	4	16	17			

(a) Write down which of the four solutions is the bubble sort, the shuttle sort, the Shell sort and the quick sort. *(3 marks)*

(b) For each of the four solutions, write down the number of comparisons and swaps (exchanges) on the first pass. *(8 marks)*

Turn over for the next question

8 Each day, a factory makes three types of hinge: basic, standard and luxury. The hinges produced need three different components: type *A*, type *B* and type *C*.

Basic hinges need 2 components of type *A*, 3 components of type *B* and 1 component of type *C*.

Standard hinges need 4 components of type *A*, 2 components of type *B* and 3 components of type *C*.

Luxury hinges need 3 components of type *A*, 4 components of type *B* and 5 components of type *C*.

Each day, there are 360 components of type *A* available, 270 of type *B* and 450 of type *C*.

Each day, the factory must use at least 720 components in total.

Each day, the factory must use at least 40% of the total components as type *A*.

Each day, the factory makes x basic hinges, y standard hinges and z luxury hinges.

In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, find five inequalities, each involving x, y and z, which must be satisfied. Simplify each inequality where possible. *(8 marks)*

END OF QUESTIONS

Type A: $2x + 4y + 3z \leqslant 360$

Type B: $3x + 2y + 4z \leqslant 270$

Type C: $x + 3y + 5z \leqslant 450$

Total: $6x + 9y + 12z \geqslant 720$

$\Rightarrow 2x + 3y + 4z \geqslant 240$

$2x + 4y + 3z \geqslant 0.4(6x + 9y + 12z) \Rightarrow 2x + 4y + 3z \geqslant 2.4x + 3.6y + 4.8z$

$0.4y \geqslant 0.4x + 1.8z$

$4y \geqslant 4x + 18z$

$2y \geqslant 2x + 9z$