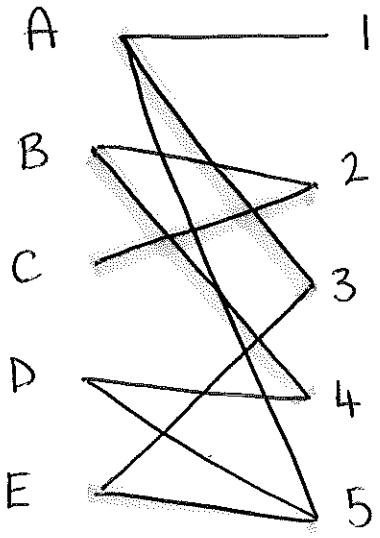
June '06

1a)



D - 5 + E - 3 + A - 1

A 1
B 4
C 2
D 5
E 3

2a)

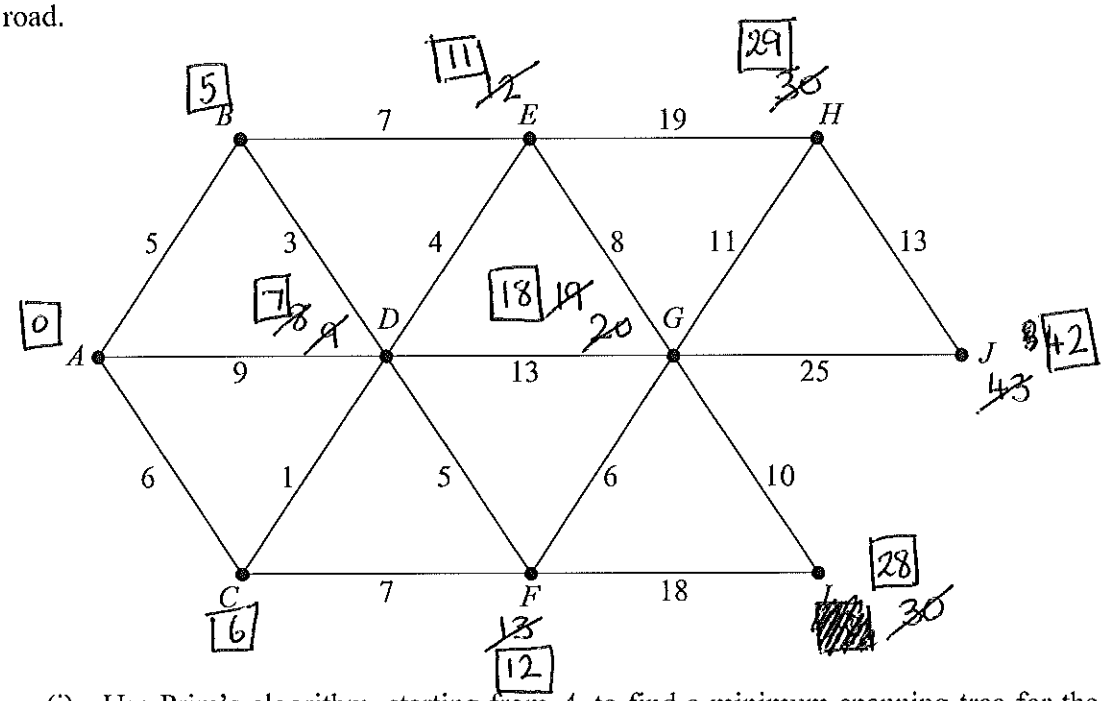
									C	S
	18	2	12	7	26	19	16	24		
P1	2	18	12	7	26	19	16	24	I	I
P2	2	12	18	7	26	19	16	24	II	I
P3	2	7	12	18	26	19	16	24	III	II
P4	2	7	12	18	26	19	16	24	I	0
P5	2	7	12	18	19	26	16	24	II	I
P6	2	7	12	16	18	19	26	24	IIII	III
P7	2	7	12	16	18	19	24	26	II	I

b)

Pass 1 — 1 comparison — 1 swap
Pass 2 — 2 comparisons — 1 swap
Pass 3 — 3 comparisons — 2 swaps

3 [**Figure 1**, printed on the insert, is provided for use in **part (b)** of this question.]

The diagram shows a network of roads. The number on each edge is the length, in kilometres, of the road.



(a) (i) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(b) (i) Use Dijkstra's algorithm on **Figure 1** to find the shortest distance from A to J. *(6 marks)*

(ii) A new road, of length x km, is built connecting I to J. The minimum distance from A to J is reduced by using this new road. Find, and solve, an inequality for x. *(2 marks)*

Turn over for the next question

3ai)

AB	5
BD	3
DC	1
DE	4
DF	5
FG	6
GI	10
GH	11
HJ	13
	58

ii) 58

bi) see sheet 42

ii) $28 + x < 42$

$x < 14$

4a) odd = A, C, D, F

AC = 18 (ABC)

AD = 32 (ABED)

AF = 12

DF = 22 (DEF)

CF = 30 (CBAF)

CD = 30

AC + DF = 18 + 22 = 40

AD + CF = 32 + 30 = 62

AF + CD = 12 + 30 = 42

164 + 40 = 204

b) A and C must be odd so add DF

164 + 22 = 186

ci) shortest pairing of odd vertices (see above) is 12 (AF).

so 164 + 12 = 176

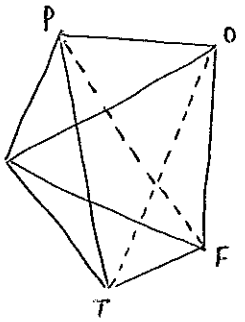
ii) if A & F now odd, can start at C or D

5ai) 7

ii) 7

bi) PF (PMF) 3

OT (OPT/ OMT) $3\frac{1}{4}$



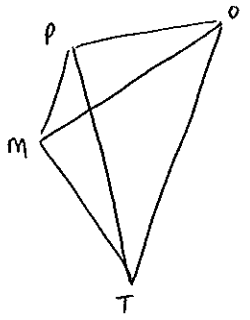
ii) F → T → P → O → M — F

$1\frac{1}{4}$ $2\frac{1}{4}$ 1 $1\frac{3}{4}$ 2 = $8\frac{1}{4}$

iii) F → T → M → P → O → F = 7

$1\frac{1}{4}$ $1\frac{1}{2}$ 1 1 $2\frac{1}{4}$

iv)



		FT FM
OP	1	$1\frac{1}{4} + 2 = 3\frac{1}{4}$
PM	1	
MT	$1\frac{1}{2}$	
	$3\frac{1}{2}$	

+ = $6\frac{3}{4}$

6a) $10 \leqslant x \leqslant 80$

$5 \leqslant y \leqslant 40$

$x + y \leqslant 100$

$20x + 60y \leqslant 3000 \quad \Rightarrow \quad x + 3y \leqslant 150$

$P = 2x + y$

b) see next sheet

c) maximum at $(80, 20)$

$P = 2(80) + (20) = 160 + 20 = £180$

d) $P = x + 4y$

maximum now at $(30, 40)$

$P = 30 + 4(40) = 30 + 160 = £190$

7ai) $m - 1$

ii) $n \geqslant m - 1$

b) Hamiltonian = [redacted] m edges

c)

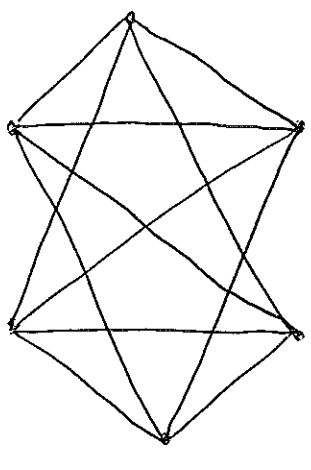


Figure 3 (for use in Question 6)