

$$\begin{aligned} \textcircled{1} \quad f(-1) &= 15(-1)^3 + 19(-1)^2 - 4 \\ &= -15 + 19 - 4 = 0 \end{aligned} \quad \begin{array}{l} \text{∴ } (x+1) \text{ is a} \\ \text{factor} \end{array}$$

$$\begin{aligned} \textcircled{ii} \quad f\left(\frac{2}{5}\right) &= 15\left(\frac{2}{5}\right)^3 + 19\left(\frac{2}{5}\right)^2 - 4 \\ &= \frac{24}{25} + \frac{76}{25} - 4 = 0 \end{aligned}$$

∴ $(5x-2)$ is a factor.

$$\begin{aligned} \textcircled{b} \quad f(x) &= (x+1)(5x-2)(ax+b) = (x+1)(5x-2)(3x+2) \\ &\quad \underline{a=3} \quad \underline{b=2} \end{aligned}$$

$$\frac{15x^2 - 6x}{f(x)} = \frac{3x(5x-2)}{(x+1)(5x-2)(3x+2)} = \frac{3x}{(x+1)(3x+2)}$$

$$\textcircled{2} \quad \cos x + 3 \sin x = R \cos(x-\alpha)$$

$$\cos(x-\alpha) = \cos x \cos \alpha + \sin x \sin \alpha$$

$$\cos x + 3 \sin x = (R \cos \alpha) \cos x + (R \sin \alpha) \sin x$$

$$\Rightarrow \quad 1 = R \cos \alpha \quad \textcircled{1} \quad 3 = R \sin \alpha \quad \textcircled{2}$$

$$\begin{aligned} \frac{\textcircled{2}}{\textcircled{1}} : \quad 3 &= \frac{\sin \alpha}{\cos \alpha} \Rightarrow \tan \alpha = 3 \\ \alpha &= \tan^{-1}(3) = 1.249 \end{aligned}$$

$$\begin{aligned} \textcircled{1}^2 + \textcircled{2}^2 : \quad 1^2 + 3^2 &= R^2 \cos^2 \alpha + R^2 \sin^2 \alpha \\ 10 &= R^2 (\cos^2 \alpha + \sin^2 \alpha) \\ R^2 &= 10 \Rightarrow R = \sqrt{10} \end{aligned}$$

$$\underline{\underline{\sqrt{10} \cos(x-1.249)}}$$

(b)(i) min value occurs at min value of $\cos(x-1.249) = -1$
 so min value is $-\sqrt{10}$

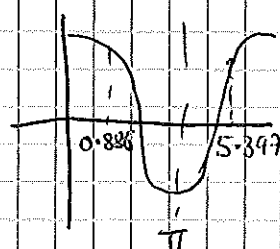
(ii) $\cos \theta = -1$ when $\theta = \pi, -\pi$
 $x - 1.249 = -\pi, \pi$
 $x = \cancel{-1.843}, \underline{4.391}$

(c) $-\sqrt{10} \cos(x-1.249) = 2$
 $\cos(x-1.249) = \frac{2}{\sqrt{10}}$

$x - 1.249 = 0.886, 5.397$
 -0.886

$x = 0.363, 2.135$

(others out of range)



(3) (a) (i) $(1+x)^{\frac{1}{3}} \approx 1 + \left(\frac{1}{3}\right)x + \frac{\left(\frac{1}{3}\right)\left(\frac{-4}{3}\right)x^2}{2!}$
 $= 1 - \frac{x}{3} + \frac{2}{9}x^2$

(ii) replace x with $\frac{3}{4}x$

$1 - \frac{1}{3}\left(\frac{3}{4}x\right) + \frac{2}{9}\left(\frac{3}{4}x\right)^2 = 1 - \frac{1}{4}x + \frac{1}{8}x^2$

(b) $\sqrt[3]{\frac{256}{4+3x}} = \sqrt[3]{\frac{\frac{1}{4}(256)}{\frac{1}{4}(4+3x)}} = \sqrt[3]{\frac{64}{1+\frac{3}{4}x}} = 4\left(1+\frac{3}{4}x\right)^{-\frac{1}{3}}$

$\approx 4\left(1 - \frac{1}{4}x + \frac{1}{8}x^2\right) = 4 - x + \frac{1}{2}x^2$

$a=4 \quad b=-1 \quad c=\frac{1}{2}$

$$\textcircled{4} \quad \frac{10x^2+8}{(x+1)(5x-1)} = 2 + \frac{A}{(x+1)} + \frac{B}{(5x-1)}$$

$$= \frac{2(x+1)(5x-1)}{(x+1)(5x-1)} + \frac{A(5x-1)}{(x+1)(5x-1)} + \frac{B(x+1)}{(x+1)(5x-1)}$$

$$\Rightarrow 10x^2+8 = 2(x+1)(5x-1) + A(5x-1) + B(x+1)$$

$$\text{let } \underline{x = -1}$$

$$10+8 = A(-6)$$

$$\Rightarrow 18 = -6A \Rightarrow \underline{\underline{A = -3}}$$

$$\text{let } \underline{x = \frac{1}{5}}$$

$$10\left(\frac{1}{5}\right)^2 + 8 = B\left(\frac{6}{5}\right)$$

$$\frac{10}{25} + 8 = \frac{6}{5}B$$

$$\frac{42}{5} = \frac{6}{5}B \Rightarrow \underline{\underline{B = 7}}$$

$$2 + \frac{-3}{(x+1)} + \frac{7}{(5x-1)}$$

$$\textcircled{b} \int 2 + \frac{-3}{(x+1)} + \frac{7}{(5x-1)} dx = 2x - 3\ln(x+1) + \frac{7}{5}\ln(5x-1) + C$$

$$\textcircled{5} \quad x^2 + xy = e^y$$

$$\frac{d}{dx}(xy) = \left(\frac{d}{dx}x\right)xy + x\left(\frac{d}{dx}y\right) = y + x \times \frac{dy}{dx} \frac{d}{dy}y$$

$$= y + x \frac{dy}{dx}$$

$$\frac{d}{dx}(e^y) = \frac{dy}{dx} \frac{d}{dy}e^y = \frac{dy}{dx} \times e^y$$

$$\Rightarrow \frac{d}{dx}(x^2 + xy) = \frac{d}{dx}(e^y)$$

$$2x + y + x \frac{dy}{dx} = e^y \frac{dy}{dx}$$

$$2x + y = \frac{dy}{dx}(e^y - x)$$

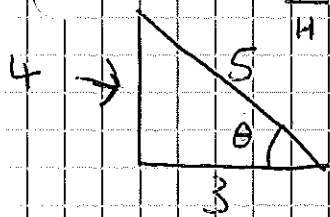
$$\frac{dy}{dx} = \frac{2x + y}{e^y - x}$$

$$x = -1 \quad y = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{-2}{1+1} = \underline{\underline{-1}}$$

$$\begin{aligned} \textcircled{6} \textcircled{c} \sin 2\theta &= \sin(\theta + \theta) = \sin\theta \cos\theta + \sin\theta \cos\theta \\ &= 2 \sin\theta \cos\theta \end{aligned}$$

$$\begin{aligned} \cos 2\theta &= \cos(\theta + \theta) = \cos\theta \cos\theta - \sin\theta \sin\theta \\ &= \cos^2\theta - \sin^2\theta \end{aligned}$$

$$\textcircled{6} \textcircled{d} \cos\theta = \frac{3}{5} \Rightarrow \sin\theta = \frac{4}{5}$$



$$\sin 2\theta = 2 \sin\theta \cos\theta = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$$

as required

$$\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} = \underline{\underline{-\frac{7}{25}}} \end{aligned}$$

$$(b)(i) \quad x = 3 \sin 2\theta \quad y = 4 \cos 2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$\frac{dy}{d\theta} = -8 \sin 2\theta$$

$$\frac{dx}{d\theta} = 6 \cos 2\theta \Rightarrow \frac{d\theta}{dx} = \frac{1}{6 \cos 2\theta}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-8 \sin 2\theta}{6 \cos 2\theta} = -\frac{4}{3} \frac{\sin 2\theta}{\cos 2\theta} \\ &= -\frac{4}{3} \tan(2\theta) \end{aligned}$$

$$(ii) \quad \cos \theta = \frac{3}{5} \quad \text{from (a):} \quad \sin 2\theta = \frac{24}{25} \quad \cos 2\theta = \frac{-7}{25}$$

$$\frac{dy}{dx} = -\frac{4}{3} \times \frac{24}{25} \times \frac{-25}{7} = \frac{96}{21} = \frac{32}{7}$$

$$x = 3 \sin 2\theta = 3 \times \frac{24}{25} = \frac{72}{25}$$

$$y = 4 \cos 2\theta = 4 \times \frac{-7}{25} = \frac{-28}{25}$$

$$y + \frac{28}{25} = \frac{32}{7} \left(x - \frac{72}{25} \right)$$

$$(7) \quad \frac{dy}{dx} = \frac{1}{y} \cos\left(\frac{x}{3}\right)$$

Separate variables:

$$\int y \, dy = \int \cos\left(\frac{x}{3}\right) \, dx$$

$$\frac{1}{2} y^2 = 3 \sin\left(\frac{x}{3}\right) + C$$

$$y^2 = 6 \sin\left(\frac{x}{3}\right) + k$$

$$y=1 \quad x = \frac{\pi}{2}$$

$$1 = 6 \sin\left(\frac{\pi}{6}\right) + k$$

$$\Rightarrow \underline{k = -2}$$

$$y^2 = 6 \sin\left(\frac{x}{3}\right) - 2$$

$$(8) \quad r = \begin{pmatrix} 2+7 \\ -1-37 \\ -5+27 \end{pmatrix}$$

If B lies on r then $0 = 2+7 \Rightarrow \underline{7 = -2}$

check other coordinates:

$$5 = -1-37$$

$$5 = -1-3(-2) = 5 \quad \checkmark$$

$$-9 = -5 + 2(-2) = -5 - 4 = -9 \quad \checkmark$$

so B lies on L.

$$(b) \quad \vec{BC} = \vec{OC} - \vec{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 12 \end{pmatrix}$$

$$(c)(i) \quad 2\vec{BC} = \begin{pmatrix} 18 \\ -6 \\ 24 \end{pmatrix}$$

$$\vec{AD} = \vec{OD} - \vec{OA} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ -5 \end{pmatrix} = \begin{pmatrix} x-2 \\ y+1 \\ z+5 \end{pmatrix}$$

$$x-2 = 18 \Rightarrow x = 20$$

$$y+1 = -6 \Rightarrow y = -7$$

$$z+5 = 24 \Rightarrow z = 19$$

as required $D \begin{pmatrix} 20 \\ -7 \\ 19 \end{pmatrix}$

$$(ii) \quad P \begin{pmatrix} 2+p \\ -1-3p \\ -5+2p \end{pmatrix}$$

$$\vec{PD} = \vec{OD} - \vec{OP} = \begin{pmatrix} 20 \\ -7 \\ 19 \end{pmatrix} - \begin{pmatrix} 2+p \\ -1-3p \\ -5+2p \end{pmatrix}$$

$$\vec{PD} = \begin{pmatrix} 18-p \\ -6+3p \\ 24-2p \end{pmatrix}$$

$\vec{PD} \perp L$ which means that

$$\begin{pmatrix} 18-p \\ -6+3p \\ 24-2p \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} = 0$$

$$\Rightarrow 18-p -3(-6+3p) + 2(24-2p) = 0$$

$$\begin{array}{rcl} 18 & - & p \\ +18 & - & 9p \\ +48 & - & 4p \end{array} = 0$$

$$84 = 14p \Rightarrow \underline{\underline{p=6}}$$

$$(a)(i) \underline{t=0} \quad h = A(1 - e^0) = 0.$$

$$(ii) \quad t=12 \quad h=57$$

$$57 = A(1 - e^{-3}) \Rightarrow A = \frac{57}{1 - e^{-3}} = 59.987$$
$$= 60 \text{ (2sf)}.$$

$$(b)(i) \quad h=48.$$

$$48 = 60(1 - e^{-\frac{1}{4}t}) \Rightarrow 1 - e^{-\frac{1}{4}t} = \frac{48}{60}$$
$$e^{-\frac{1}{4}t} = 1 - \frac{4}{5} = \frac{1}{5}$$

$$\ln(e^{-\frac{1}{4}t}) = \ln\left(\frac{1}{5}\right)$$

$$-\frac{1}{4}t = \ln\left(\frac{1}{5}\right)$$

$$t = -4 \ln\left(\frac{1}{5}\right) = 4 \ln\left(\frac{1}{5}\right)^{-1} = \underline{\underline{4 \ln 5}}$$

$$(ii) \quad h = 60 - 60e^{-\frac{1}{4}t}$$

$$\begin{aligned}\frac{dh}{dt} &= -60 \times \left(-\frac{1}{4}\right) e^{-\frac{1}{4}t} \\ &= 15e^{-\frac{1}{4}t}\end{aligned}$$

$$\frac{h}{4} = 15 - 15e^{-\frac{1}{4}t}$$

$$\frac{h}{4} = 15 - \frac{dh}{dt} \quad \Rightarrow \quad \frac{dh}{dt} = 15 - \frac{h}{4}$$

$$(iii) \quad \text{at } \frac{dh}{dt} = 13$$

$$13 = 15 - \frac{h}{4}$$

$$\frac{h}{4} = 2 \quad \Rightarrow \quad \underline{\underline{h = 8}}$$