

Core 4 - January 2007

① a) i) $x = 1 + 2t$ $y = 1 - 4t^2$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = -8t$$

ii) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = -8t \times \frac{1}{2} = -4t$

b) $t=1$, $x = 1 + 2(1) = 3$

$$y = 1 - 4(1)^2 = -3$$

$$\frac{dy}{dt} = -4(1) = -4, \text{ therefore gradient normal} = \frac{1}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{1}{4}(x - 3)$$

$$4y + 12 = x - 3$$

$$x - 4y - 15 = 0$$

c) $x = 1 + 2t \rightarrow x - 1 = 2t \rightarrow t = \left(\frac{x-1}{2}\right)$

$$y = 1 - 4t^2$$

$$y = 1 - 4\left(\frac{x-1}{2}\right)^2$$

② a) $2x-3 \rightarrow$ sub in $x = \frac{3}{2}$

$$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 7\left(\frac{3}{2}\right)^2 + 13 = 4$$

b) $f\left(\frac{3}{2}\right) = 0$ if $2x-3$ is a factor

$$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 7\left(\frac{3}{2}\right)^2 + 13 + d = 0$$

$$4 + d = 0 \rightarrow d = -4$$

c) $(2x-3)(x^2-2x+3) = 2x^3 - 7x^2 + 9$

$$\begin{array}{r} x^2 - 2x - 3 \\ (2x-3) \overline{) 2x^3 - 7x^2 + 0x + 9} \\ \underline{2x^3 - 3x^2} \\ -4x^2 + 0x \\ \underline{-4x^2 + 6x} \\ -6x + 9 \end{array}$$

$$\begin{aligned} a &= -2 \\ b &= -3 \end{aligned}$$

$$\begin{aligned} &-4x^2 + 6x \\ &-6x + 9 \end{aligned}$$

③ a) $\cos(2x) = 1 - 2\sin^2(x)$

b) i) $3\sin(x) - \cos(2x)$

$= 3\sin(x) - (1 - 2\sin^2(x))$

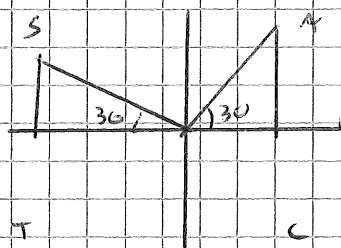
$= 3\sin(x) - 1 + 2\sin^2(x)$

$= 2\sin^2(x) + 3\sin(x) - 1$

ii) $(2\sin(x) - 1)(\sin(x) + 2) = 0$

$\sin(x) = 1/2$

$x = 30^\circ$



$x = 30^\circ, 150^\circ$

$\sin(x) = -2$

$x = \text{NO REAL SOLUTIONS}$

Be careful!!!
 $\begin{cases} 3\sin(x) - \cos(2x) = 1 \\ = 2\sin^2(x) + 3\sin(x) - 1 = 1 \\ = 2\sin^2(x) + 3\sin(x) - 2 = 0 \end{cases}$

c) $\int \sin^2(x) dx$

$= 1/2 \int (1 - \cos(2x)) dx$

$= 1/2 [x - 1/2 \sin(2x)]$

$= x/2 - 1/4 \sin(2x)$

$\sin^2(x) \cos(2x) = 1 - 2\sin^2(x)$

$2\sin^2(x) = 1 - \cos(2x)$

$\sin^2(x) = 1/2 (1 - \cos(2x))$

④ a) i) $\frac{3x-5}{x-3} = A + \frac{B}{x-3}$

$3x-5 = A(x-3) + B$

$\boxed{x=3} \quad 4 = B$

$\boxed{x=0} \quad -5 = -3A + 4$

$3A = 9 \rightarrow A = 3$

$= 3 + \frac{4}{x-3}$

ii) $\int 3 + \frac{4}{x-3} = 3x + 4 \ln(x-3) + C$

$$b) i) \frac{6x-5}{(2x+5)(2x-5)} = \frac{P}{(2x+5)} + \frac{Q}{(2x-5)}$$

$$6x-5 = P(2x-5) + Q(2x+5)$$

~~3/2/8/1~~ ~~2/8~~

$$\boxed{x = 5/2} \quad 10 = 10Q \rightarrow Q = 1$$

$$\boxed{x = -5/2} \quad -20 = -10P \rightarrow P = 2$$

$$\rightarrow \frac{2}{2x+5} + \frac{1}{2x-5}$$

$$ii) \int \frac{2}{2x+5} + \int \frac{1}{2x-5}$$

$$= \ln(2x+5) + \frac{1}{2} \ln(2x-5) + C$$

$$(5) a) (1+x)^{1/3} \approx 1 + \left(\frac{1}{3}\right)(x) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)x^2}{2}$$

$$= 1 + x/3 - x^2/9$$

$$b) i) (8+3x)^{1/3} = 8^{1/3} (1 + 3/8 x)^{1/3}$$

$$= 2 \left[1 + \left(\frac{1}{3}\right)\left(\frac{3}{8}x\right) - \frac{1}{9} \left(\frac{3}{8}x\right)^2 \right]$$

$$= 2 \left[1 + \frac{1}{8}x - \frac{1}{64}x^2 \right]$$

$$= 2 + \frac{1}{4}x - \frac{1}{32}x^2$$

$$ii) (8+3x)^{1/3} \quad \text{if } x = 1/3 \rightarrow 9^{1/3}$$

$$\approx 2 + \frac{1}{4}\left(\frac{1}{3}\right) - \frac{1}{32}\left(\frac{1}{3}\right)^2$$

$$= \frac{599}{288}$$

$$(6) c) i) \vec{BA} = \vec{BO} + \vec{OA}$$

$$= \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix}$$

$$ii) \vec{BC} = \vec{BO} + \vec{OC} = \begin{pmatrix} -5 \\ -4 \\ 0 \end{pmatrix} + \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix}$$

$$|\vec{BA}| = \sqrt{(-2)^2 + (-6)^2 + (4)^2} = \sqrt{56}$$

$$|\vec{BC}| = \sqrt{56}$$

$$\vec{BA} \cdot \vec{BC} = \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 2 \\ -4 \end{pmatrix} = -12 - 12 - 16 = -40$$

$$\cos(\angle ABC) = \frac{a \cdot b}{|\vec{a}| |\vec{b}|} = \frac{-40}{\sqrt{56} \sqrt{56}} = \frac{-40}{56} = -5/7$$

$$\therefore \angle ABC = \cos^{-1}(-5/7)$$

$$b) \vec{r} = \begin{pmatrix} 8 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$$

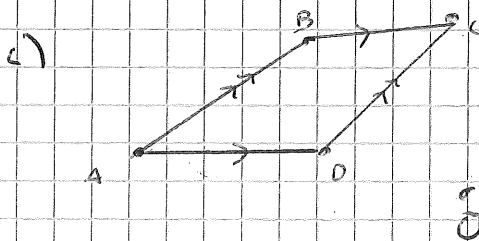
$$i) \vec{c} = \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix} \quad \begin{array}{l} 8 + 1\lambda = 11 \\ -3 + 3\lambda = 6 \\ 2 - 2\lambda = -4 \end{array}$$

All satisfied by $\lambda = 3$

$\therefore C$ lies on line ℓ

$$ii) \vec{AB} = \begin{pmatrix} 2 \\ 6 \\ -4 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \therefore \text{parallel to } \ell$$

as same direction



$$\begin{aligned} \vec{OD} &= \vec{OC} + \vec{BA} \\ &= \begin{pmatrix} 11 \\ 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ 0 \end{pmatrix} \end{aligned}$$

$$D = (9, 0, 0)$$

$$\begin{aligned} 7) a) A=B &\rightarrow \tan(2x) = \frac{\tan(x) + \tan(x)}{1 - \tan(x)\tan(x)} \\ &= x \\ &= \frac{2 \tan(x)}{1 - \tan^2(x)} \end{aligned}$$

$$\begin{aligned} b) 2 - 2 \tan(x) &= \frac{2 \tan(x)}{\tan(2x)} \\ &= 2 - 2 \tan(x) = 2 \tan(x) \frac{(1 - \tan^2(x))}{2 \tan(x)} \\ &= 2 - 2 \tan(x) = (1 - \tan^2(x)) \\ &= 2 - 2 \tan(x) - 1 + \tan^2(x) \\ &= \tan^2(x) - 2 \tan(x) + 1 \\ &= (\tan(x) - 1)(\tan(x) - 1) \\ &= (\tan(x) - 1)^2 \end{aligned}$$

(8) a) i) $\frac{dy}{dt} = y \sin(t)$

$$\int \frac{1}{y} dy = \int \sin(t) dt$$

$$\ln(y) = -\cos(t) + C$$

$$y = e^{-\cos(t) + C}$$

$$y = A e^{-\cos(t)}$$

where
 $A = e^C$

ii) $y = 50, t = \pi \rightarrow 50 = A e^{-\cos(\pi)}$

$$50 = A e^1 \rightarrow A = 50/e$$

$$\begin{aligned} y &= 50/e e^{-\cos(t)} \\ &= 50 e^{-1} e^{-\cos(t)} \\ &= 50 e^{(-1 + -\cos(t))} \\ &= 50 e^{-(1 + \cos(t))} \end{aligned}$$

b) i) $y = 50 e^{-(1 + \cos(t))}$

$$t = 6 \rightarrow y = 50 e^{-(1 + \cos(6))}$$

$$= 7.0417 \dots = 7.04 \text{ cm (2dp)}$$

$$= 7 \text{ cm (nearest cm)}$$

ii) $\frac{d^2y}{dt^2} = \frac{dy}{dt} \sin(t) + y \cos(t)$

$$\frac{d^2y}{dt^2} = \frac{dy}{dt} \sin(t) + y \cos(t)$$

$$u = y \quad v = \sin(t)$$

$$\frac{du}{dy} = \frac{dy}{dt} \quad \frac{dv}{dt} = \cos(t)$$

PRODUCT RULE

when $t = \pi$ $\frac{dy}{dt} = 50 \times 0 = 0$

$\therefore$ stationary point

$$\begin{aligned} \text{when } t = \pi \quad \frac{d^2y}{dt^2} &= 0 + 50 \times \cos(\pi) \\ &= -50 \end{aligned}$$

$\therefore$ MAXIMUM