

## Core 4 - January 2006

① a) i)  $f(1) = 3(1)^3 + 2(1)^2 - 7(1) + 2 = 0$

ii)  $f(-2) = 3(-2)^3 + 2(-2)^2 - 7(-2) + 2 =$

$$= -24 + 8 + 14 + 2 = 0$$

iii)  $3x^3 + 2x^2 - 7x + 2 = (x+2)(x-1)(3x-1)$

check:  $f(1/3) = 0 \checkmark$

to get  $3x^3$       to get  $+2$

$$\frac{(x-1)(x+2)}{(x+2)(x-1)(3x-1)} = \frac{1}{3x-1}$$

b)  $g(x) = 3x^3 + 2x^2 - 7x + d$

$$g(1/3) = 3(1/3)^3 + 2(1/3)^2 - 7(1/3) + d = 2$$

$$1/9 + 2/9 - 7/3 + d = 2$$

$$-2 + d = 2 \rightarrow d = 4$$

② a)  $x = 3 - 4t$

$$y = 1 + 2/t = 1 + 2t^{-1}$$

$$dx/dt = -4$$

$$dy/dt = -2t^{-2} = -2/t^2$$

$$dy/dx = dy/dt \times dt/dx$$

$$= 2/t^2 \times -1/4 = 1/2t^2$$

b) when  $t = 2$ :

$$x = 3 - 4(2) = -5$$

$$y = 1 + 2/2 = 2$$

$$dy/dx = 1/2(2)^2 = 1/8$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1/8(x + 5)$$

$$8y - 16 = x + 5$$

$$\rightarrow x - 8y + 21 = 0$$

③  $3 \cos(\theta) - 2 \sin(\theta) = R \cos(\theta + \alpha)$

$$= R[\cos\theta \cos\alpha - \sin\theta \sin\alpha]$$

$$3 = R \cos(\alpha)$$

$$2 = R \sin(\alpha)$$

a)  $R = \sqrt{3^2 + 2^2} = \sqrt{13}$

b)  $\tan(\alpha) = 2/3 \rightarrow \alpha = 33.690^\circ \approx 33.7^\circ$

$$c) \sqrt{13} \cos(\theta + 33.7^\circ)$$

stretch SF  $\sqrt{13}$  in y direction

$$\text{translation } \begin{pmatrix} -33.7 \\ 0 \end{pmatrix}$$

$$\text{Max value} = \sqrt{13}$$

cos has max values at  $0, 360, 720 \dots$

$$\text{So following translation, max value is at } 360 - 33.7 \\ = 326.3^\circ$$

$$(4) V = A k^t$$

$$a) \text{ when } t = 0, V = 80 \rightarrow A = 80$$

$$b) \text{ when } t = 56, V = 5000 \rightarrow 5000 = 80 k^{56}$$

$$5000/80 = k^{56}$$

$$\ln(5000/80) = k^{56}$$

$$k = \sqrt[56]{\frac{5000}{80}} = 1.076636 \dots$$

$$c) i) V = 80 \times 1.07663^{106} = \pounds 200,647.75$$

$$ii) 800000 = 80 \times 1.07663^t$$

$$\frac{800000}{80} = 1.07663^t$$

$$\ln(10000) = t \ln(1.07663)$$

$$\frac{\ln(10000)}{\ln(1.07663)} = t = 124.7 \text{ years} = 2024$$

$$(5) a) i) (1-x)^{-1} \approx 1 + (-1)(-x) + \frac{(-1)(-2)(-x)^2}{2}$$

$$\approx 1 + x + x^2$$

$$ii) \frac{1}{3-2x} = (3-2x)^{-1} = 3^{-1} (1 - \frac{2}{3}x)^{-1}$$

$$= \frac{1}{3} \left[ 1 + \frac{2}{3}x + \left(\frac{2}{3}x\right)^2 \right]$$

$$= \frac{1}{3} \left[ 1 + \frac{2}{3}x + \frac{4}{9}x^2 \right]$$

$$= \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

$$b) \frac{1}{(1-x)^2} = (1-x)^{-2} \approx 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2$$

$$\approx 1 + 2x + 3x^2$$

$$c) \frac{2x^2-3}{(3-2x)(1-x)^2} = \frac{A}{(3-2x)} + \frac{B}{(1-x)} + \frac{C}{(1-x)^2}$$

$$2x^2-3 = A(1-x)^2 + B(3-2x)(1-x) + C(3-2x)$$

$$\boxed{x=1} \quad -1 = C(1) \rightarrow C = -1$$

$$\boxed{x=3/2} \quad 3/2 = A(1/2) \rightarrow A = 6$$

$$\boxed{x=0} \quad -3 = 6(1) + B(3)(1) + (-1)(3)$$

$$-3 = 6 + 3B - 3$$

$$-6 = 3B \rightarrow B = -2$$

$$\rightarrow \frac{6}{3-2x} - \frac{2}{(1-x)} - \frac{1}{(1-x)^2}$$

$$d) \bullet 6(3-2x)^{-1} = 6/3 + 12/9 x + 24/27 x^2 \quad (\text{FROM a)})$$

$$\bullet -2(1-x)^{-1} = -2 - 2x - 2x^2 \quad (\text{FROM a)})$$

$$\bullet -1(1-x)^{-2} = \frac{-1 - 2x - 3x^2}{(1-x)^2} \quad (\text{FROM b)})$$

$$\text{Add terms together} \rightarrow -1 - 8/3 x - 37/9 x^2$$

$$b) a) \cos(2x) = 2\cos^2(x) - 1$$

$$b) \int_0^{\pi/2} \cos^2(x) dx$$

$$= \frac{1}{2} \int_0^{\pi/2} (\cos(2x) + 1) dx$$

$$= \frac{1}{2} \left[ \frac{1}{2} \sin(2x) + x \right]_0^{\pi/2}$$

$$= \frac{1}{2} \left[ \frac{1}{2} \sin(\pi) + \frac{\pi}{2} - \frac{1}{2} \sin(0) - 0 \right]$$

$$= \frac{1}{2} \left[ 0 + \frac{\pi}{2} \right]$$

$$= \frac{\pi}{4}$$

$$2\cos^2(x) - 1 = \cos(2x)$$

$$2\cos^2(x) = \cos(2x) + 1$$

$$\cos^2(x) = \frac{1}{2} (\cos(2x) + 1)$$

$$\textcircled{7} \text{ a) i) } \vec{AB} = \vec{AO} + \vec{OB} \\ = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$$

$$\text{ii) } \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{therefore parallel}$$

$$\text{iii) } \begin{cases} 6 + 1\lambda = 2 \\ 1 + 1\lambda = -3 \\ -1 + 0\lambda = -1 \end{cases} \quad \left. \vphantom{\begin{cases} 6 + 1\lambda = 2 \\ 1 + 1\lambda = -3 \\ -1 + 0\lambda = -1 \end{cases}} \right\} \text{ all satisfied when } \lambda = -4$$

$$\text{b) i) } \vec{OM} = \vec{OB} + \vec{BM} \\ = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

$$\vec{L}_2 = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

$$\text{ii) } \vec{AO} = \vec{AO} + \vec{OC} \\ = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} + \begin{pmatrix} 6 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$$

$$\boxed{\text{Scalar}} \quad \vec{a} \cdot \vec{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix} = 8 + 0 - 8 = 0$$

$\therefore$  Angle is  $90^\circ$  as vectors are perpendicular

$$\textcircled{8} \text{ a) } dx/dt = -2(x-b)^{1/2}$$

$$\frac{1}{(x-b)^{1/2}} dx = -2 dt$$

$$\int (x-b)^{-1/2} dx = \int -2 dt$$

$$2(x-b)^{1/2} = -2t + C$$

$$x=70, t=0 \rightarrow 2\sqrt{64} = C \rightarrow C = 16$$

$$2\sqrt{x-b} = -2t + 16$$

$$2t = 16 - 2\sqrt{x-b}$$

$$t = 8 - \sqrt{x-b}$$

b) i)  $x = 6 \rightarrow \frac{dx}{dt} = 0 \rightarrow$  fuel stops glowing

$$\begin{aligned} \text{ii) } x = 22 \quad t &= 8 - \sqrt{x-6} \\ &= 8 - \sqrt{22-6} = 8 - \sqrt{16} = 4 \end{aligned}$$

time = 4 minutes