Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature	WRITTEN SOLUTIONS		

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MPC3

Unit Pure Core 3

Friday 5 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_{1.5}^{5.5} e^{2-x}\ln(3x-2)\,dx$, giving your answer to three decimal places.

[4 marks]

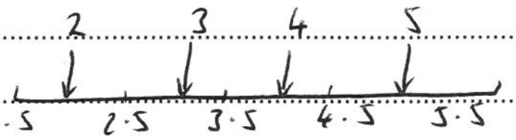
(b) Find the exact value of the gradient of the curve $y = e^{2-x}\ln(3x-2)$ at the point on the curve where $x = 2$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

1a) $h = \frac{5.5 - 1.5}{4} = 1$



x	2	3	4	5
y	1·38629	0·71586	0·31112	0·12770

$\int \approx h\,(y_{1/2} + y_{3/2} + \ldots y_{n/2})\,dx$

$1\,(1{\cdot}38629 + 0{\cdot}715.. + 0{\cdot}311.. + 0{\cdot}127..)$

$= \underline{2{\cdot}541}$ (3dp)

b) $y = e^{2-x}\ln(3x-2)$

$u = e^{2-x}$ $\quad v = \ln(3x-2)$

$\frac{du}{dx} = -e^{2-x}$ $\quad \frac{dv}{dx} = \frac{3}{3x-2}$

$\frac{dy}{dx} = -e^{2-x}\ln(3x-2) + e^{2-x}\frac{3}{3x-2}$

when $x = 2$

$\frac{dy}{dx} = -e^{0}\ln 4 + e^{0}\frac{3}{4}$

$= \underline{-\ln 4 + \frac{3}{4}}$ or $\underline{\frac{3}{4} - \ln 4}$

2 (a) Sketch, on the axes below, the curve with equation $y = 4 - |2x + 1|$, indicating the coordinates where the curve crosses the axes.

[4 marks]

(b) Solve the equation $x = 4 - |2x + 1|$.

[3 marks]

(c) Solve the inequality $x < 4 - |2x + 1|$.

[2 marks]

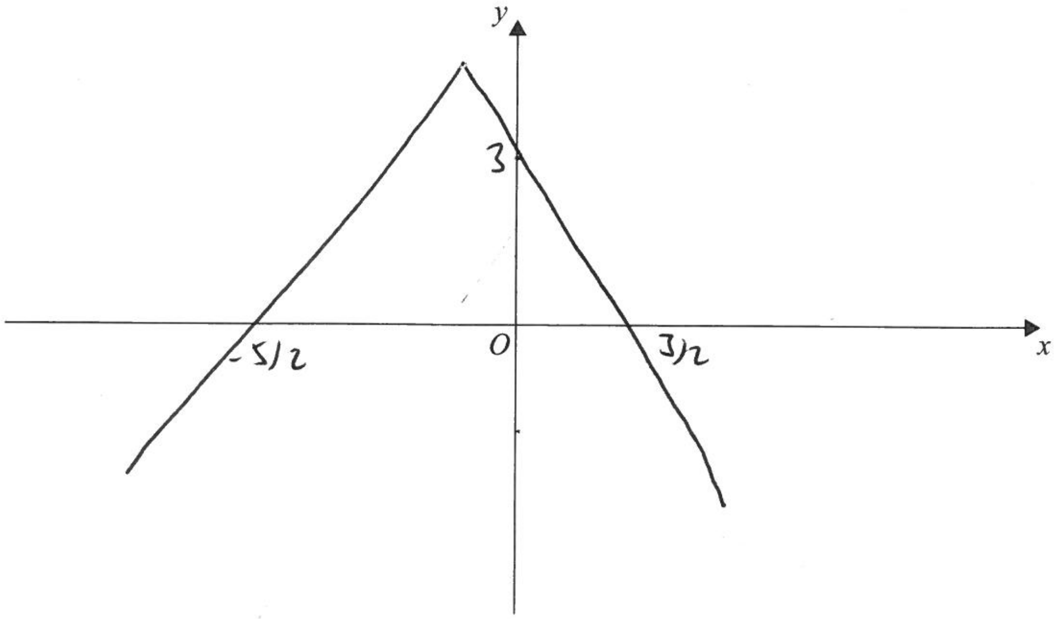
(d) Describe a sequence of two geometrical transformations that maps the graph of $y = |2x + 1|$ onto the graph of $y = 4 - |2x + 1|$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

(a)



2a) $y = 2x+1$ → reflected in y axis
→ Modulus ~~so~~ reflected in positive y (~~y~~axis)
→ reflected in x axis
→ translation $\begin{pmatrix}0\\4\end{pmatrix}$

when $x = 0$, $y = 4 - |1|$
$y = 3$ $(0, 3)$

when $y = 0$ $4 - |2x+1| = 0$
$|2x+1| = 4$

$2x+1 = 4$ or $2x+1 = -4$
$x = 3/2$ $x = -5/2$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 2

b) $x = 4 - |2x+1|$

$|2x+1| = 4 - x$

$2x+1 = 4 - x$ OR $2x+1 = -(4-x)$

$3x = 3$ $\quad$ $2x+1 = -4 + x$

$\underline{x = 1}$ $\quad$ $\underline{x = -5}$

c) $x < 4 - |2x+1|$

$y = x$ lower than $y = 4 - |2x+1|$

when $\underline{-5 < x < 1}$

d) $y = |2x+1| \rightarrow y = 4 - |2x+1|$

Reflection in ~~x~~ axis followed by translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

Turn over ▶

Do not write outside the box

3 (a) It is given that the curves with equations $y = 6\ln x$ and $y = 8x - x^2 - 3$ intersect at a single point where $x = \alpha$.

(i) Show that α lies between 5 and 6.

[2 marks]

(ii) Show that the equation $x = 4 + \sqrt{13 - 6\ln x}$ can be rearranged into the form

$$6\ln x + x^2 - 8x + 3 = 0$$

[3 marks]

(iii) Use the iterative formula

$$x_{n+1} = 4 + \sqrt{13 - 6\ln x_n}$$

with $x_1 = 5$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(b) A curve has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 6\ln x + x^2 - 8x + 3$.

(i) Find the exact values of the coordinates of the stationary points of the curve.

[5 marks]

(ii) Hence, or otherwise, find the exact values of the coordinates of the stationary points of the curve with equation

$$y = 2\mathrm{f}(x - 4)$$

[2 marks]

QUESTION PART REFERENCE	Answer space for question 3
3a i)	$6\ln x = 8x - x^2 - 3$
	$f(x) = 6\ln x - 8x + x^2 + 3$
	$f(5) = -2.3$
	$f(6) = 1.75$
	change of sign $\therefore$ $5 < \alpha < 6$
ii)	$x = 4 + \sqrt{13 - 6\ln x}$
	$x - 4 = \sqrt{13 - 6\ln x}$
	$(x-4)^2 = 13 - 6\ln x$
	$x^2 - 8x + 16 = 13 - 6\ln x$
	$6\ln x + x^2 - 8x + 3 = 0$ (as req)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

iii) $x_1 = 5$

$x_2 = \underline{5.828}$ (3dp)

$x_3 = \underline{5.557}$ (3dp)

bi) $f(x) = 6\ln x + x^2 - 8x + 3$

$\frac{dy}{dx} = \frac{6}{x} + 2x - 8$

stationary point at $\frac{dy}{dx} = 0$

$\frac{6}{x} + 2x - 8 = 0 \quad (\times x)$

$6 + 2x^2 - 8x = 0$

$2x^2 - 8x + 6 = 0 \quad (\div 2)$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or $x = 3$

$y = 6\ln(1) + 1^2 - 8(1) + 3$, $\quad y = 6\ln 3 + 3^2 - 8(3) + 3$

$y = -4 \qquad y = 6\ln 3 - 12$

$\underline{(1, -4)} \qquad \underline{(3, 6\ln 3 - 12)}$

ii) $y = 2f(x-4) \rightarrow x$ value '+4'

y value '×2'

$\underline{(5, -8)} \qquad \underline{(7, 12\ln 3 - 24)}$

Turn over ▶

Do not write outside the box

4 The functions f and g are defined by

$$f(x) = 5 - e^{3x}, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x-3}, \quad \text{for } x \neq 1.5$$

(a) Find the range of f.

[2 marks]

(b) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$.

[3 marks]

(ii) Solve the equation $f^{-1}(x) = 0$.

[1 mark]

(c) Find an expression for $gg(x)$, giving your answer in the form $\dfrac{ax+b}{cx+d}$, where a, b, c and d are integers.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

4a) $f(x) = 5 - e^{3x}$ for $x \in \mathbb{R}$

$e^{3x} > 0$

so $5 - e^{3x} < 5$

$\underline{f(x) < 5}$

b) $y = 5 - e^{3x}$

$x = 5 - e^{3y}$

$e^{3y} = 5 - x$

$3y = \ln(5-x)$

$y = \frac{1}{3}\ln(5-x)$

$\underline{f^{-1}(x) = \frac{1}{3}\ln(5-x)}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

(ii) $\frac{1}{3}\ln(5-x) = 0$

$\ln(5-x) = 0$

$5-x = e^0$

$5-x = 1$

$\underline{x = 4}$

(c) $g\left(\frac{1}{2x-3}\right) = \dfrac{1}{2\left(\frac{1}{2x-3}\right) - 3}$

$= \dfrac{1}{\frac{2}{2x-3} - 3}$

$= \dfrac{1}{\frac{2 - 3(2x-3)}{2x-3}}$

$= \dfrac{1}{\frac{2 - 6x + 9}{2x-3}}$

$= \dfrac{1}{\frac{11-6x}{2x-3}}$

$= \underline{\dfrac{2x-3}{11-6x}}$

Turn over ▶

Do not write outside the box

5 (a) By writing $\tan x$ as $\dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{d}{dx}(\tan x) = \sec^2 x$.

[2 marks]

(b) Use integration by parts to find $\int x \sec^2 x \, dx$.

[4 marks]

(c) The region bounded by the curve $y = (5\sqrt{x}) \sec x$, the x-axis from 0 to 1 and the line $x = 1$ is rotated through 2π radians about the x-axis to form a solid.

Find the value of the volume of the solid generated, giving your answer to two significant figures.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

5a)

$y = \dfrac{\sin x}{\cos x}$ $\quad u = \sin x \quad v = \cos x$

$\dfrac{du}{dx} = \cos x \quad \dfrac{dv}{dx} = -\sin x$

$\dfrac{dy}{dx} = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x} = \sec^2 x$ (as req.)

b)

$\int x \sec^2 x \, dx$

$\boxed{u = x \quad \dfrac{dv}{dx} = \sec^2 x \quad \dfrac{du}{dx} = 1 \quad v = \tan x}$

$x \tan x - \int \tan x \, dx$

$x \tan x - \ln \sec x + c$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5
c)	$\pi\int_0^1 y^2\,dx$ $y = (5\sqrt{x})\sec x$
	$y^2 = 25x\sec^2 x$
	$\pi\int_0^1 25x\sec^2 x\,dx$
	$25\pi\int_0^1 x\sec^2 x\,dx$
	$25\pi\left[x\tan x - \ln\sec x\right]_0^1$
	$25\pi\left(\left(1\tan 1 - \ln\sec 1\right) - \left(0\tan 0 - \ln\sec 0\right)\right)$
	$25\pi\left(1\tan 1 - \ln\sec 1\right)$
	$= \underline{\underline{74}}$ (2sf)

Turn over ▶

Do not write outside the box

6 (a) Sketch, on the axes below, the curve with equation $y = \sin^{-1}(3x)$, where y is in radians.

State the exact values of the coordinates of the end points of the graph.

[3 marks]

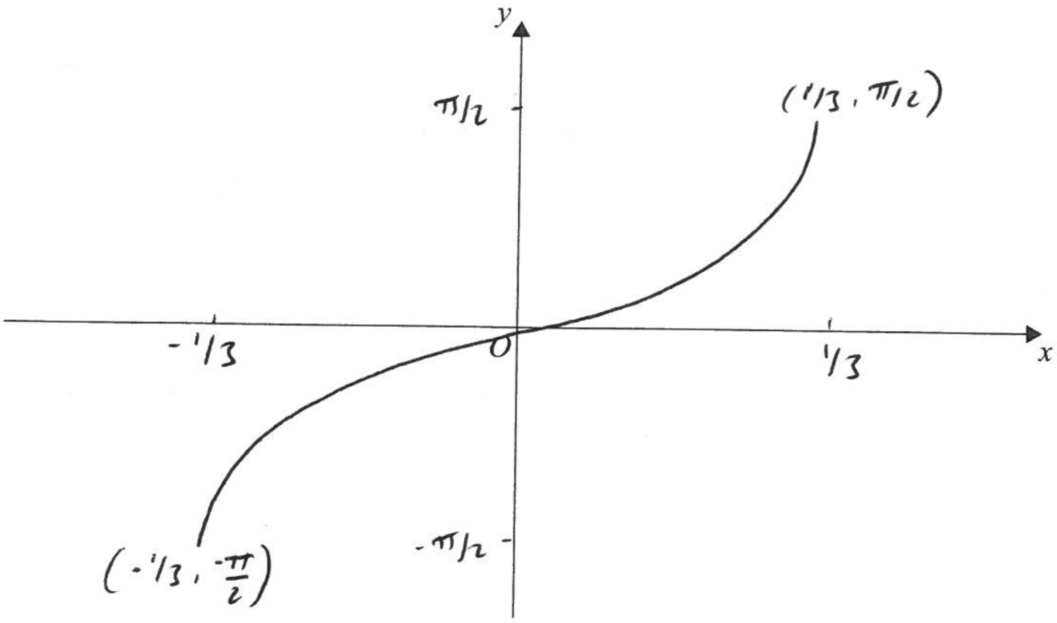
(b) Given that $x = \frac{1}{3}\sin y$, write down $\frac{dx}{dy}$ and hence find $\frac{dy}{dx}$ in terms of y.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

(a)



6a) $y = \sin^{-1} x$ (inverse sin graph) → reflect $y = \sin x$ in $y = x$

$y = \sin^{-1}(3x)$ (x value $\div 3$)

b) $x = \frac{1}{3}\sin y$

$\frac{dx}{dy} = \frac{1}{3}\cos y$

$\frac{dy}{dx} = \frac{1}{\frac{1}{3}\cos y} = \frac{3}{\cos y}$

7 Use the substitution $u = 6 - x^2$ to find the value of $\int_1^2 \frac{x^3}{\sqrt{6-x^2}}\,dx$, giving your answer in the form $p\sqrt{5} + q\sqrt{2}$, where p and q are rational numbers.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 7

7) $u = 6 - x^2 \rightarrow \boxed{x^2 = 6 - u}$

$\frac{dy}{dx} = -2x$

$\boxed{dx = \frac{du}{-2x}}$ when $x = 2 \rightarrow u = 2$

$x = 1 \rightarrow u = 5$

$\int_5^2 \frac{x^3}{u^{1/2}} \frac{du}{-2x}$

$-\frac{1}{2}\int_5^2 \frac{x^2}{u^{1/2}}\,du$

$-\frac{1}{2}\int_5^2 \frac{6-u}{u^{1/2}}\,du$

$-\frac{1}{2}\int_5^2 \frac{6}{u^{1/2}} - \frac{u}{u^{1/2}}\,du$

$-\frac{1}{2}\int_5^2 6u^{-1/2} - u^{1/2}\,du$

$-\frac{1}{2}\left[\frac{6u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right]_5^2$

$-\frac{1}{2}\left(12u^{1/2} - \frac{2}{3}u^{3/2}\right]_5^2$

$\left[-6u^{1/2} + \frac{1}{3}u^{3/2}\right]_5^2$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7

$$\left(-6(2)^{1/2} + \frac{1}{3}(2)^{3/2}\right) - \left(-6(5)^{1/2} + \frac{1}{3}(5)^{3/2}\right)$$

$$-6\sqrt{2} + \frac{1}{3}(2)\sqrt{2} + 6\sqrt{5} - \frac{1}{3}(5)\sqrt{5})$$

$$-6\sqrt{2} + \frac{2}{3}\sqrt{2} + 6\sqrt{5} - \frac{5}{3}\sqrt{5}$$

$$-\frac{16}{3}\sqrt{2} + \frac{13}{3}\sqrt{5}$$

$$\underline{\frac{13}{3}\sqrt{5} - \frac{16}{3}\sqrt{2}}$$

Turn over ▶

Do not write outside the box

8 (a) Show that the equation $4\operatorname{cosec}^2\theta - \cot^2\theta = k$, where $k \neq 4$, can be written in the form

$$\sec^2\theta = \frac{k-1}{k-4}$$

[5 marks]

(b) Hence, or otherwise, solve the equation

$$4\operatorname{cosec}^2(2x + 75°) - \cot^2(2x + 75°) = 5$$

giving all values of x in the interval $0° < x < 180°$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 8

8a) $4\operatorname{cosec}^2\theta - \cot^2\theta = k$

$4(1 + \cot^2\theta) - \cot^2\theta = k$

$4 + 4\cot^2\theta - \cot^2\theta = k$

$4 + 3\cot^2\theta = k$

$3\cot^2\theta = k - 4$

$\cot^2\theta = \dfrac{k-4}{3}$

$\tan^2\theta = \dfrac{3}{k-4}$

$\sec^2\theta - 1 = \dfrac{3}{k-4}$

$\sec^2\theta = \dfrac{3}{k-4} + 1$

$\sec^2\theta = \dfrac{3 + k - 4}{k-4}$

$\sec^2\theta = \dfrac{k-1}{k-4}$ (as req)

QUESTION PART REFERENCE

Answer space for question 8

$k = 5$

b) $\sec^2\theta = \dfrac{5-1}{5-4}$

$\sec^2\theta = 4$ (let $\theta = 2x + 75$)

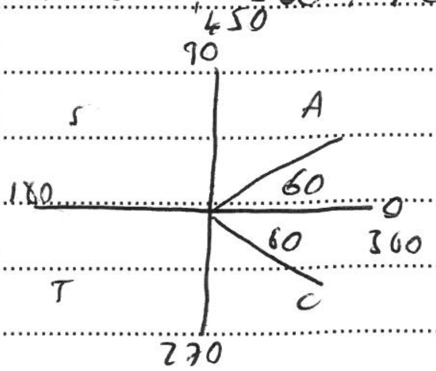
$\sec\theta = \pm\sqrt{4}$

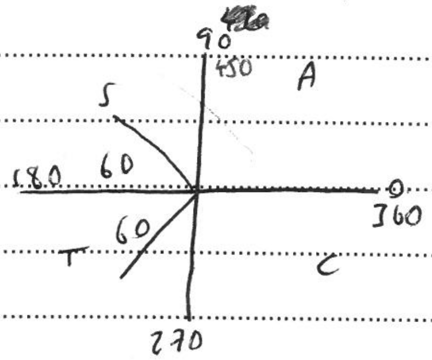
$\boxed{75 < 2x + 75 < 425}$

$\sec\theta = 2$ OR $\sec\theta = -2$

$\cos\theta = \dfrac{1}{2}$ $\cos\theta = -\dfrac{1}{2}$

$\theta = 60^\circ, 300, 420^\circ$ $\theta = \cancel{60}\ 120^\circ, 240$





$2x + 75 = 120, 240, 300, 420$

$x = 22.5, 82.5, 112.5, 172.5^\circ$