Centre Number						Candidate Number				
Surname	WRITTEN SOLUTIONS									
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MPC3

Unit Pure Core 3

Tuesday 10 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^{\pi} x^{\frac{1}{2}} \sin x \, dx$$

Give your answer to four significant figures.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

1) $h = \frac{\pi - 0}{4} = \frac{\pi}{4}$

	y_0	y_1	y_2	y_3	y_4
x	0	$\pi/4$	$\pi/2$	$\frac{3\pi}{4}$	π
y	0	0.62665	1.2533	1.0854	0

odds even

$\frac{1}{3} \times h \times \left(y_0 + y_4 + 4\left(y_1 + y_3\right) + 2\left(y_2\right)\right)$

$\frac{1}{3} \times \frac{\pi}{4} \times \left(0 + 0 + 4\left(0.62665\ldots + 1.0854\right) + 2\left(1.25\right)\right)$

$2.449097\ldots$

$\underline{\underline{2.449}}$ (4 sf)

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

Do not write outside the box

2 A curve has equation $y = 2\ln(2e - x)$.

(a) Find $\frac{dy}{dx}$.

[2 marks]

(b) Find an equation of the normal to the curve $y = 2\ln(2e - x)$ at the point on the curve where $x = e$.

[4 marks]

(c) The curve $y = 2\ln(2e - x)$ intersects the line $y = x$ at a single point, where $x = \alpha$.

(i) Show that α lies between 1 and 3.

[2 marks]

(ii) Use the recurrence relation

$$x_{n+1} = 2\ln(2e - x_n)$$

with $x_1 = 1$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(iii) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = 2\ln(2e - x)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 2

2a) $y = 2\ln(2e - x)$ $u = 2e - x$ $y = 2\ln u$

$\frac{du}{dx} = -1$ $\frac{dy}{dx} = \frac{2}{u}$

$\frac{dy}{dx} = -1 \times \frac{2}{u}$

$= \frac{-2}{2e - x}$

b) when $x = e$, $y = 2\ln(2e - e)$, $\frac{dy}{dx} = \frac{-2}{2e - e} = \frac{-2}{e}$

$= 2\ln e = 2$

$y - 2 = \frac{e}{2}(x - e)$ normal $= \frac{e}{2}$ (m)

$2y - 4 = ex - e^2$ (oe)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

c i) $y = 2\ln(2e - x)$ intersects $y = x$

$f(x) = 2\ln(2e - x) - x$

$f(1) = 2\ln(2e - 1) - 1 = 1.98$

$f(3) = 2\ln(2e - 3) - 3 = -1.22$

change of sign therefore $1 < \alpha < 3$

ii) $x_{n+1} = 2\ln(2e - x_n)$

$x_1 = 1$

$x_2 = 2.97976 = 2.980$ (3dp)

$x_3 = 1.7977... = 1.798$ (3dp)

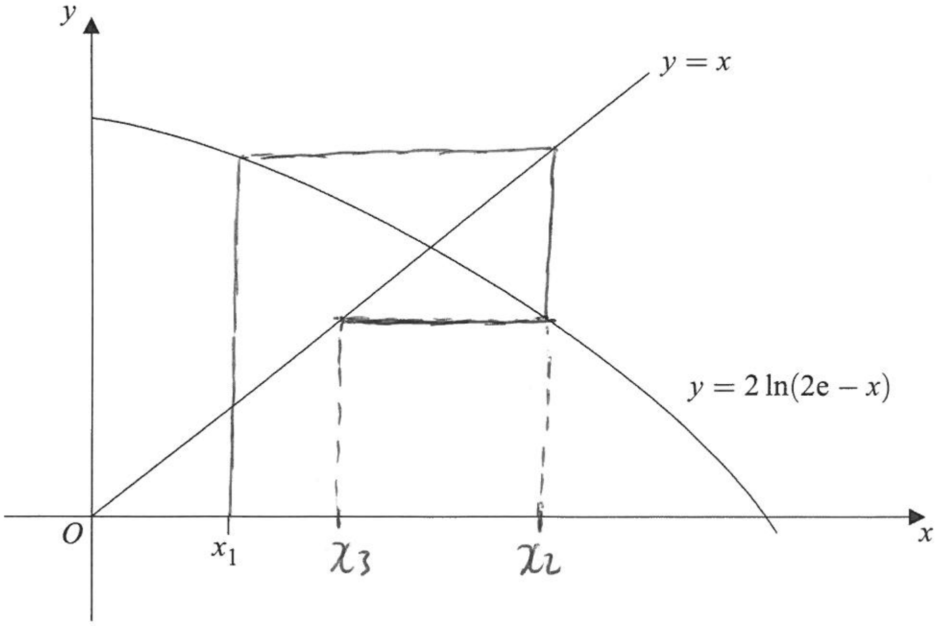
(using calculator)

iii) diagram → up to curve, across to $y = x$ to find next x_n

cobweb diagram (values oscillate)

(c)(iii)

Figure 1



Turn over ▶

3 (a) (i) Differentiate $(x^2+1)^{\frac{5}{2}}$ with respect to x.

[2 marks]

(ii) Given that $y = e^{2x}(x^2+1)^{\frac{5}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 0$.

[3 marks]

(b) A curve has equation $y = \frac{4x-3}{x^2+1}$. Use the quotient rule to find the x-coordinates of the stationary points of the curve.

[5 marks]

Answer space for question 3

3ai) let $y = (x^2+1)^{5/2}$ $\qquad u = x^2+1$

$y = u^{5/2}$ $\qquad \frac{du}{dx} = 2x$

$\frac{dy}{du} = \frac{5}{2}u^{3/2}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \frac{5}{2}u^{3/2} \times 2x$

$= \underline{\underline{5x(x^2+1)^{3/2}}}$

ii) $y = e^{2x}(x^2+1)^{5/2}$ $\qquad u = e^{2x}$ $\qquad v = (x^2+1)^{5/2}$

$\frac{du}{dx} = 2e^{2x}$ $\qquad \frac{dv}{dx} = 5x(x^2+1)^{3/2}$

$\frac{dy}{dx} = 5xe^{2x}(x^2+1)^{3/2} + 2e^{2x}(x^2+1)^{5/2}$

when $x=0$, $\frac{dy}{dx} = 0 + 2(1)$

$= \underline{\underline{2}}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 3

b)

$$y = \frac{4x-3}{x^2+1} \qquad u = 4x-3 \qquad v = x^2+1$$

$$\frac{du}{dx} = 4 \qquad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{4(x^2+1) - 2x(4x-3)}{(x^2+1)^2}$$

$$= \frac{4x^2+4-8x^2+6x}{(x^2+1)^2}$$

$$= \frac{-4x^2+6x+4}{(x^2+1)^2}$$

$\frac{dy}{dx} = 0$ at stationary points :-

$$\frac{-4x^2+6x+4}{(x^2+1)^2} = 0$$

$$-4x^2+6x+4 = 0$$

$$4x^2-6x-4 = 0 \quad (\div 2)$$

$$2x^2-3x-2 = 0$$

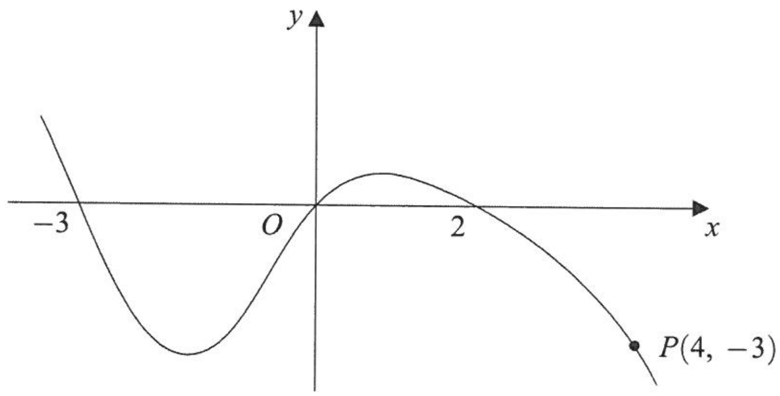
$$(2x+1)(x-2) = 0$$

$$2x+1 = 0 \quad \text{or} \quad x-2 = 0$$

$$\underline{x = -1/2} \qquad \underline{x = 2}$$

Turn over ▶

4 The sketch shows part of the curve with equation $y = \mathrm{f}(x)$.



(a) On **Figure 2** below, sketch the curve with equation $y = -|\mathrm{f}(x)|$.

[3 marks]

(b) On **Figure 3** on the page opposite, sketch the curve with equation $y = \mathrm{f}(|2x|)$.

[2 marks]

(c) (i) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x + 2)$.

[4 marks]

(ii) Find the coordinates of the image of the point $P(4,\ -3)$ under the sequence of transformations given in part **(c)(i)**.

[2 marks]

QUESTION PART REFERENCE

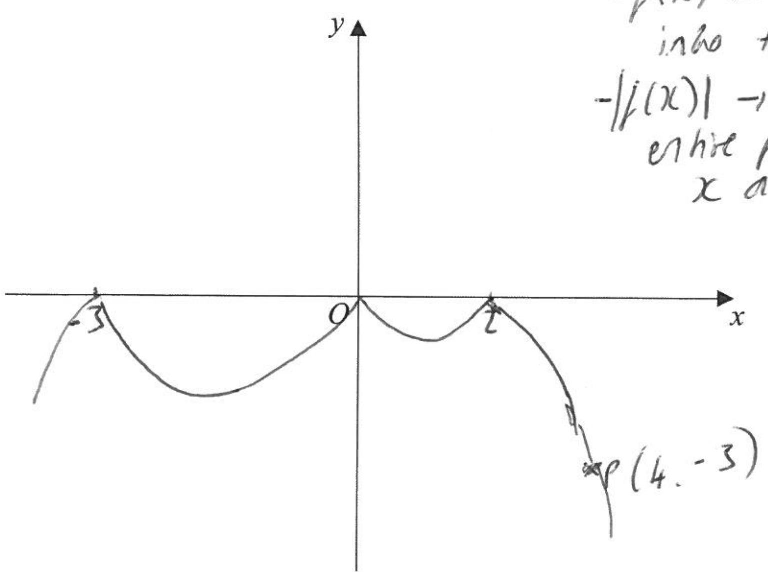
Answer space for question 4

(a)

$|f(x)| \rightarrow$ reflect $-f(x)$ above x axis into $+f(x)$

$-|f(x)| \rightarrow$ reflect entire function in x axis

Figure 2



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QUESTION PART REFERENCE	Answer space for question 4

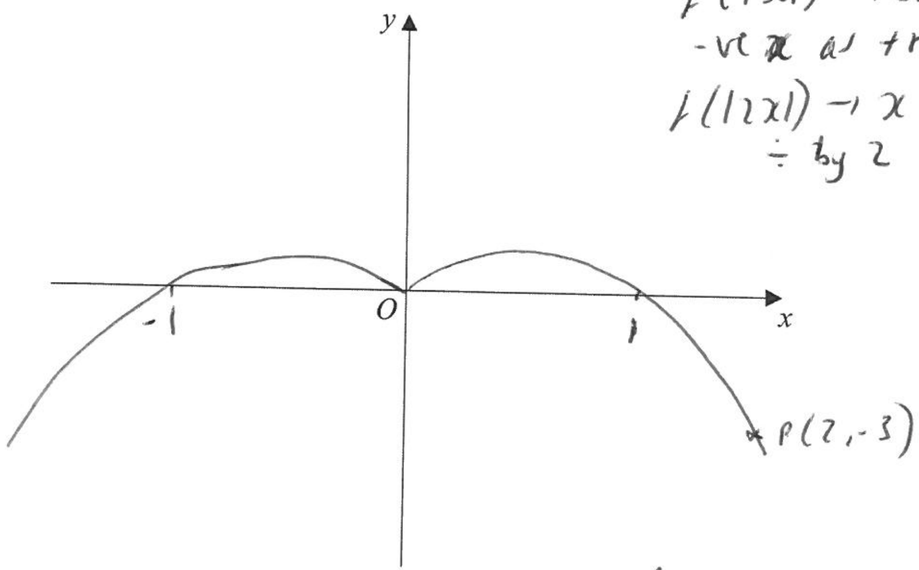
(b)

Figure 3

$f(|2x|)$

$f(|x|) \rightarrow$ same $-ve$ ~~x~~ as $+ve$ x

$f(|2x|) \rightarrow x$ values $\div$ by 2



(symmetrical about y axis)

c i) <u>either</u> $y = f(2x+2) \rightarrow$ translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

<u>followed by</u> stretch s.f. $1/2$ about x axis

<u>or</u> $y = f(2(x+1)) \rightarrow$ stretch s.f. $1/2$ about x axis

<u>followed by</u> translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

ii) $(1, -3)$ $\rightarrow$ x value $\rightarrow -2 \rightarrow \div 2$

$4 \rightarrow 2 \rightarrow 1$

<u>or</u> $\rightarrow x$ value $\rightarrow \div 2 \rightarrow -1$

$4 \rightarrow 2 \rightarrow 1$

y value is unchanged

Turn over ▶

Do not write outside the box

5 The functions f and g are defined with their respective domains by

$$f(x) = x^2 - 6x + 5, \quad \text{for } x \geqslant 3$$

$$g(x) = |x - 6|, \quad \text{for all real values of } x$$

(a) Find the range of f.

[2 marks]

(b) The inverse of f is f^{-1}.

Find $f^{-1}(x)$. Give your answer in its simplest form.

[4 marks]

(c) (i) Find $gf(x)$.

[1 mark]

(ii) Solve the equation $gf(x) = 6$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 5

5) $f(x) = x^2 - 6x + 5 \qquad x \geqslant 3$

a) if $x \geqslant 3$, $f(3) = 3^2 - 6(3) + 5$

$= 9 - 18 + 5$

$= -4$

graph is U shaped, so $f(x) \geqslant -4$

b) $y = x^2 - 6x + 5$ (complete the square)

$y = (x-3)^2 - 9 + 5$

$y = (x-3)^2 - 4 \qquad (+4)$

$y + 4 = (x-3)^2 \qquad (\surd)$

$\pm\sqrt{y+4} = x - 3 \qquad (+3)$

$3 \pm \sqrt{y+4} = x$

$f^{-1}(x) = 3 \pm \sqrt{x+4}$ but $x \geqslant 3$

so, $f^{-1}(x) = 3 + \sqrt{x+4}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5
(i)	$f(x) = x^2 - 6x + 5$
	$g(x) = \lvert x - 6 \rvert$
	$gf(x) = g(x^2 - 6x + 5)$
	$= \lvert x^2 - 6x + 5 - 6 \rvert$
	$gf(x) = \lvert x^2 - 6x - 1 \rvert$
(ii)	$gf(x) = 6$
	$\lvert x^2 - 6x - 1 \rvert = 6$
	$x^2 - 6x - 1 = 6$ or $x^2 - 6x - 1 = -6$
	$x^2 - 6x - 7 = 0$ $x^2 - 6x + 5 = 0$
	$(x - 7)(x + 1) = 0$ $(x - 5)(x - 1) = 0$
	$x = 7$ or $x = -1$ $x = 5$ or $x = 1$
	but $x \geq 3$
	so, $x = 5$, $x = 7$

Do not write outside the box

6 (a) By using integration by parts twice, find

$$\int x^2 \sin 2x \, dx$$

[6 marks]

(b) A curve has equation $y = x\sqrt{\sin 2x}$, for $0 \leqslant x \leqslant \frac{\pi}{2}$.

The region bounded by the curve and the x-axis is rotated through 2π radians about the x-axis to generate a solid.

Find the exact value of the volume of the solid generated.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 6

6a) $\int x^2 \sin 2x \, dx$

$\boxed{u = x^2 \quad \frac{dv}{dx} = \sin 2x \quad \frac{du}{dx} = 2x \quad v = -\frac{1}{2}\cos 2x}$

$-\frac{x^2}{2}\cos 2x - \int -x\cos 2x \, dx$

$-\frac{x^2}{2}\cos 2x + \int x\cos 2x \, dx$

$\boxed{u = x \quad \frac{dv}{dx} = \cos 2x \quad \frac{du}{dx} = 1 \quad v = \frac{1}{2}\sin 2x}$

$-\frac{x^2}{2}\cos 2x + \left[\frac{x}{2}\sin 2x - \int \frac{1}{2}\sin 2x \, dx\right]$

$-\frac{x^2}{2}\cos 2x + \left[\frac{x}{2}\sin 2x - \left(-\frac{1}{4}\cos 2x\right)\right] + c$

$-\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + c$

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QUESTION PART REFERENCE

Answer space for question 6

b) $V = \pi \int y^2 \, dx$

$y = x\sqrt{\sin 2x}$

$y^2 = x^2 \sin 2x$

$V = \pi \int_0^{\pi/2} x^2 \sin 2x \, dx$

$= \pi \left[-\frac{x^2}{2} \cos 2x + \frac{x}{2} \sin 2x + \frac{1}{4} \cos 2x \right]_0^{\pi/2}$

$= \pi \left[\left(-\frac{(\pi/2)^2}{2} \cos 2(\pi/2) + \frac{\pi/2}{2} \sin 2(\pi/2) + \frac{1}{4} \cos 2(\pi/2) \right) - \left(\frac{0}{2} \cos 0 + \frac{0}{2} \sin 0 + \frac{1}{4} \cos 0 \right) \right]$

$= \pi \left[\left(-\frac{\pi^2}{8}(-1) + 0 \cancel{+} -\frac{1}{4} \right) - \left(0 + 0 + \frac{1}{4} \right) \right]$

$= \pi \left[\frac{\pi^2}{8} - \frac{1}{4} - \frac{1}{4} \right]$

$= \pi \left(\frac{\pi^2}{8} - \frac{1}{2} \right)$

Turn over ▶

Do not write outside the box

7 Use the substitution $u = 3 - x^3$ to find the exact value of $\int_0^1 \frac{x^5}{3 - x^3}\,dx$.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 7

7) $\int_0^1 \frac{x^5}{3-x^3}\,dx$

$u = 3 - x^3$

$\frac{du}{dx} = -3x^2$

$\boxed{dx = \frac{du}{-3x^2}}$

$\int_3^2 \frac{\cancel{x^5}^{\,x^3}}{u} \frac{du}{-3\cancel{x^2}}$

$\int_3^2 \frac{x^3}{u} \frac{du}{-3}$

$x = 1,\ u = 3 - 1 = 2$

$x = 0,\ u = 3 - 0 = 3$

$-\frac{1}{3}\int_3^2 \frac{3-u}{u}\,du$

$\boxed{x^3 = 3 - u}$

$-\frac{1}{3}\int_3^2 \frac{3}{u} - \frac{u}{u}\,du$

$-\frac{1}{3}\int_3^2 \frac{3}{u} - 1\,du$

$-\frac{1}{3}\left[3\ln u - u\right]_3^2$

$-\frac{1}{3}\left((3\ln 2 - 2) - (3\ln 3 - 3)\right)$

$-\frac{1}{3}\left(3\ln 2 - 3\ln 3 - 2 + 3\right)$

$-\frac{1}{3}\left(3\ln 2 - 3\ln 3 + 1\right) = -\ln 2 + \ln 3 - \frac{1}{3}$

$= \ln\frac{3}{2} - \frac{1}{3}$ (o.e.)

1 4

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QUESTION PART REFERENCE	Answer space for question 7

Turn over ▶

Do not write outside the box

8 (a) Show that the expression $\dfrac{1-\sin x}{\cos x}+\dfrac{\cos x}{1-\sin x}$ can be written as $2\sec x$.

[4 marks]

(b) Hence solve the equation

$$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=\tan^2 x-2$$

giving the values of x to the nearest degree in the interval $0° \leqslant x < 360°$.

[6 marks]

(c) Hence solve the equation

$$\frac{1-\sin(2\theta-30°)}{\cos(2\theta-30°)}+\frac{\cos(2\theta-30°)}{1-\sin(2\theta-30°)}=\tan^2(2\theta-30°)-2$$

giving the values of θ to the nearest degree in the interval $0° \leqslant \theta \leqslant 180°$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 8

8a) $\dfrac{1-\sin x}{\cos x}+\dfrac{\cos x}{1-\sin x}$

$\dfrac{(1-\sin x)^2+\cos^2 x}{\cos x(1-\sin x)}$

$\dfrac{1-2\sin x+\sin^2 x+\cos^2 x}{\cos x(1-\sin x)}$ $\boxed{\sin^2 x+\cos^2 x=1}$

$\dfrac{1-2\sin x+1}{\cos x(1-\sin x)} = \dfrac{2-2\sin x}{\cos x(1-\sin x)}$

$= \dfrac{2\cancel{(1-\sin x)}}{\cos x\cancel{(1-\sin x)}}$

$= \dfrac{2}{\cos x}$

$= 2\sec x$ (as req)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

b) $\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} = \tan^2 x - 2$

$2\sec x = \tan^2 x - 2$

$\tan^2 x - 2\sec x - 2 = 0$

$\boxed{\tan^2 x = \sec^2 x - 1}$

$\sec^2 x - 1 - 2\sec x - 2 = 0$

$\sec^2 x - 2\sec x - 3 = 0$

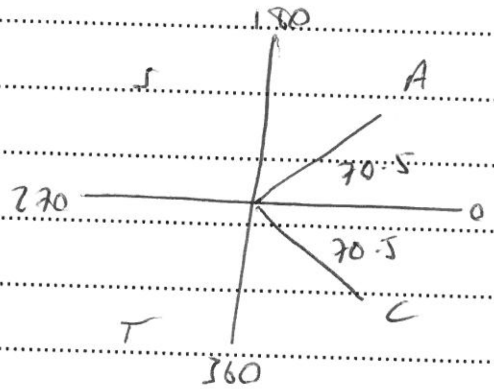
$(\sec x - 3)(\sec x + 1) = 0$

$\sec x = 3$ or $\sec x = -1$

$\cos x = \frac{1}{3}$ $\cos x = -1$

$x = \cos^{-1}\left(\frac{1}{3}\right)$ $x = \cos^{-1}(-1)$

$x = 70.5\ldots, 289.4\ldots$ $x = 180°$



so, $x = 71°, 180°, 289°$ (nearest °)

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8
c)	let $x = 2\theta - 30$ $\quad\quad 0 \leq \theta \leq 180$
	$-30 \leq 2\theta - 30 \leq 330$
	$2\theta - 30 = 70.5,\ 180,\ 289.5$ *
	$2\theta = 100.5,\ 210,\ 319.5$
	$\theta = 50.25,\ 105,\ 159.75$
	$\theta = \underline{50^\circ},\ \underline{105^\circ},\ \underline{160^\circ}$ (nearest degree)
	* keep <u>unrounded</u> answers to start with *

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QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS

There are no questions printed on this page

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