For Examiner's Use	
Examiner's Initials	

Question	Mark
1	
2	
3	
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10	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics   MPC3

Unit Pure Core 3

Thursday 6 June 2013   9.00 am to 10.30 am

For this paper you must have:
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed
• 1 hour 30 minutes

Instructions
• Use black ink or black ball-point pen. Pencil should only be used for drawing.
• Fill in the boxes at the top of this page.
• Answer **all** questions.
• Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
• You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
• Do not write outside the box around each page.
• Show all necessary working; otherwise marks for method may be lost.
• Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
• The marks for questions are shown in brackets.
• The maximum mark for this paper is 75.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.
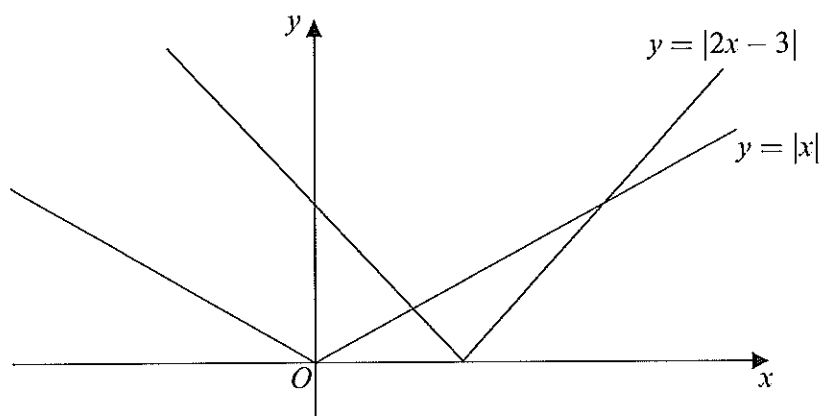• You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1   The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.

(a)   Find the x-coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$.

(3 marks)

(b)   Hence, or otherwise, solve the inequality

$$|2x - 3| \geq |x|$$

(2 marks)

QUESTION PART REFERENCE	Answer space for question 1
(a)	$2x - 3 = x$ $\quad\quad$ $2x - 3 = -x$
	$x = 3$ $\quad\quad\quad\quad$ $3x = 3$
	$\quad\quad\quad\quad\quad\quad\quad$ $x = 1$
b)	$x \leq 1$, $\quad$ $x \geq 3$
	(As $y = |2x - 3|$ is higher than $y = |x|$
	at those points)

QUESTION PART REFERENCE	Answer space for question 1

Turn over ▶

2 (a) Given that $y = x^4 \tan 2x$, find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Find the gradient of the curve with equation $y = \dfrac{x^2}{x-1}$ at the point where $x = 3$. *(3 marks)*

QUESTION PART REFERENCE	Answer space for question 2

2a) $y = x^4 \tan 2x$ $\qquad u = x^4 \qquad v = \tan 2x$

$\qquad\qquad\qquad\qquad\qquad \dfrac{du}{dx} = 4x^3 \quad \dfrac{dv}{dx} = 2\sec^2 2x$

$\dfrac{dy}{dx} = 4x^3 \tan 2x + 2x^4 \sec^2 2x$

b) $y = \dfrac{x^2}{(x-1)} \qquad u = x^2 \qquad v = x+1$

$\qquad\qquad\qquad\qquad \dfrac{du}{dx} = 2x \qquad \dfrac{dv}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{2x(x+1) - x^2}{(x+1)^2}$

$= \dfrac{2x^2 + 2x - x^2}{(x+1)^2}$

$= \dfrac{x^2 + 2x}{(x+1)^2}$

when $x = 3$,

$\dfrac{dy}{dx} = \dfrac{3^2 + 2(3)}{(3+1)^2} = \dfrac{3}{4}$

QUESTION PART REFERENCE	Answer space for question 2

Turn over ▶

0 5

3 (a) The equation $e^{-x} - 2 + \sqrt{x} = 0$ has a single root, α.

Show that α lies between 3 and 4. *(2 marks)*

(b) Use the recurrence relation $x_{n+1} = (2 - e^{-x_n})^2$, with $x_1 = 3.5$, to find x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(c) The diagram on the opposite page shows parts of the graphs of $y = (2 - e^{-x})^2$ and $y = x$, and a position of x_1.

On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

QUESTION PART REFERENCE	**Answer space for question 3**

3a) $e^{-x} - 2 + \sqrt{x} = 0$

$f(3) = -0.218$

$f(4) = 0.0183$

change of sign $\therefore$ $3 < \alpha < 4$
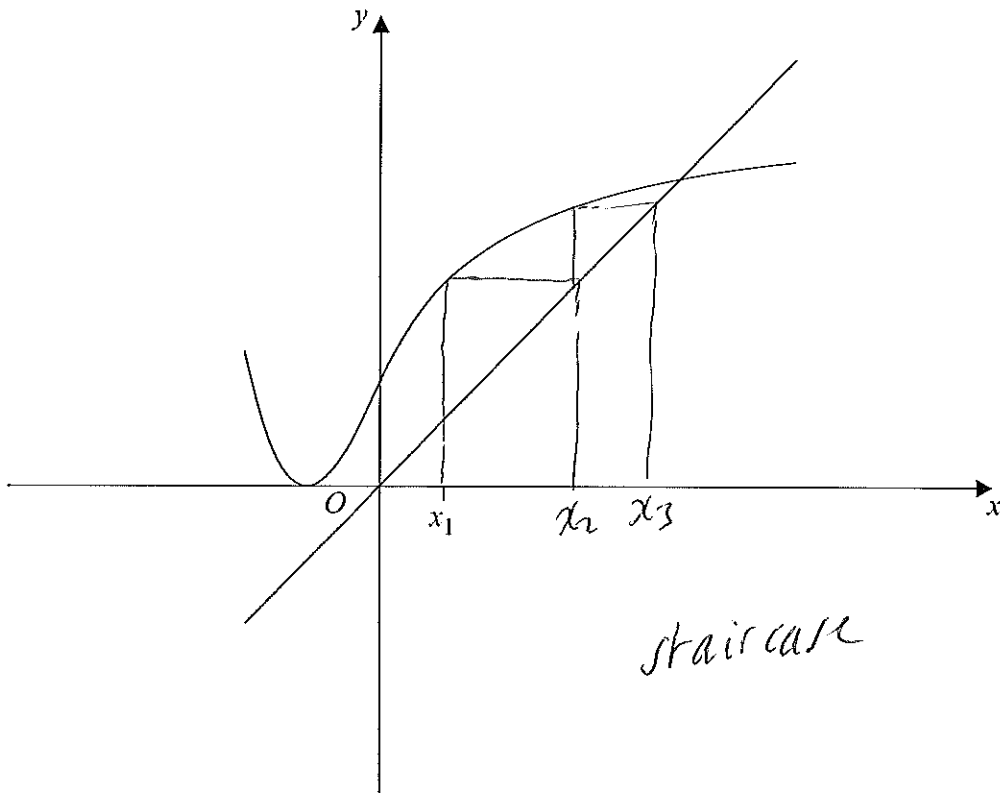
b) $x_1 = 3.5$

$x_2 = 3.880$

$x_3 = 3.918$

c)

QUESTION PART REFERENCE	Answer space for question 3

(c)

staircase

4 By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures.

(7 marks)

QUESTION PART REFERENCE	Answer space for question 4

4) $\sec^2 x = \tan^2 x + 1$ $\boxed{\tan^2 x = \sec^2 x - 1}$

$8 \sec x - 2 \sec^2 x = \sec^2 x - 1 - 2$

$8 \sec x - 2 \sec^2 x = \sec^2 x - 3$

$3 \sec^2 x - 8 \sec x - 3 = 0$

$(3 \sec x + 1)(\sec x - 3) = 0$
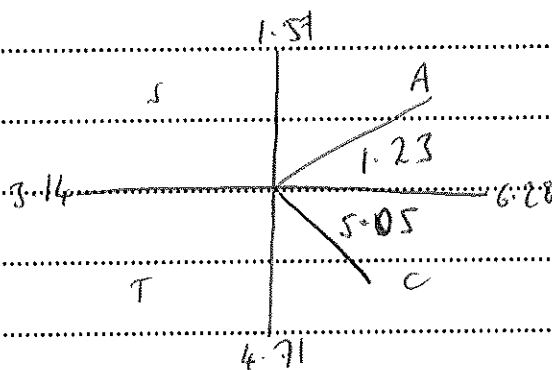
$3 \sec x = -1 \qquad \sec x = 3$

$\sec x = -1/3 \qquad \cos x = 1/3$

$\cos x = -3 \times$
n not a real solution $\qquad x = \cos^{-1}(1/3)$

$x = 1.23, \ 5.05$

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QUESTION PART REFERENCE	Answer space for question 4

5   The diagram shows a sketch of the graph of $y = \sqrt{27 + x^3}$.
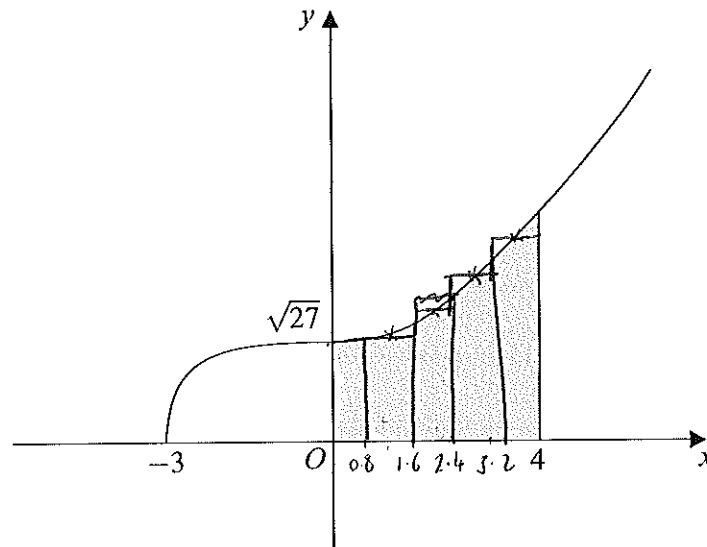
(a)   The area of the shaded region, bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$, is given by $\int_0^4 \sqrt{27 + x^3}\, dx$.

Use the mid-ordinate rule with **five** strips to find an estimate for this area. Give your answer to three significant figures. *(4 marks)*

(b)   With the aid of a diagram, explain whether the mid-ordinate rule applied in part **(a)** gives an estimate which is smaller than or greater than the area of the shaded region. *(2 marks)*

QUESTION PART REFERENCE	**Answer space for question 5**

5a) $\int_0^4 \sqrt{27 + x^3}\, dx$ $h = \dfrac{4 - 0}{5} = 4/5$

x	0.4	1.2	2.0	2.8	3.6
y	5.20231	5.35785	5.91608	6.99657	8.58231

$= 0.8 (5.202 + 5.359 + 5.916 + 6.996 + 8.582)$

$= 25.6$

b) Smaller as rectangles generally below the curve

| QUESTION PART REFERENCE	Answer space for question 5

Turn over ▶

6 (a)   Sketch the graph of $y = \cos^{-1} x$, where y is in radians. State the coordinates of the
end points of the graph.   *(2 marks)*

(b)   Sketch the graph of $y = \pi - \cos^{-1} x$, where y is in radians. State the coordinates of
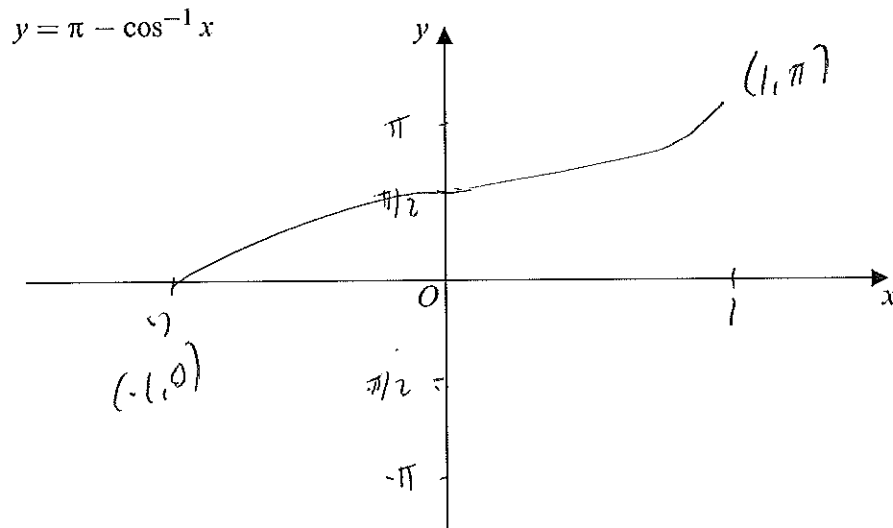the end points of the graph.   *(2 marks)*

Answer space for question 6

(a)

$y = \cos^{-1} x$

$(-1, \pi)$

$(1, 0)$

(b)

$y = \pi - \cos^{-1} x$

$(1, \pi)$

$(-1, 0)$

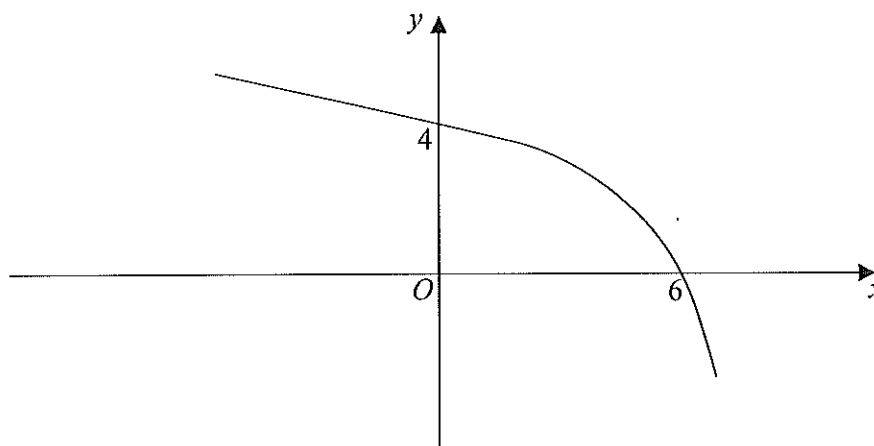| QUESTION PART REFERENCE	Answer space for question 6

Turn over ▶
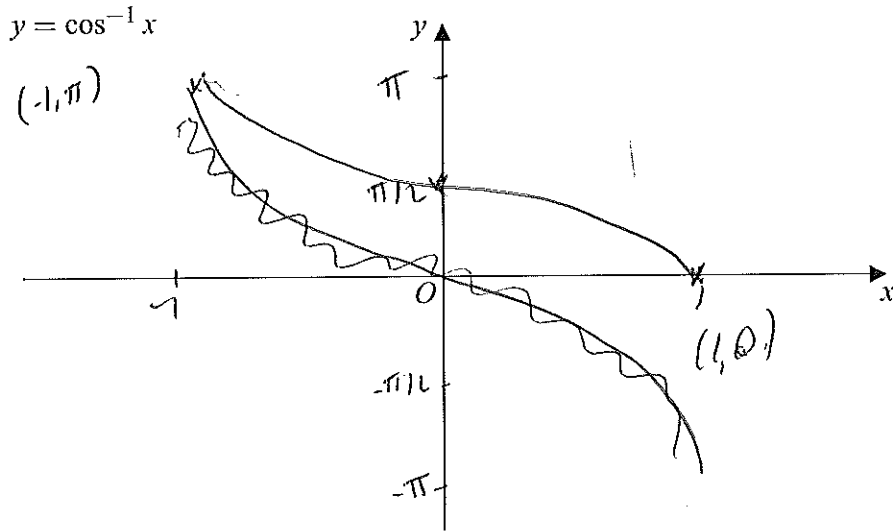
7 The diagram shows a sketch of the curve with equation $y = f(x)$.

(a) On **Figure 1**, below, sketch the curve with equation $y = -f(3x)$, indicating the values where the curve cuts the coordinate axes. *(2 marks)*

(b) On **Figure 2**, on the opposite page, sketch the curve with equation $y = f(|x|)$, indicating the values where the curve cuts the coordinate axes. *(3 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f\left(-\frac{1}{2}x\right)$. *(4 marks)*
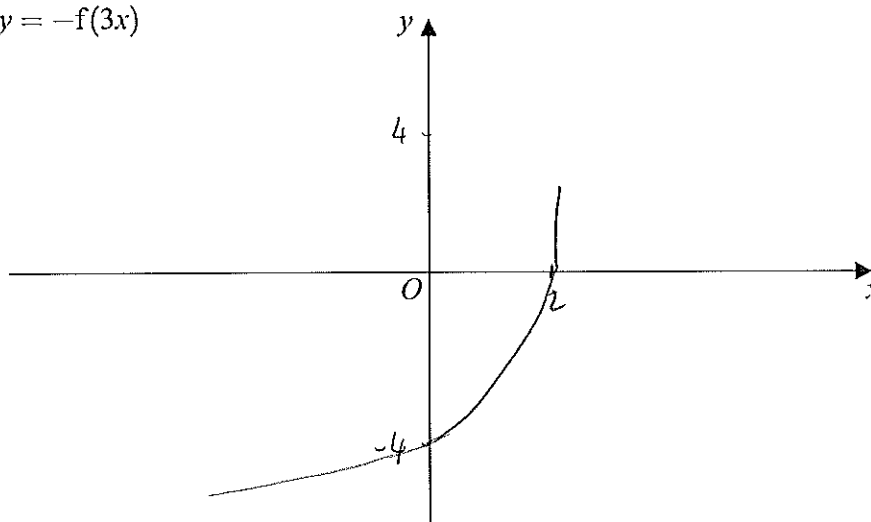
QUESTION PART REFERENCE	Answer space for question 7

Figure 1

(a) $y = -f(3x)$

QUESTION PART REFERENCE

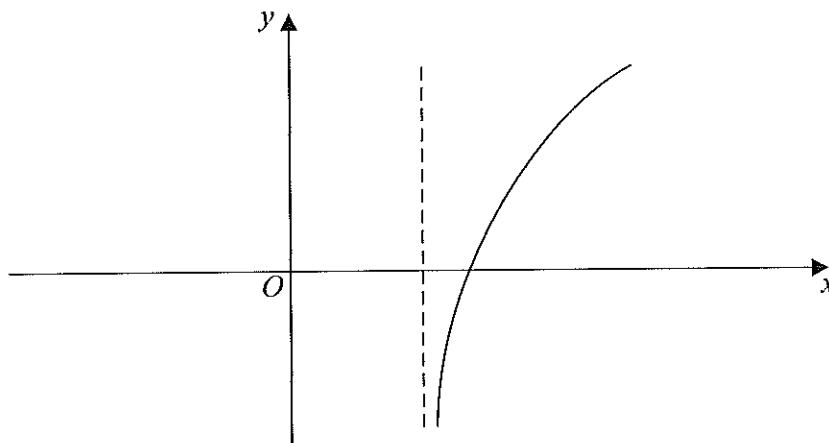**Answer space for question 7**

(b)

Figure 2

$y = f(|x|)$
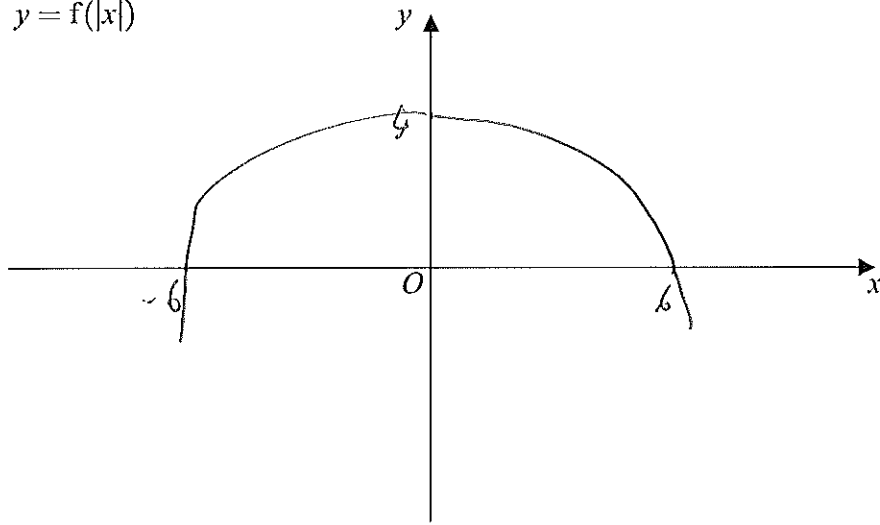
(c)

reflection in y axis

stretch 1/2 in x direction

8   The curve with equation $y = f(x)$, where $f(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.

(a)   The inverse of f is f^{-1}.

(i)   Find $f^{-1}(x)$.   *(3 marks)*

(ii) State the range of f^{-1}.   *(1 mark)*

(iii) Sketch, on the axes given on the opposite page, the curve with equation $y = f^{-1}(x)$, indicating the value of the y-coordinate of the point where the curve intersects the y-axis.   *(2 marks)*

(b)   The function g is defined by

$$g(x) = e^{2x} - 4, \quad \text{for all real values of } x$$

(i)   Find $gf(x)$, giving your answer in the form $(ax - b)^2 - c$, where a, b and c are integers.   *(3 marks)*

(ii) Write down an expression for $fg(x)$, and hence find the exact solution of the equation $fg(x) = \ln 5$.   *(3 marks)*

QUESTION PART REFERENCE	Answer space for question 8
(ai)	$y = \ln(2x - 3)$
	$x = \ln(2y - 3)$
	$e^x = 2y - 3$ $f^{-1}(x) = \dfrac{e^x + 3}{2}$ or $\dfrac{1}{2}(e^x + 3)$
	$2y = e^x + 3$
	$y = \dfrac{e^x + 3}{2}$

QUESTION PART REFERENCE	**Answer space for question 8**

ii) range of $f^{-1}(x)$ same as domain of $f(x)$

$\therefore f^{-1}(x) > 3/2$

b) $g(x) = e^{2x} - 4$

$gf(x) = g'(\ln(2x-3))$
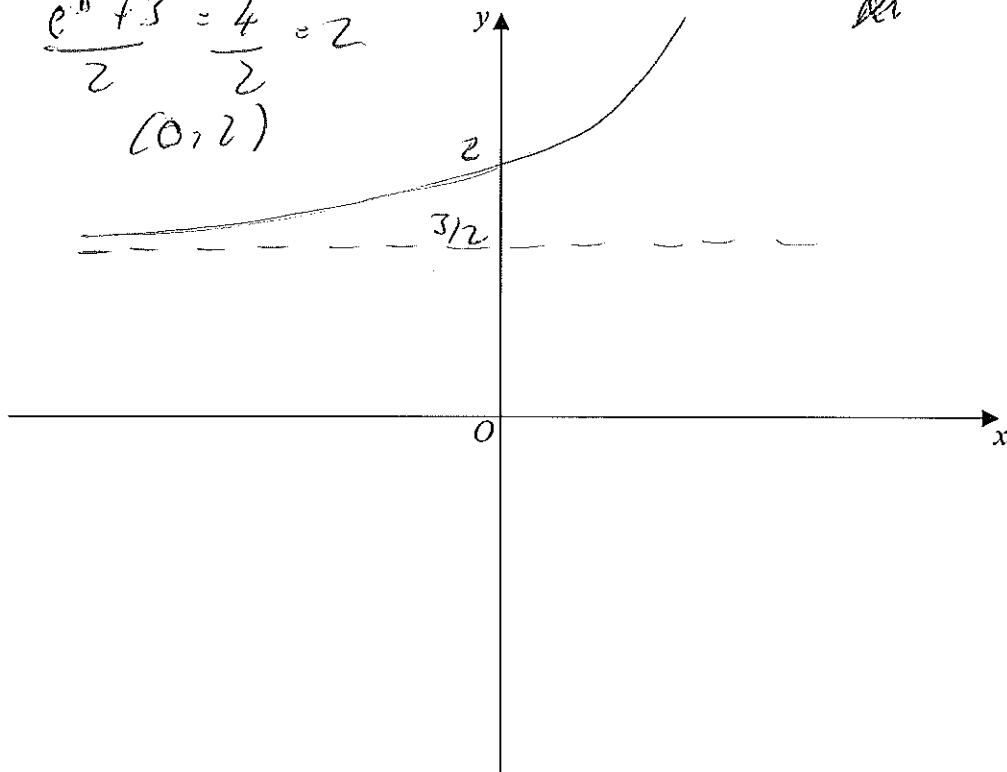
$= e^{2(\ln(2x-3))} - 4$

$= (2x-3)^2 - 4 \quad (\text{as req})$

ii) $fg(x) = f(e^{2x} - 4)$

$= \ln(2(e^{2x}-4) - 3)$

$\ln 5 = \ln(2e^{2x} - 11)$

$5 = 2e^{2x} - 11$

$2e^{2x} = 16$

$e^{2x} = 8$

$2x = \ln 8$

$x = \frac{1}{2} \ln 8$
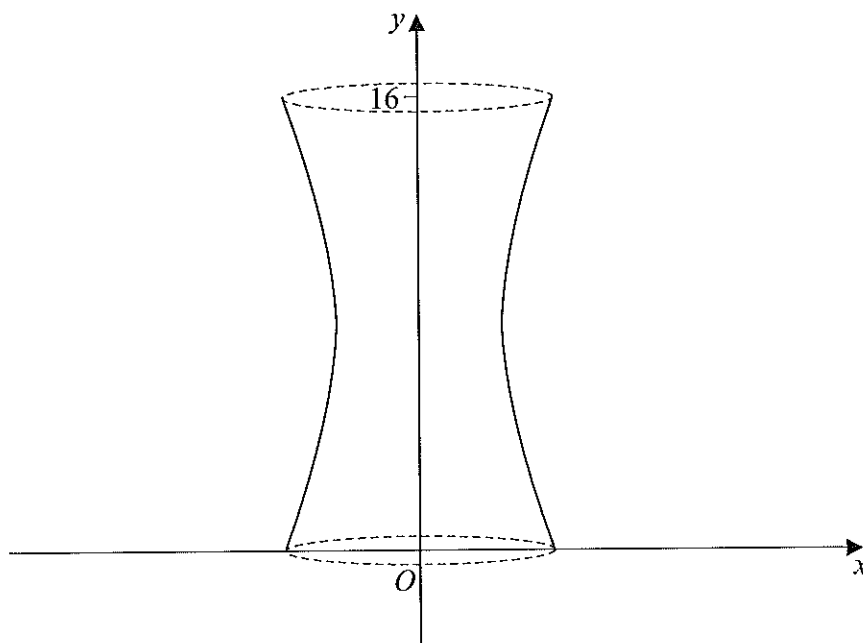
(a)(iii)

$\frac{e^0 + 3}{2} = \frac{4}{2} = 2$

$(0, 2)$

9 The shape of a vase can be modelled by rotating the curve with equation
$16x^2 - (y-8)^2 = 32$ between $y = 0$ and $y = 16$ completely **about the *y*-axis**.

The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value.

(5 marks)

QUESTION PART REFERENCE	**Answer space for question 9**

9) $\pi \int_0^{16} x^2 \, dy$

$16x^2 - (y-8)^2 = 32$

$16x^2 = 32 + (y-8)^2$

$\pi \int_0^{16} \frac{1}{16}(32 + (y-8)^2) \, dy$

$x^2 = \frac{1}{16}(32 + (y-8)^2)$

$\frac{\pi}{16} \int_0^{16} 32 + y^2 - 16y + 64 \, dy$

$\frac{\pi}{16} \int_0^{16} y^2 - 16y + 96 \, dy$

$\frac{\pi}{16} \left[\frac{y^3}{3} - \frac{16y^2}{2} + 96y \right]_0^{16} = \frac{\pi}{16} \left[\frac{y^3}{3} - 8y^2 + 96y \right]_0^{16}$

Answer space for question 9

$$\frac{\pi}{16}\left[\left(\frac{16^3}{3} - 8(16)^2 + 96(16)\right) - 0\right]$$

$$\frac{\pi}{16}\left(\frac{4096}{3} - 2048 + 1536\right)$$

$$\frac{160\pi}{3}$$

10 (a) (i) By writing $\ln x$ as $(\ln x) \times 1$, use integration by parts to find $\int \ln x \, dx$. *(4 marks)*

(ii) Find $\int (\ln x)^2 \, dx$. *(4 marks)*

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$$ *(7 marks)*

| QUESTION PART REFERENCE | Answer space for question 10 |

$u = \ln x$

10(a)i) $\int \ln x \, dx$ $\qquad \frac{du}{dx} = \frac{1}{x} \qquad \frac{dv}{dx} = 1$

$x \ln x - \int \frac{x}{x} \, dx \qquad\qquad v = x$

$x \ln x - \int 1 \, dx$

$x \ln x - x + C$

ii) $\int (\ln x)^2 \, dx \qquad u = (\ln x)^2 \quad w = \ln x \qquad u = w^2$

$\qquad\qquad\qquad\qquad \frac{du}{dx} = 2 \ln x \quad \frac{dw}{dx} = \frac{1}{x} \quad \frac{du}{dw} = 2w$

$x (\ln x)^2 - \int \frac{2 x \ln x}{x} \, dx$

$\qquad\qquad\qquad\qquad\qquad \frac{dv}{dx} = 1 \quad v = x$

$x (\ln x)^2 - \int 2 \ln x \, dx$

$x (\ln x)^2 - 2 x \ln x - 2x + C$

Answer space for question 10

b) $\int_1^4 \frac{1}{x+\sqrt{x}} \, dx$ $\qquad$ $u = \sqrt{x}$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$

$\int \frac{1}{x+\sqrt{x}} \, 2\sqrt{x} \, du$ $\qquad$ $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$\int_1^2 \frac{1}{u^2+u} \, 2u \, du$ $\qquad$ $dx = 2\sqrt{x} \, du$

$u^2 = x$

$\int_1^2 \frac{2u}{u^2+u} \, du$

$\int_1^2 \frac{2}{u+1} \, du$ $\qquad$ $x = 4, \quad u = 2$

$x = 1, \quad u = 1$

$\left[2\ln(u+1) \right]_1^2$

$2\ln 3 - 2\ln 2$

$OR \quad 2\ln\left(\frac{3}{2}\right)$

$\ln\left(\frac{3}{2}\right)^2 = \ln\left(\frac{9}{4}\right) \quad OR \quad 2\ln\frac{3}{2}$

QUESTION PART REFERENCE	Answer space for question 10

END OF QUESTIONS