Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	WRITTEN SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MPC3

Unit Pure Core 3

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

J A N 1 3 M P C 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Show that the equation $x^3 - 6x + 1 = 0$ has a root α, where $2 < \alpha < 3$. *(2 marks)*

(b) Show that the equation $x^3 - 6x + 1 = 0$ can be rearranged into the form

$$x^2 = 6 - \frac{1}{x}$$ *(1 mark)*

(c) Use the recurrence relation $x_{n+1} = \sqrt{6 - \frac{1}{x_n}}$, with $x_1 = 2.5$, to find the value of x_3, giving your answer to four significant figures. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1a) $x^3 - 6x + 1 = 0$

$f(x) = x^3 - 6x + 1$

$f(2) = (2)^3 - 6(2) + 1$

$= 8 - 12 + 1 = -3$

$f(3) = (3)^3 - 6(3) + 1$

$= 27 - 18 + 1 = 10$

change of sign therefore $2 < \alpha < 3$

b) $x^3 - 6x + 1 = 0$

$x^3 = 6x - 1 \quad (\div x)$

$\underline{x^2 = 6 - 1/x}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

c) $x_{n+1} = \sqrt{6 - \frac{1}{x_n}}$

$x_1 = 2.5$

$x_2 = 2.366431913$

$x_3 = 2.361656807$

$= \underline{\underline{2.362}}$ (4 d.p.)

Turn over ▶

2 (a) Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^4 \frac{x}{x^2+2}\,dx$$

Give your answer to four significant figures. *(4 marks)*

(b) Show that the exact value of $\int_0^4 \frac{x}{x^2+2}\,dx$ is $\ln k$, where k is an integer. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2a) $\int_0^4 \frac{x}{x^2+2}\,dx$

$h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	$1/3$	$1/3$	$3/11$	$2/9$
	y_0	y_1	y_2	y_3	y_4

$\frac{1}{3} \times 1 \left(0 + \frac{2}{9} + 4\left(\underbrace{1/3 + 3/11}_{\text{odds}}\right) + 2\left(\underbrace{1/3}_{\text{evens}}\right)\right)$

$= \frac{1}{3}\left(\frac{2}{9} + 2.4242\ldots + 2/3\right)$

$= 1.104377104$

$= \underline{1.104}$ (4 s.f.)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

b) $\int_0^4 \frac{x}{x^2+2}\,dx$

$u = x^2+2$

$\frac{du}{dx} = 2x$

$\boxed{dx = \frac{du}{2x}}$

$\int \frac{1}{2u}\,du$

$= \frac{1}{2}\ln u$

$= \left[\frac{1}{2}\ln(x^2+2)\right]_0^4$

$= \frac{1}{2}\left(\ln 18 - \ln 2\right)$

$= \frac{1}{2}\ln\frac{18}{2}$

$= \frac{1}{2}\ln 9$

$= \underline{\underline{\ln 3}}$ $(\ln 9^{1/2} \to \ln\sqrt{9})$

3 (a) Find $\dfrac{dy}{dx}$ when

$$y = e^{3x} + \ln x$$

(2 marks)

(b) (i) Given that $u = \dfrac{\sin x}{1 + \cos x}$, show that $\dfrac{du}{dx} = \dfrac{1}{1 + \cos x}$. *(3 marks)*

(ii) Hence show that if $y = \ln\left(\dfrac{\sin x}{1 + \cos x}\right)$, then $\dfrac{dy}{dx} = \operatorname{cosec} x$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3) $y = e^{3x} + \ln x$

$\dfrac{dy}{dx} = 3e^{3x} + \dfrac{1}{x}$

bi) $u = \dfrac{\sin x}{1+\cos x}$

$\boxed{w = \sin x \quad v = 1+\cos x \quad \dfrac{dw}{dx} = \cos x \quad \dfrac{dv}{dx} = -\sin x}$

$\dfrac{du}{dx} = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$= \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$

$\boxed{\cos^2 x + \sin^2 x = 1}$

$= \dfrac{\cos x + 1}{(1+\cos x)(1+\cos x)}$

$= \dfrac{1}{1+\cos x}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

ii) $y = \ln\left(\frac{\sin x}{1+\cos(x)}\right)$ $\qquad u = \frac{\sin x}{1+\cos x}$

$\frac{du}{dx} = \frac{1}{1+\cos x}$

$y = \ln u$

$\frac{dy}{du} = \frac{1}{u} = \frac{1+\cos x}{\sin x}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

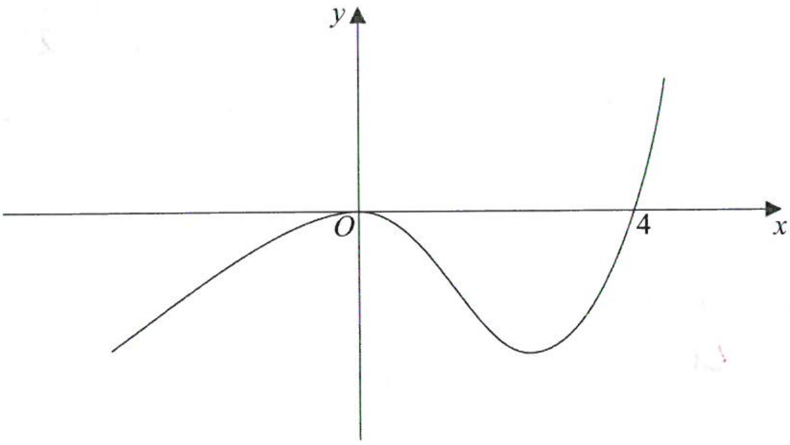
$= \frac{1}{u} \times \frac{1}{1+\cos x}$

$= \frac{\cancel{1+\cos x}}{\sin x} \times \frac{1}{\cancel{1+\cos x}}$

$= \frac{1}{\sin x} = \underline{\underline{\operatorname{cosec} x}}$ (as required)

Turn over ▶

4 The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$.



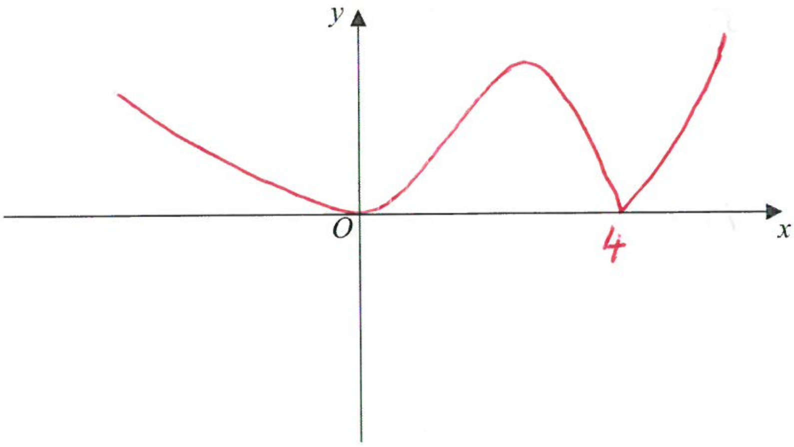
(a) On the axes below, sketch the curve with equation $y = |\mathrm{f}(x)|$. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x - 1)$. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 4

(a)



4a) $y = |f(x)|$ - reflect -ve y in x axis

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

b) $y = f(x) \rightarrow y = f(2x+1)$
$= f(2(x+0.5))$

stretch scale factor $\frac{1}{2}$ in x axis
translation $\begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$

OR
translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
stretch scale factor $1/2$ in x axis

5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \quad \text{for real values of } x, \textbf{ where } \boldsymbol{x \leqslant 0}$$

(a) State the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

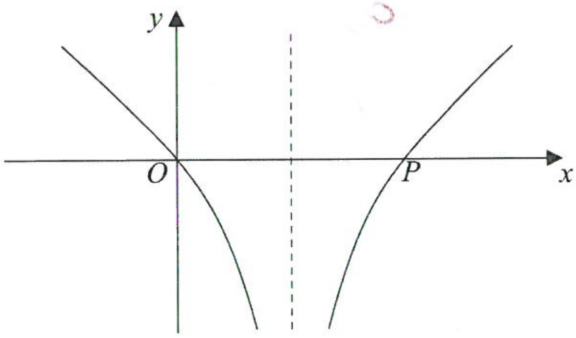
(i) Write down the domain of f^{-1}. *(1 mark)*

(ii) Find an expression for $f^{-1}(x)$. *(3 marks)*

(c) The function g is defined by

$$g(x) = \ln|3x - 1|, \quad \text{for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x-axis at the origin and at the point P.

Find the x-coordinate of P. *(2 marks)*

(ii) State whether the function g has an inverse. Give a reason for your answer. *(1 mark)*

(iii) Show that $gf(x) = \ln|x^2 - k|$, stating the value of the constant k. *(2 marks)*

(iv) Solve the equation $gf(x) = 0$. *(4 marks)*

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5
5)	$f(x) = \frac{x^2-4}{3}$ $\quad x \leq 0$
a)	when $x=0$, $f(x) = \frac{-4}{3}$
	$x=-1$, $f(x) = \frac{-3}{3}$
	$f(x) \geq \frac{-4}{3}$
bi)	$x \geq \frac{-4}{3}$ (same as range of $f(x)$)
ii)	$y = \frac{x^2-4}{3}$ $(\times 3)$
	$3y = x^2 - 4$ $(+4)$
	$3y + 4 = x^2$ $(\sqrt{\ })$
	$\pm\sqrt{3y+4} = x$
	$f^{-1}(x) = \pm\sqrt{3x+4}$
	$\therefore f^{-1}(x) = -\sqrt{3x+4}$ $\quad$ ($x \leq 0$) range of $f^{-1}(x)$ same as domain of $f(x)$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

(i) intersects x axis when $y = 0$

$g(x) = 0$

$\ln|3x-1| = 0$

$|3x-1| = e^0$

$|3x-1| = 1$

$3x-1 = 1$ OR $3x-1 = -1$

$3x = 2$ $3x = 0$

$x = 2/3$ $x = 0$

$P \rightarrow \underline{x = 2/3}$

(ii) g has no inverse as it is not one to one

(iii) $gf(x) = g\left(\frac{x^2-4}{3}\right)$

$= \ln\left|3\left(\frac{x^2-4}{3}\right)-1\right|$

$= \ln|x^2-4-1|$

$= \underline{\ln|x^2-5|}$ $k = 5$

(iv) $gf(x) = 0$

$\ln|x^2-5| = 0$

$|x^2-5| = e^0$

$|x^2-5| = 1$

$x^2-5 = 1$ OR $x^2-5 = -1$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

$x^2 = 6$ $\qquad$ $x^2 = 4$

$x = \pm\sqrt{6}$ $\qquad$ $x = \pm 2$

$x \leq 0$ so, $\underline{x = -2}$ OR $\underline{x = -\sqrt{6}}$

6 (a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$. *(3 marks)*

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180° < x < 180°$. *(6 marks)*

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60°)}{(\sec(2\theta - 60°) + 1)(\sec(2\theta - 60°) - 1)} = \operatorname{cosec}(2\theta - 60°) + 3$$

giving the values of θ to the nearest degree in the interval $0° < \theta < 90°$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6a) $\dfrac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \dfrac{\sec^2 x}{\sec^2 x - 1}$

$\boxed{\tan^2 x = \sec^2 x - 1}$

$= \dfrac{\sec^2 x}{\tan^2 x}$

$= \sec^2 x \div \tan^2 x$

$= \dfrac{1}{\cos^2 x} \times \dfrac{1}{\tan^2 x}$

$= \dfrac{1}{\cancel{\cos^2 x}} \times \dfrac{\cancel{\cos^2 x}}{\sin^2 x}$

$= \dfrac{1}{\sin^2 x} = \operatorname{cosec}^2 x$ (as req)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

b) $\operatorname{cosec}^2 x = \operatorname{cosec} x + 3$

$\operatorname{cosec}^2 x - \operatorname{cosec} x - 3 = 0$

$a = 1, \quad b = -1, \quad c = -3$

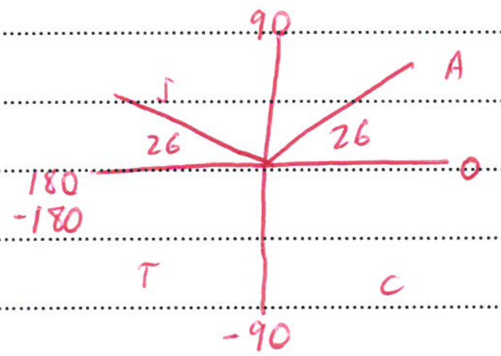
$$\operatorname{cosec} x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$$

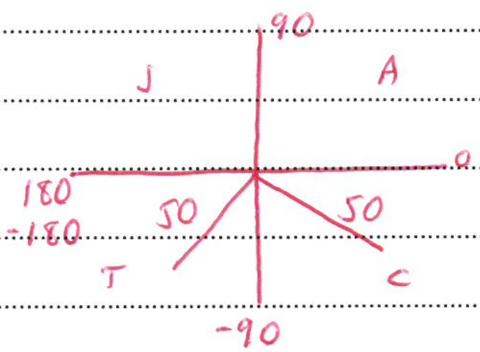
$$= \frac{1 \pm \sqrt{13}}{2}$$

$$\sin x = \frac{2}{1 \pm \sqrt{13}}$$

$\sin x = 0.434\ldots$ OR $-0.768\ldots$

$x = \underline{26°}, \underline{154°}$ $\qquad$ $x = \underline{-50°}, \underline{-130°}$





c) let $x = 2\theta - 60$ $\qquad$ $0 < \theta < 90$

$2\theta - 60 = -50, \ 26$ $\qquad$ $-60 < 2\theta - 60 < 120$

$2\theta = 10, \ 86$

$\theta = \underline{5°}, \underline{43°}$

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

Do not write outside the box

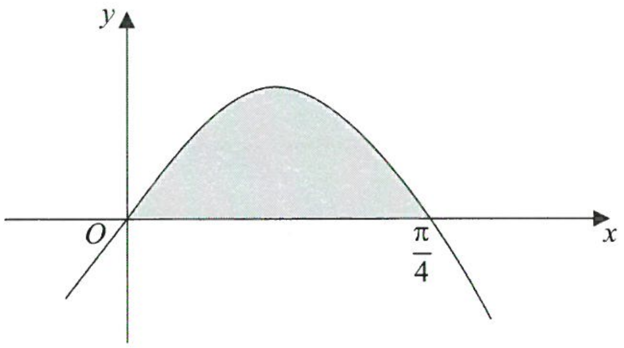
QUESTION PART REFERENCE

Answer space for question 6

7 A curve has equation $y = 4x\cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \dfrac{\pi}{4}$. *(5 marks)*

(b) The region shaded on the diagram below is bounded by the curve $y = 4x\cos 2x$ and the x-axis from $x = 0$ to $x = \dfrac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region. *(5 marks)*

QUESTION PART REFERENCE	**Answer space for question 7**
7)	$y = 4x\cos 2x$ $\boxed{u = 4x \quad v = \cos 2x \quad \frac{du}{dx} = 4 \quad \frac{dv}{dx} = -2\sin 2x}$
	$\frac{dy}{dx} = 4\cos 2x - 8x\sin 2x$
	when $x = \frac{\pi}{4}$, $\frac{dy}{dx} = 4\cos 2\left(\frac{\pi}{4}\right) - 8\left(\frac{\pi}{4}\right)\sin 2\left(\frac{\pi}{4}\right)$
	$= 4\cos\frac{\pi}{2} - 2\pi\sin\frac{\pi}{2}$
	$= 4(0) - 2\pi(1)$
	$= \underline{-2\pi}$ (gradient)
	when $x = \frac{\pi}{4}$, $y = 4\left(\frac{\pi}{4}\right)\cos 2\left(\frac{\pi}{4}\right)$
	$y = 0$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7
	$y - y_1 = m(x - x_1)$
	$y = -2\pi\left(x - \frac{\pi}{4}\right)$
or	$y = -2\pi x + \frac{\pi}{2}$
b)	$\int_0^{\pi/4} 4x\cos 2x \, dx$
	$u = 4x$ $\quad \frac{dv}{dx} = \cos 2x$
	$\frac{du}{dx} = 4$ $\quad v = \frac{1}{2}\sin 2x$
	$4x \times \frac{1}{2}\sin 2x - \int 4 \times \frac{1}{2}\sin 2x \, dx$
	$2x\sin 2x - \int 2\sin 2x \, dx$
	$2x\sin 2x - -\cos 2x + c$
	$\left[2x\sin 2x + \cos 2x\right]_0^{\pi/4}$
	$\left(2\left(\frac{\pi}{4}\right)\sin 2\left(\frac{\pi}{4}\right) + \cos 2\left(\frac{\pi}{4}\right)\right) - \left(2(0)\sin 0 + \cos 0\right)$
	$\left(\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - (1)$
	$\left(\frac{\pi}{2}(1) + 0\right) - (1)$
	$= \frac{\pi}{2} - 1$

Turn over ▶

Do not write outside the box

8 (a) Show that

$$\int_0^{\ln 2} e^{1-2x}\,dx = \frac{3}{8}e$$ *(4 marks)*

(b) Use the substitution $u = \tan x$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x\sqrt{\tan x}\,dx$$ *(8 marks)*

QUESTION PART REFERENCE	Answer space for question 8
8a)	$\int_0^{\ln 2} e^{1-2x}\,dx$
	$= \left[-\frac{1}{2}e^{1-2x}\right]_0^{\ln 2}$
	$= \left(-\frac{1}{2}e^{1-2(\ln 2)}\right) - \left(-\frac{1}{2}e^{1-0}\right)$
	$= -\frac{1}{2}\left(\frac{1}{4}e\right) + \frac{1}{2}e$
	$= -\frac{1}{8}e + \frac{4}{8}e$
	$= \frac{3}{8}e$ (as required)

Do not write outside the box

n13/MPC3

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

b) $\int_0^{\pi/4} \sec^4 x \sqrt{\tan x}\, dx$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$\int_0^1 \cancel{\sec^4 x}^{\sec^2 x} \sqrt{u}\ \frac{du}{\cancel{\sec^2 x}}$

$\boxed{dx = \frac{du}{\sec^2 x}}$

when $x = \frac{\pi}{4}$, $u = \tan\frac{\pi}{4} = 1$

$\int_0^1 (u^2+1)u^{1/2}\, du$

$x = 0$, $u = \tan 0 = 0$

$\int_0^1 u^{5/2} + u^{1/2}\, du$

$\boxed{\begin{array}{l}\sec^2 x = \tan^2 x + 1 \\ \sec^2 x = u^2 + 1\end{array}}$

$\left[\frac{u^{7/2}}{7/2} + \frac{u^{3/2}}{3/2}\right]_0^1$

$\left[\frac{2}{7}u^{7/2} + \frac{2}{3}u^{3/2}\right]_0^1$

$\left(\frac{2}{7}(1)^{7/2} + \frac{2}{3}(1)^{3/2}\right) - (0+0)$

$\frac{2}{7} + \frac{2}{3} = \frac{6}{21} + \frac{14}{21}$

$= \frac{20}{21}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS