Core 3 - January 2009

① 5 ordinates, 4 strips :

x	y
1	0.5
3	0.3660
5	0.3090
7	0.2743
9	0.25

with $(h) = 2$

$$\therefore \int = \frac{1}{3} \times 2 \times \left[0.5 + 4(0.3660 + 0.2743) + 2(0.3090) + 0.25 \right]$$

$$= 2.62 \ (3sf)$$

② $y = \sqrt{(x-2)^5}$

$$V = \pi \int_3^4 y^2 \, dx \qquad y^2 = (x-2)^5$$

$$\Rightarrow \pi \int_3^4 (x-2)^5 \, dx$$

$$= \pi \left[\frac{(x-2)^6}{6} \right]_3^4 = \pi \left[\frac{2^6}{6} - \frac{1^6}{6} \right]$$

$$= 10.5\pi$$

③ a) Let $f(x) = x^3 + 5x - 4$

$f(0.5) = 0.5^3 + 5(0.5) - 4 = -11/8$ or -1.375

$f(1) = 1^3 + 5(1) - 4 = 2$

change of sign $\therefore$ root lies between 0.5 and 1

b) $x^3 + 5x - 4 = 0$

$5x = 4 - x^3$

$x = 1/5 (4 - x^3)$

c) $x_1 = 0.5$

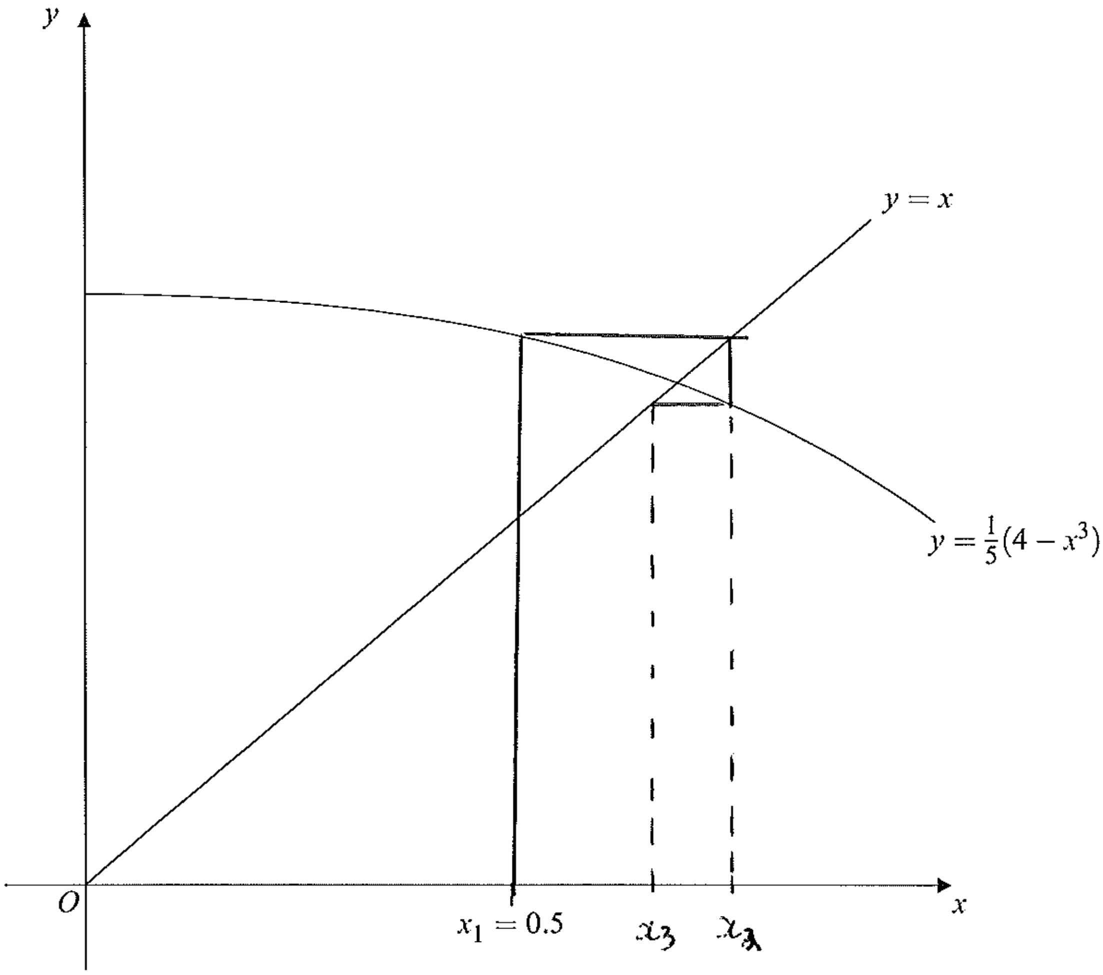
$x_2 = 1/5 (4 - 0.5^3) = 31/40$ or 0.775

$x_3 = 1/5 (4 - 0.775^3) = 0.70690...$

$= 0.707 \ (3dp)$

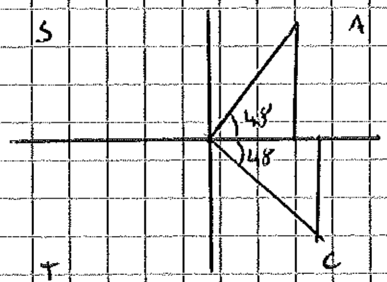
d) see Insert.

Figure 1 (for use in Question 3)



④ a) $\sec(x) = 3/2 \Rightarrow \cos(x) = 2/3$

$x = \cos^{-1}(2/3) = 48.19$



$x = 48.19, \; 311.81^\circ$

$= 48^\circ, \; 312^\circ$ (nearest degree)

b) $2\tan^2(x) = 10 - 5\sec(x)$ $\quad \{\tan^2(x) = \sec^2(x) - 1\}$

$2(\sec^2(x) - 1) = 10 - 5\sec(x)$

$2\sec^2(x) - 2 = 10 - 5\sec(x)$

$2\sec^2(x) + 5\sec(x) - 12 = 0$

$(2\sec(x) - 3)(\sec(x) + 4) = 0$

$2\sec(x) - 3 = 0$

$\sec(x) = 3/2$

$x = 48, 312^\circ$

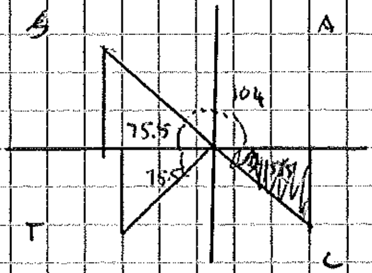
from (a)

$\sec(x) + 4 = 0$

$\sec(x) = -4$

$\cos(x) = -1/4$

$x = 104.47...^\circ$,



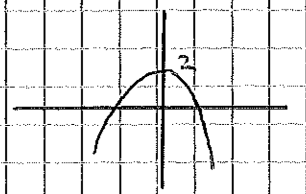
$x = 104.5, \; 255.5$

$x = 104^\circ, \; 256^\circ$

$x = 48^\circ, \; 312^\circ$

$104^\circ, \; 256^\circ$

⑤ a) $f(x) = 2 - x^4$



Range $f(x) \leq 2$

b) $f(x)$ is <u>NOT</u> a one-to-one function

c) i) $fg(x) = f\left(\frac{1}{x-4}\right) = 2 - \left(\frac{1}{x-4}\right)^4$

ii) $2 - \left(\frac{1}{x-4}\right)^4 = -14$

$$16 = \left(\frac{1}{x-4}\right)^4$$

$$\sqrt[4]{16} = \left(\frac{1}{x-4}\right)$$

$$\pm 2 = \frac{1}{x-4}$$

$$x-4 = \pm \tfrac{1}{2}$$

$x = 4 + \tfrac{1}{2} = 4\tfrac{1}{2}$ or $x = 4 - \tfrac{1}{2} = 3\tfrac{1}{2}$

(6) a) $y = e^{2x}(x^2 - 4x - 2)$

PRODUCT RULE $u = e^{2x}$ $v = x^2 - 4x - 2$

$du/dx = 2e^{2x}$ $dv/dx = 2x - 4$

$dy/dx = 2e^{2x}(x^2 - 4x - 2) + e^{2x}(2x - 4)$

At st point, $dy/dx = 0$

$\rightarrow 2e^{2x}(x^2 - 4x - 2) + e^{2x}(2x - 4) = 0$

$= e^{2x}(2x^2 - 8x - 4) + e^{2x}(2x - 4) = 0$

$e^{2x}[2x^2 - 8x - 4 + 2x - 4] = 0$

$e^{2x}[2x^2 - 6x - 8] = 0$

$(\div 2)$ $e^{2x}[x^2 - 3x - 4] = 0$

$e^{2x}(x - 4)(x + 1) = 0$

cannot $= 0$ $x = 4$ $x = -1$

b) i) $dy/dx = e^{2x}(2x^2 - 6x - 8)$

PRODUCT RULE $u = e^{2x}$ $v = 2x^2 - 6x - 8$

$du/dx = 2e^{2x}$ $dv/dx = 4x - 6$

$d^2y/dx^2 = 2e^{2x}(2x^2 - 6x - 8) + e^{2x}(4x - 6)$

ii) when $x = 4 \rightarrow d^2y/dx^2 = 10e^{8} = +ve$ $\therefore$ MINIMUM

when $x = -1 \rightarrow d^2y/dx^2 = -10e^{-2} = -ve$ $\therefore$ MAXIMUM

7) a) $3e^{x} = 4$

$e^{x} = 4/3$

$x = \ln(4/3)$

b) i) $3e^{x} + 20e^{-x} = 19$

$3e^{x} + \frac{20}{e^{x}} = 19$ $\quad y = e^{x}$

$3y + \frac{20}{y} = 19$

$\Rightarrow 3y^2 + 20 = 19y$

$\Rightarrow 3y^2 - 19y + 20 = 0$

ii) $3y^2 - 19y + 20 = 0$

$(3y - 4)(y - 5) = 0$

$3y - 4 = 0$ $\qquad$ $y - 5 = 0$

$\Rightarrow y = 4/3$ $\qquad$ $\Rightarrow y = 5$

$\Rightarrow e^{x} = 4/3$ $\qquad$ $\Rightarrow e^{x} = 5$

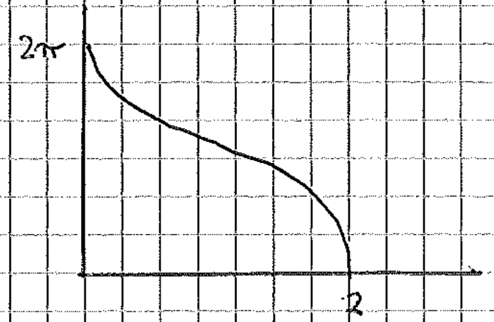
$\Rightarrow x = \ln(4/3)$ $\qquad$ $\Rightarrow x = \ln(5)$

8) a) $P = (-1, \pi)$, $Q = (1, 0)$

b) ① Translation by vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

② Stretch, scale factor 2, y direction

c)



d) i) $2\cos^{-1}(x-1) = y$

$\cos^{-1}(x-1) = y/2$

$x - 1 = \cos(y/2)$

$x = \cos(y/2) + 1$

d) ii) $dx/dy = -1/2 \sin(y/2)$

when $y = 2$, $dx/dy = -1/2 \sin(1)$

(9) a) $y = \frac{4x}{4x-3}$

$u = 4x$ $\quad$ $v = 4x-3$

$du/dx = 4$ $\quad$ $dv/dx = 4$

$dy/dx = \frac{4(4x-3) - 4(4x)}{(4x-3)^2}$

$= \frac{16x - 12 - 16x}{(4x-3)^2}$

$= \frac{-12}{(4x-3)^2}$

b) i) $y = x\ln(4x-3)$

$\boxed{\text{Product Rule}}$

$u = x$ $\quad$ $v = \ln(4x-3)$

$du/dx = 1$ $\quad$ $dv/dx = \cancel{4}\ \frac{4}{4x-3}$

$dy/dx = \ln(4x-3) + \frac{4(x)}{4x-3}$

ii) when $x = 1$:

$y = (1)\ln(4-3) = 0$

$dy/dx = \ln(4-3) + \frac{4}{4-3} = 4$

$\rightarrow \quad y - y_1 = m(x - x_1)$

$y = 4(x-1)$ $\quad$ or $\quad y = 4x - 4$

c) i) $\int \frac{4x}{4x-3}\,dx$

$u = 4x-3$

$du/dx = 4$

$\rightarrow dx = 1/4\ du$

$\rightarrow \int \frac{4x}{u}\ 1/4\ du$

$= \int \frac{x}{u}\,du$

If $u = 4x-3$

$u + 3 = 4x$

$\frac{u+3}{4} = x$

$\rightarrow 1/4 \int \frac{u+3}{u}\,du$

$= 1/4 \int \frac{u}{u} + \frac{3}{u}\,du = 1/4 \int 1 + \frac{3}{u}\,du$

$$= \frac{1}{4}\left[u + 3\ln(u)\right]$$

$$= \frac{1}{4}\left[(4x-3) + 3\ln(4x-3)\right] + c$$

ii) $\int \ln(4x-3)$

$$= \int 1 \times \ln(4x-3)$$

$u = \ln(4x-3)$ $\quad dv/dx = 1$

$du/dx = \frac{4}{4x-3}$ $\quad v = x$

$$= uv - \int v \, du/dx$$

$$= x\ln(4x-3) - \int \frac{4x}{4x-3}\,dx$$

FROM PART i)

$$= x\ln(4x-3) - \frac{1}{4}\left[(4x-3) + 3\ln(4x-3)\right] + c$$