

Core 3 - January 2008

① a) i)  $y = (2x^2 - 5x + 1)^{20} \rightarrow \frac{dy}{dx} = 20(4x - 5)(2x^2 - 5x + 1)^{19}$

ii)  $y = x \cos(x)$

$\frac{dy}{dx} = \cos(x) - x \sin(x)$

$u = x$	$v = \cos(x)$
$\frac{du}{dx} = 1$	$\frac{dv}{dx} = -\sin(x)$

Product rule!

b)  $y = \frac{x^3}{x-2}$

$u = x^3$

$v = x-2$

$\frac{du}{dx} = 3x^2$

$\frac{dv}{dx} = 1$

$\frac{dy}{dx} = \frac{3x^2(x-2) - x^3}{(x-2)^2}$

$= \frac{3x^3 - 6x^2 - x^3}{(x-2)^2}$

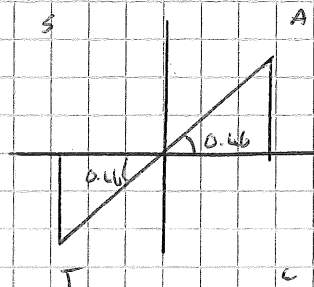
$= \frac{2x^3 - 6x^2}{(x-2)^2}$

$= \frac{2x^2(x-3)}{(x-2)^2}$

② a)  $\cot(x) = 2 \rightarrow \tan(x) = 1/2$

$\rightarrow x = 0.4636, \dots$

$x = 6.46, 3.61$



b)  $\csc^2(x) = \frac{3 \cot(x) + 4}{2}$

$1 + \cot^2(x) = \frac{3 \cot(x) + 4}{2}$

$2 + 2 \cot^2(x) = 3 \cot(x) + 4$

$2 \cot^2(x) - 3 \cot(x) - 2 = 0$

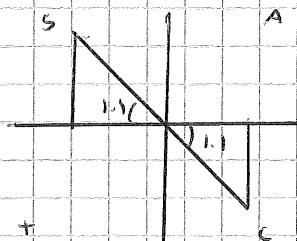
c)  $(2 \cot(x) + 1)(\cot(x) - 2) = 0$

$2 \cot(x) + 1 = 0$   
 $\cot(x) = -1/2$

$\tan(x) = -2$  [ $x = -1.107, \dots$ ]

$\cot(x) = 2$

$x = 0.46, 3.61$  (from a)



$x = 2.034, \dots, 5.176, \dots$

Final answer:  $x = 0.46, 3.61, 2.03, 5.17$

③ a) Let  $f(x) = x + (1 + 3x)^{1/4}$

$$f(-0.33) = -0.33 + (1 + 3(-0.33))^{1/4} = -0.01377$$

$$f(-0.32) = -0.32 + (1 + 3(-0.32))^{1/4} = 0.1272$$

change of sign,  $\therefore$  root lies between

b)  $x + (1 + 3x)^{1/4} = 0$

$$x = -(1 + 3x)^{1/4}$$

$$x^4 = (1 + 3x)$$

$$x^4 = 1 + 3x$$

$$3x = x^4 - 1 \quad \rightarrow \quad x = \frac{1}{3}(x^4 - 1)$$

raise both sides to power 4  
 $\rightarrow$  both positive

c)  $x_1 = -0.3$

$$x_2 = \frac{1}{3}(-0.3^4 - 1) = -0.3306$$

$$x_3 = \frac{1}{3}(-0.33^4 - 1) = -0.3293$$

$$x_4 = \frac{1}{3}(-0.329^4 - 1) = -0.3294 = -0.329 \text{ (3sf)}$$

④ a) Range:  $f(x) \in \mathbb{R}$  all real numbers

b) i)  $f \circ g(x) = f\left(\frac{1}{x-3}\right) = \left(\frac{1}{x-3}\right)^3$

ii)  $\left(\frac{1}{x-3}\right)^3 = 64 \quad \rightarrow \quad \frac{1}{x-3} = 4$

$$1 = 4(x-3)$$

$$x-3 = \frac{1}{4} \quad \rightarrow \quad x = 3\frac{1}{4}$$

c) i) let  $y = \frac{1}{x-3}$

$$y(x-3) = 1 \quad \rightarrow \quad x-3 = \frac{1}{y}$$

$$\rightarrow x = \frac{1}{y} + 3$$

$$\rightarrow y = \frac{1}{x+3} = g^{-1}(x)$$

ii) Range  $g^{-1}(x) = \text{Domain of } g(x)$

$\rightarrow$  Range  $\neq 3$

(5) a) i)  $y = 2x^2 - 8x + 3 \rightarrow \frac{dy}{dx} = 4x - 8$

ii)  $\int_{x=2}^{x=4} \frac{dx}{2x^2 - 8x + 3} = \frac{1}{4} \int_{2x^2 - 8x + 3} dx$

$$= \left[ \frac{1}{4} \ln(2x^2 - 8x + 3) \right]_2^4 = \frac{1}{4} \ln(2 \cdot 4^2 - 8 \cdot 4 + 3) - \frac{1}{4} \ln(2 \cdot 2^2 - 8 \cdot 2 + 3)$$

$$= \frac{1}{4} \ln(27) - \frac{1}{4} \ln(3)$$

$$= \frac{1}{4} \ln\left(\frac{27}{3}\right) = \frac{1}{4} \ln(9) = \frac{1}{4} \ln(3^2)$$

$$= \frac{3}{4} \ln(3) = \frac{1}{2} \ln(3)$$

b)  $\int x \sqrt{3x-1} dx$

$$u = 3x - 1$$

$$\frac{du}{dx} = 3 \rightarrow dx = \frac{1}{3} du$$

$$\int x u^{1/2} \frac{1}{3} du$$

$$u = 3x - 1$$

$$u + 1 = 3x$$

$$x = \frac{1}{3}(u + 1)$$

$$\frac{1}{3} \int (u+1) u^{1/2} \frac{1}{3} du$$

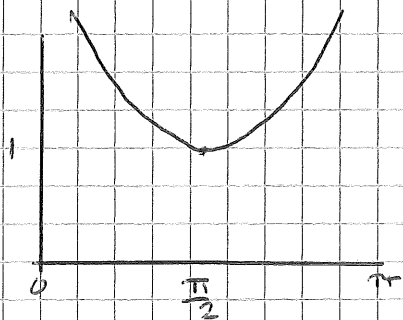
$$= \frac{1}{9} \int u^{3/2} + u^{1/2} du$$

$$= \frac{1}{9} \left[ \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} \right]$$

$$= \frac{1}{9} \left[ \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]$$

$$= \frac{2}{45} u^{5/2} + \frac{2}{27} u^{3/2} = \frac{2}{45} (3x-1)^{5/2} + \frac{2}{27} (3x-1)^{3/2}$$

(6) a)



b)

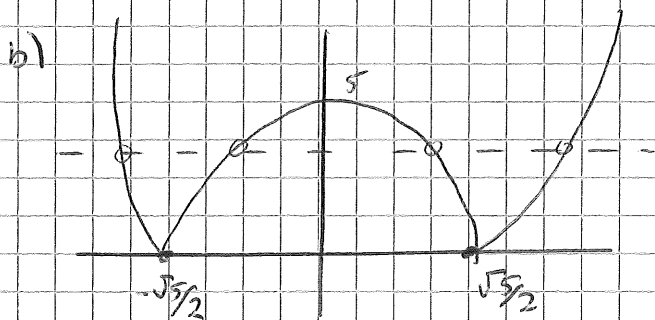
$x$	$y$
0.15	6.642
0.25	4.042
0.35	2.916
0.45	2.249

$$h = 0.1$$

$$A = 0.1 \times 15.949$$

$$= 1.5949 \text{ (358)}$$

- 7) a) 1st  $4x^2 \rightarrow$  stretch y direction scale factor 4  
2nd  $-5 \rightarrow$  translation  $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$



$$y = 4x^2 - 5$$

crosses x when  $y = 0$

$$\rightarrow 0 = 4x^2 - 5$$

$$5 = 4x^2$$

$$x = \pm \sqrt{4 \cdot 5/4}$$

$$= \pm \sqrt{5}/2$$

a) i) From graph above:

$$4x^2 - 5 = 4$$

$$4x^2 = 9$$

$$x^2 = 9/4$$

$$x = \pm 3/2$$

AND

$$-4x^2 + 5 = 4$$

$$4x^2 = 1$$

$$x^2 = 1/4$$

$$x = \pm 1/2$$

Solutions:  $x = -3/2, -1/2, 1/2, 3/2$

ii) From graph:  $x \geq 3/2, x \leq -3/2$

$$-1/2 \leq x \leq 1/2$$

8) a)  $e^{-2x} = 3$

$$-2x = \ln(3)$$

$$x = -1/2 \ln(3)$$

b)  $\int x e^{-2x} dx$

$$u = x$$

$$du/dx = 1$$

$$dv/dx = e^{-2x}$$

$$v = -1/2 e^{-2x}$$

$$= uv - \int v du/dx$$

$$= -1/2 x e^{-2x} - \int -1/2 e^{-2x}$$

$$= -1/2 x e^{-2x} + 1/2 \int e^{-2x}$$

$$= -1/2 x e^{-2x} - 1/4 e^{-2x} + C$$

$$c) \text{ i) } y = e^{-2x} + 6x$$

$$\text{At st point, } dy/dx = 0$$

$$dy/dx = -2e^{-2x} + 6 = 0$$

$$e^{-2x} = 3 = 0$$

$$e^{-2x} = 3$$

$$-2x = \ln(3)$$

$$x = -1/2 \ln(3)$$

$$y = e^{-2(-1/2 \ln(3))} + 6(-1/2 \ln(3))$$

$$= e^{\ln(3)} - 3 \ln(3)$$

$$= 3 - 3 \ln(3)$$

$$\therefore \text{ (0-ordinates) } = (-1/2 \ln(3), 3 - 3 \ln(3))$$

$$\text{ii) } d^2y/dx^2 = 4e^{-2x}$$

$$x = -1/2 \ln(3)$$

$$\rightarrow 4e^{-2(-1/2 \ln(3))}$$

$$= 4e^{\ln(3)} = 4 \times 3 = 12$$

$12 > 0$   $\therefore$  minimum point.

$$\text{iii) } V = \pi \int y^2 dx$$

$$V = \pi \int e^{-4x} + \boxed{12xe^{-2x}} + 36x^2$$

$$= \pi \left[ -1/4 e^{-4x} - 6xe^{-2x} - 3e^{-2x} + 12x^3 \right]_0^1$$

$$= \pi \left[ -1/4 e^{-4} - 6e^{-2} - 3e^{-2} + 12 \right]$$

$$- \pi \left[ -1/4 e^0 - 0 - 3e^0 + 0 \right]$$

$$= \pi \left[ 15\frac{1}{4} - 9e^{-2} - 1/4 e^{-4} \right]$$

$$= 44.1 \text{ (3sf)}$$

$$y = e^{-2x} + 6x$$

$$y^2 = (e^{-2x} + 6x)(e^{-2x} + 6x)$$

$$= e^{-4x} + 12xe^{-2x} + 36x^2$$

$$\boxed{12xe^{-2x}}$$

Use integration by parts:

(answer from a)

$$\int xe^{-2x} = -1/2 xe^{-2x} - 1/4 e^{-2x}$$

$$\text{so } \int 12xe^{-2x} = -6xe^{-2x} - 3e^{-2x}$$