Core 3 - January 2007

①

x	y
1.5	0.7115
2.5	0.5218
3.5	0.4439
4.5	0.3993

$h = 1$

Area $\approx 1 \times (0.7115 + 0.5218 + 0.4439 + 0.3993)$

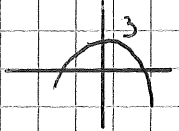
$= 2.08$ (3sf)

② $y = \sec(x)$

$(3x) \rightarrow$ stretch, scale factor $\frac{1}{3}$, x direction

$(+1) \rightarrow$ translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

③ a) $f(x) = 3 - x^2$



Range: $f(x) \leq 3$

b) $g(x) = \dfrac{2}{x+1}$

i) $y = \dfrac{2}{x+1} \rightarrow y(x+1) = 2$

$\rightarrow x + 1 = \frac{2}{y}$

$\rightarrow x = \frac{2}{y} - 1$

$\therefore y = \frac{2}{x} - 1 = g^{-1}(x)$

ii) Range $g^{-1}(x)$ = Domain $g(x)$ $\Rightarrow$ Range $\neq -1$

c) i) $gf(x) = g(3 - x^2)$

$= \dfrac{2}{(3 - x^2) + 1} = \dfrac{2}{4 - x^2} = \dfrac{2}{(2-x)(2+x)}$

ii) Domain $x \in \mathbb{R}$, $x \neq 2$, $x \neq -2$ (or denominator $= 0$)

4) a) $\int x \sin(x)\, dx$ $\qquad u = x \qquad dv/dx = \sin(x)$

$du/dx = 1 \qquad v = -\cos(x)$

$\rightarrow uv - \int v\, du/dx$

$= -x\cos(x) - \int -\cos(x)$

$= -x\cos(x) + \int \cos(x) = -x\cos(x) + \sin(x) + c$

b) $\int x\sqrt{x^2+5}\, dx$

$u = x^2 + 5$

$du/dx = 2x \rightarrow dx = du \ 1/2x$

$\int x\, u^{1/2}\, du \ 1/2x$

$= \frac{1}{2}\int u^{1/2}\, du = \frac{1}{2}\left[u^{3/2} / \tfrac{3}{2} \right]$

$= \frac{1}{2}\left[\tfrac{2}{3} u^{3/2} \right]$

$= \frac{1}{3} u^{3/2}$

$= \frac{1}{3}\sqrt{(x^2+5)^3} + C$

c) $y = x^2 - 9$

$x^2 = y + 9$

$\pi \int_1^2 x^2\, dy$

$= \pi \int_1^2 y + 9\, dy$

$= \pi \left[y^2/2 + 9y \right]_1^2$

$= \pi \left[4/2 + 18 - 1/2 - 9 \right]$

$= \pi \left[10\tfrac{1}{2} \right) = 10\tfrac{1}{2}\pi$

5) a) i) $2\cot^2(x) + 5\operatorname{cosec}(x) = 10$

$2(\operatorname{cosec}^2(x) - 1) + 5\operatorname{cosec}(x) = 10$ $\qquad \operatorname{cosec}^2 x = \cot^2(x) + 1$

$2\operatorname{cosec}^2(x) - 2 + 5\operatorname{cosec}(x) = 10$ $\qquad \cot^2(x) = \operatorname{cosec}^2(x) - 1$

$2\operatorname{cosec}^2(x) + 5\operatorname{cosec}(x) - 12 = 0$

ii) $(2\operatorname{cosec}(x) - 3)(\operatorname{cosec}(x) + 4) = 0$

$2\operatorname{cosec}(x) - 3 = 0$ $\qquad \operatorname{cosec}(x) + 4 = 0$

$\operatorname{cosec}(x) = 3/2$ $\qquad \operatorname{cosec}(x) = -4$

$\sin(x) = 2/3$ $\qquad \sin(x) = -1/4$

b) $2\operatorname{cosec}^2(t) + 5\operatorname{cosec}(t) - 12 = 0$

$t = \theta - 0.1$

$-\pi < \theta < \pi$

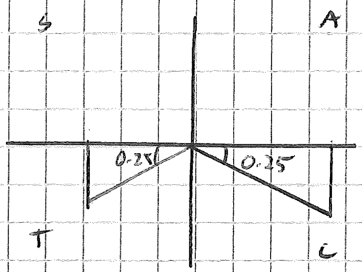
$-\pi - 0.1 < t < \pi - 0.1$

$\theta = t + 0.1$

From a)

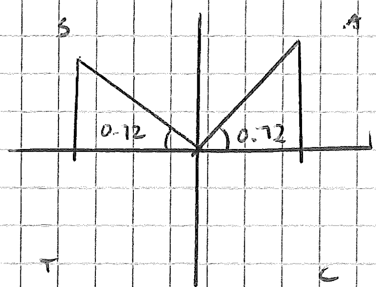
$\sin t = -1/4$ $\quad\quad$ $\sin t = 2/3$

$\boxed{\sin t = -1/4}$ $\quad t = -0.252...$



$t = -0.252, -2.888$

$\boxed{\sin t = 2/3}$ $\quad t = 0.7297..$



$t = 0.730, 2.412$

$\theta = t + 0.1 \rightarrow \theta = 0.83, 2.51, -0.15, -2.79$

6 a) i) $y = (4x^2 + 3x + 2)^{10}$

$dy/dx = 10 \times (8x + 3)(4x^2 + 3x + 2)^9$

$= (80x + 30)(4x^2 + 3x + 2)^9$

ii) $y = x^2 \tan(x)$

$dy/dx = 2x\tan(x) + x^2\sec^2(x)$

$u = x^2$	$v = \tan(x)$
$du/dx = 2x$	$dv/dx = \sec^2(x)$

b) i) $x = 2y^3 + \ln(y)$

$dx/dy = 6y^2 + 1/y$

ii) $x = 2$

$y = 1$

$dx/dy = 6(1)^2 + 1 = 7 \rightarrow dy/dx = 1/7 = m$

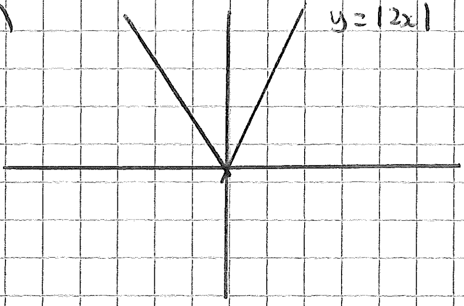
$y - y_1 = m(x - x_1)$

$y - 1 = 1/7(x - 2)$

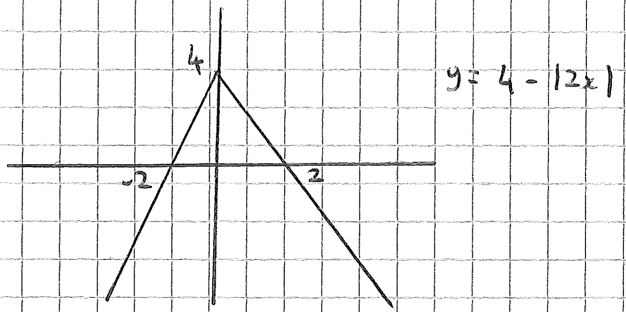
$7y - 7 = x - 2$

$x - 7y + 5 = 0$

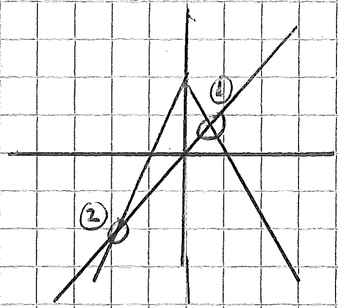
(7) a)



b)



c)



(1) $x = 4 - 2x$

$3x = 4$

$x = 4/3$

(2) $x = 4 + 2x$

$0 = 4 + x$

$x = -4$

d) From graph: $-4 < x < 4/3$

(8) a) A $(-1, \pi)$ B $(0, \pi/2)$

b) $\cos^{-1}(x) = 3x + 1$

$f(x) = \cos^{-1}(x) - 3x - 1 = 0$

$f(0.1) \to \cos^{-1}(0.1) - 3(0.1) - 1 = 0.1706...$

$f(0.2) \to \cos^{-1}(0.2) - 3(0.2) - 1 = -0.2305...$

Sign change, therefore root between 0.1 and 0.2

c) $x_1 = 0.1$

$x_2 = 1/3\,[\cos^{-1}(0.1) - 1] = 0.15687...$

$x_3 = 1/3\,[\cos^{-1}(0.1568...) - 1] = 0.13775...$

$x_4 = 1/3\,[\cos^{-1}(0.1377...) - 1] = 0.14420...$

$= 0.144$ (3dp)

(9) a) i) $\int 4 - e^{2x} = 4x - 1/2\, e^{2x} + c$

ii) $\left[4x - 1/2\, e^{2x}\right]_0^{\ln(2)} = 4\ln(2) - 1/2\, e^{2\ln(2)} - 0 + 1/2\, e^0$

$= 4\ln(2) - 1/2 \times 4 + 1/2$

$= 4\ln(2) - 3/2$

b) i) $x = 0$, $y = 4 - e^0 = 3 \rightarrow A = (0, 3)$

ii) At B, $y = 0$

$\rightarrow 4 - e^{2x} = 0$

$4 = e^{2x}$

$\ln(4) = 2x$

$x = \frac{1}{2}\ln(4)$

$= \ln(\sqrt{4}) = \ln(2)$

c) At B: $x = \ln(2)$

$y = 0$

$dy/dx = \cancel{4 \ldots e^{2x}} \; -2e^{2x}$

when $x = \ln(2)$, $dy/dx = -2e^{2\ln 2} = -2 \times 4 = -8$

$\therefore$ gradient of normal $= 1/8$

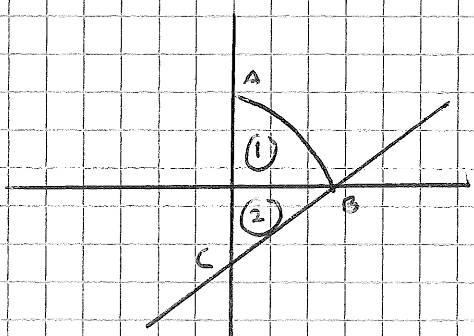
$y - y_1 = m(x - x_1)$

$y - 0 = 1/8\,(x - \ln(2))$

$8y = x - \ln(2)$

$x - 8y - \ln(2) = 0$ OR $y = 1/8\,x - 1/8\ln(2)$

d)



Area ① $= 4\ln(2) - 3/2$ [From a)]

Point C: $x = 0$

$\rightarrow y = -1/8\ln(2)$

Area ② $= 1/2\,bh$

$= 1/2 \times 1/8\ln(2) \times \ln(2)$

$= 1/16 \times [\ln(2)]^2$

Total Area $= 4\ln(2) - 3/2 + 1/16 \times [\ln 2]^2$

$= 1.30$ (3sf)