

Core 3 - January 2006

① a) $y = \tan(3x)$

$$\frac{dy}{dx} = 3 \sec^2(3x)$$

chain rule

b) $y = \frac{3x+1}{2x+1}$

$$u = 3x+1$$

$$v = 2x+1$$

$$\frac{du}{dx} = 3$$

$$\frac{dv}{dx} = 2$$

quotient rule

$$\frac{dy}{dx} = \frac{3(2x+1) - 2(3x+1)}{(2x+1)^2}$$

$$= \frac{6x+3-6x-2}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

②

x	1	1.5	2	2.5	3	
y	0.7071	0.4781	0.3333	0.2453	0.1840	$h = 0.5$

$$\text{Area} = \frac{1}{3} \times 0.5 \times [0.7071 + 0.1840 + 4(0.4781 + 0.2453) + 2(0.3333)]$$
$$= 0.743 \text{ (3sf)}$$

③ a) i) $f(x) = x^4 + 2x$

$$f'(x) = 4x^3 + 2$$

ii) $\int \frac{2x^3+1}{x^4+2x} = \frac{1}{2} \int \frac{4x^3+2}{x^4+2x} = \frac{1}{2} \ln(x^4+2x)$

b) i) $\int x\sqrt{2x+1} dx$

$$u = 2x+1$$

$$\frac{du}{dx} = 2 \rightarrow dx = \frac{1}{2} du$$

$$\int x u^{1/2} \frac{1}{2} du$$

$$u = 2x+1$$

$$u-1 = 2x$$

$$\rightarrow x = \frac{u-1}{2}$$

$$\int \left(\frac{u-1}{2}\right) (u^{1/2}) \frac{1}{2} du$$

$$= \frac{1}{4} \int (u-1) u^{1/2} du$$

$$= \frac{1}{4} \int (u^{3/2} - u^{1/2}) du$$

ii) $\int_0^4 x\sqrt{2x+1} dx$

$$= \frac{1}{4} \int_1^9 (u^{3/2} - u^{1/2}) du$$

change limits

$$u = 2x+1$$

$$(x=0) \quad u = 1$$

$$(x=4) \quad u = 9$$

$$= \frac{1}{4} \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} \right]_1^9$$

$$= \frac{1}{4} \left[\frac{9^{5/2}}{5/2} - \frac{9^{3/2}}{3/2} - \frac{1^{5/2}}{5/2} + \frac{1^{3/2}}{3/2} \right]$$

$$= \frac{1}{4} \times 79.46666 = 19.86666 = 19.9 \text{ (3sf)}$$

④ $2 \operatorname{cosec}^2(x) = 5 - 5 \cot(x)$

a) $\operatorname{cosec}^2(x) = 1 + \cot^2(x)$

$$\rightarrow 2(1 + \cot^2(x)) = 5 - 5 \cot(x)$$

$$2 + 2 \cot^2(x) = 5 - 5 \cot(x)$$

$$\rightarrow 2 \cot^2(x) + 5 \cot(x) - 3 = 0$$

b) $(2 \cot(x) - 1)(\cot(x) + 3) = 0$

$$\downarrow$$

$$\cot(x) = 1/2$$

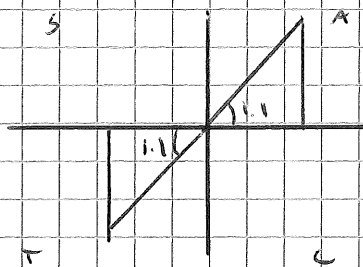
$$\rightarrow \tan(x) = 2$$

$$\downarrow$$

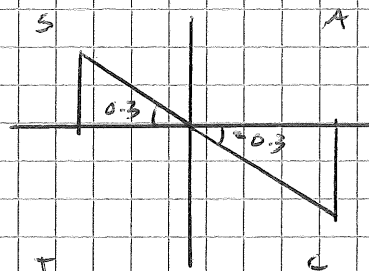
$$\cot(x) = -3$$

$$\rightarrow \tan(x) = -1/3$$

c) $x = 1.107...$



$x = -0.3217...$



$$x = 1.107...$$

$$x = -2.034...$$

$$x = -0.3217$$

$$x = 2.8148$$

$$x = 1.1, -2.0, -0.3, 2.8 \text{ (1dp)}$$

⑤ a) $y = e^{2x} - 9$

① $x = 0 \rightarrow y = 1 - 9 = -8 = a$

② $y = e^{2x} - 9$ when $y = 0$

$$e^{2x} - 9 = 0$$

$$e^{2x} = 9$$

$$2x = \ln(9) \rightarrow x = 1/2 \ln(9) \rightarrow x = \ln(\sqrt{9}) = \ln(3)$$

$$b) y^2 = (e^{2x} - 9)^2 = e^{4x} - 9e^{2x} - 9e^{2x} + 81$$

$$= e^{4x} - 18e^{2x} + 81$$

$$c) V = \pi \int_0^{\ln(3)} y^2 dx$$

$$= \pi \int_0^{\ln(3)} (e^{4x} - 18e^{2x} + 81) dx$$

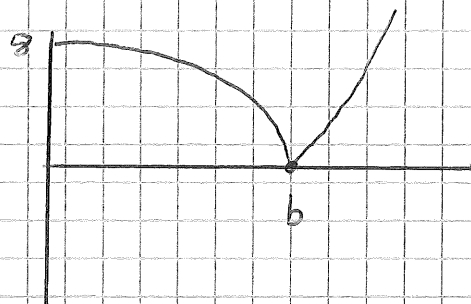
$$= \pi \left[\frac{1}{4} e^{4x} - 9e^{2x} + 81x \right]_0^{\ln(3)}$$

$$= \pi \left[\frac{1}{4} e^{4 \ln(3)} - 9e^{2 \ln(3)} + 81 \ln(3) - \frac{1}{4} e^0 + 9e^0 + 81(0) \right]$$

$$= \pi \left[81/4 - 9 + 81 \ln(3) - 1/4 + 9 + 0 \right]$$

$$= \pi \left[-52 + 81 \ln(3) \right] = \pi (81 \ln(3) - 52)$$

d)



⑥ a) $f(x) = x^3 + 4x - 3$

$$f(0.5) = 0.5^3 + 4(0.5) - 3 = -7/8$$

$$f(1) = 1^3 + 4(1) - 3 = 2$$

Sign change, therefore root lies between 0.5 and 1

b) $x^3 + 4x - 3 = 0$

$$4x = 3 - x^3$$

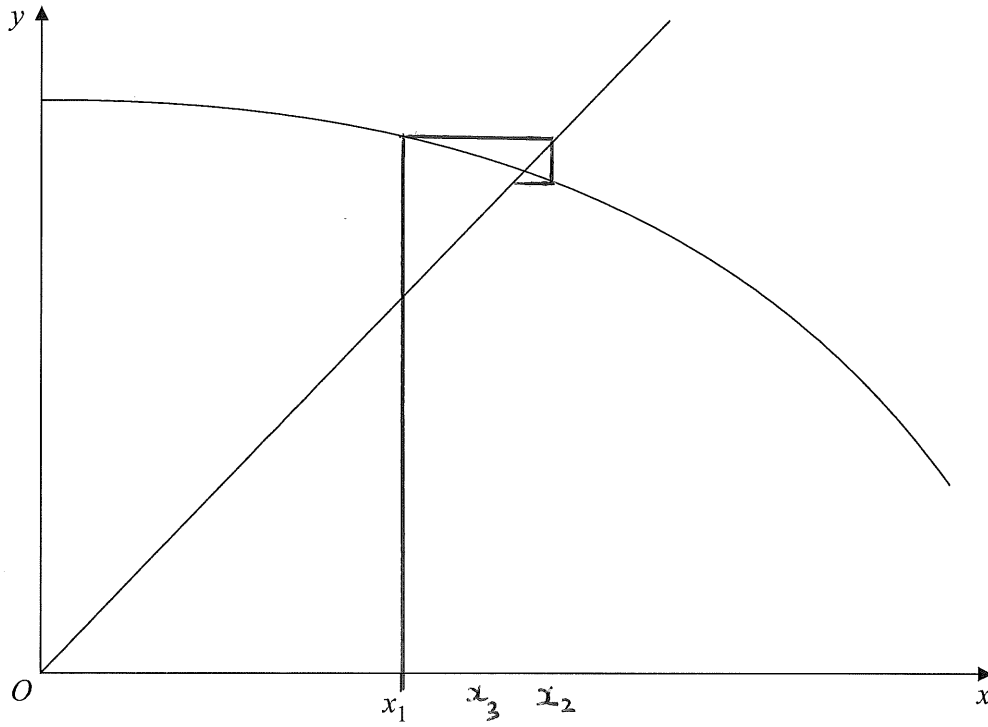
$$x = \frac{3 - x^3}{4}$$

c) $x_1 = 0.5$

$$x_2 = \frac{3 - 0.5^3}{4} = 0.71875$$

$$x_3 = \frac{3 - 0.71875^3}{4} = 0.65713... = 0.66 \text{ (2dp)}$$

Figure 1 (for Question 6)



$$9) a) y = x^{-2} \ln(x)$$

$$u = x^{-2}$$

$$v = \ln(x)$$

$$\frac{du}{dx} = -2x^{-3}$$

$$\frac{dv}{dx} = \frac{1}{x}$$

Product Rule

$$\frac{dy}{dx} = -2x^{-3} \ln(x) + x^{-2} \cdot \frac{1}{x}$$

$$= \frac{-2 \ln(x)}{x^3} + \frac{1}{x^3} = \frac{1 - 2 \ln(x)}{x^3}$$

$$b) \int x^{-2} \ln(x)$$

$$u = \ln(x)$$

$$\frac{du}{dx} = x^{-2}$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$v = -x^{-1}$$

$$= -x^{-1} \ln(x) - \int -x^{-1} \cdot \frac{1}{x} dx$$

$$= -\frac{1}{x} \ln(x) - \int -x^{-2} dx$$

$$= -\frac{1}{x} \ln(x) - [+ x^{-1}]$$

$$= -\frac{1}{x} \ln(x) - \frac{1}{x} + c$$

$$c) i) \text{ at st point, } \frac{dy}{dx} = 0$$

$$\frac{1 - 2 \ln(x)}{x^3} = 0$$

$$\rightarrow 1 - 2 \ln(x) = 0$$

$$1 = 2 \ln(x)$$

$$\frac{1}{2} = \ln(x)$$

$$e^{1/2} = x \quad \rightarrow$$

$$ii) \text{ from b) } \left[-\frac{1}{x} \ln(x) - \frac{1}{x} \right]_1^5$$

$$= -\frac{1}{5} \ln(5) - \frac{1}{5} + \ln(1) + 1$$

$$= -\frac{1}{5} \ln(5) + \frac{4}{5}$$

$$= \frac{1}{5} (4 - \ln(5))$$