Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics MPC2

Unit Pure Core 2

Wednesday 20 May 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

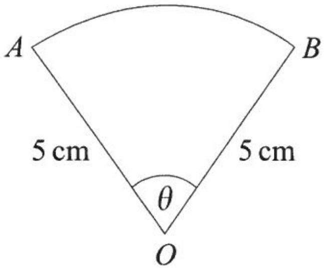
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The diagram shows a sector OAB of a circle with centre O and radius 5 cm.



The angle AOB is θ radians and the area of the sector is 15 cm^2.

Find the **perimeter** of the sector.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

area $= \frac{1}{2}r^2\theta$

$15 = \frac{1}{2} \times 5^2\theta$

$30 = 5^2\theta$

$\theta = \frac{30}{25} = \frac{6}{5}$

arc length $= r\theta$

$= 5 \times \frac{6}{5}$

$= \frac{30}{5} = 6$

perimeter $= 5 + 5 + 6 = \underline{\underline{16\text{ cm}}}$

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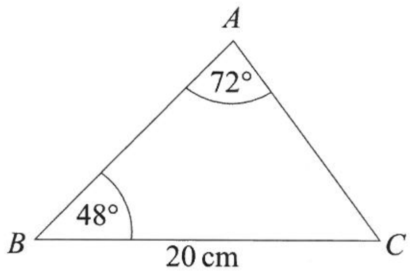
2 The diagram shows a triangle ABC.



The size of angle BAC is $72°$ and the size of angle ABC is $48°$. The length of BC is 20 cm.

(a) Show that the length of AC is 15.6 cm, correct to three significant figures.

[3 marks]

(b) The midpoint of BC is M. Calculate the length of AM, giving your answer, in cm, to three significant figures.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

2a)

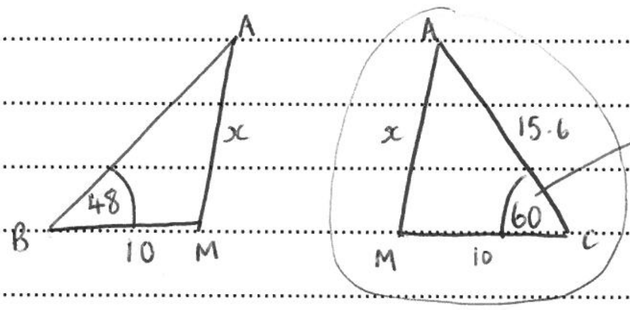
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{20}{\sin 72} = \frac{a}{\sin 48}$$

$$a = \frac{20}{\sin 72} \times \sin 48$$

$$a = 15.627774\ldots = 15.6 \ (3\text{ sf})$$

b)



angles in a Δ add to 180°

$180 - 48 - 72 = 60$

$$x^2 = 15.6^2 + 10^2 - (2 \times 15.6 \times 10 \times \cos 60)$$

$$x^2 = 187.36$$

$$x = 13.7 \ (3\text{ sf})\ \text{cm}$$

Do not write outside the box

3 The first term of an infinite geometric series is 48. The common ratio of the series is 0.6.

(a) Find the third term of the series.

[2 marks]

(b) Find the sum to infinity of the series.

[2 marks]

(c) The nth term of the series is u_n. Find the value of $\sum_{n=4}^{\infty} u_n$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 3

3a) $u_3 = ar^{n-1}$

$u_3 = 48 \times 0.6^2$

$u_3 = 17.28$

b) $S_\infty = \frac{\cancel{\ldots}}{1-} \quad \frac{48}{1-0.6} = \frac{48}{0.4} = 120$

c) $\sum_{n=4}^{\infty} u_n$ is sum to infinity - sum of first 3 terms

$120 - (48 + 28.8 + 17.28)$

$120 - 94.08 = 25.92$

$u_1 = 48$

$u_2 = 28.8$

$u_3 = 17.28$

Do not write outside the box

4 A curve is defined for $x > 0$. The gradient of the curve at the point (x, y) is given by

$$\frac{dy}{dx} = \frac{2}{x^2} - \frac{x}{4}$$

(a) Find $\frac{d^2y}{dx^2}$.

[3 marks]

(b) The curve has a stationary point M whose y-coordinate is $\frac{5}{2}$.

(i) Find the x-coordinate of M.

[2 marks]

(ii) Use your answers to parts **(a)** and **(b)(i)** to show that M is a maximum point.

[1 mark]

(iii) Find the equation of the curve.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

a) $\frac{dy}{dx} = 2x^{-2} - \frac{1}{4}x$

$\frac{d^2y}{dx^2} = -4x^{-3} - \frac{1}{4}$ $\left(= \frac{-4}{x^3} - \frac{1}{4} \right)$

b) $2x^{-2} - \frac{1}{4}x = 0$

$2x^{-2} = \frac{1}{4}x$

$(\div x)$ $(\div x)$

$2x^{-3} = \frac{1}{4}$

$\frac{2}{x^3} = \frac{1}{4}$

$2 = \frac{x^3}{4}$

$x^3 = 8$

$x = 2$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

bii) sub $x=2$ into $\frac{d^2y}{dx^2}$: $\frac{-4}{2^3} - \frac{1}{4}$

$= \frac{-4}{8} - \frac{1}{4} = -\frac{3}{4} < 0 \therefore$ maximum point

iii) $\int 2x^{-2} - \frac{1}{4}x \, dx = \frac{2x^{-1}}{-1} - \frac{1}{4}\frac{x^2}{2} + c$

$(y) = -2x^{-1} - \frac{x^2}{8} + c$

sub in $(2, \frac{5}{2})$ to find c:

$y = -2x^{-1} - \frac{x^2}{8} + c$

$\frac{5}{2} = \frac{-2}{2} - \frac{(2)^2}{8} + c$

$\frac{5}{2} = -1 - \frac{4}{8} + c$

$\frac{5}{2} = -1\frac{1}{2} + c$

$c = \frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$

$y = -2x^{-1} - \frac{x^2}{8} + 4$

Turn over ▶

5 The nth term of a sequence is u_n.

The sequence is defined by $u_{n+1} = pu_n + q$, where p and q are constants.

The second term of the sequence is 160. The third term of the sequence is 132.

The limit of u_n as n tends to infinity is 20.

(a) Find the value of p and the value of q.

[5 marks]

(b) Hence find the value of the first term of the sequence.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 5

a) $u_3 = pu_2 + q$ $\qquad L = pL + q$

$132 = 160p + q$ $\qquad 20 = 20p + q$

$132 = 160p + q$

$20 = 20p + q$

$112 = 140p$

$p = \frac{112}{140} = \frac{4}{5}$

$132 = 160\left(\frac{4}{5}\right) + q$

$132 = 128 + q$

$q = 4$

b) $u_2 = pu_1 + q$

$160 = \frac{4}{5}u_1 + 4$

$156 = \frac{4}{5}u_1$

$195 = u_1$

6 (a) Solve the equation $\sin(x + 0.7) = 0.6$ in the interval $-\pi < x < \pi$, giving your answers in radians to two significant figures.

[3 marks]

(b) It is given that $5\cos^2\theta - \cos\theta = \sin^2\theta$.

(i) By forming and solving a suitable quadratic equation, find the possible values of $\cos\theta$.

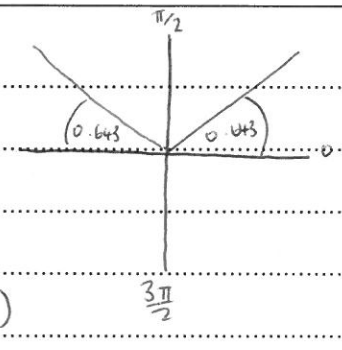
[4 marks]

(ii) Hence show that a possible value of $\tan\theta$ is $2\sqrt{2}$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 6



6a) $x + 0.7 = \sin^{-1}(0.6)$

$x + 0.7 = 0.643\ldots,\ 2.498\ldots$

$x = -0.0564\ldots,\ 1.798\ldots$

$x = -0.056\ (2\text{sf}),\ 1.8\ (2\text{sf})$

bi) $5\cos^2\theta - \cos\theta = \sin^2\theta$

$5\cos^2\theta - \cos\theta = 1 - \cos^2\theta$

$6\cos^2\theta - \cos\theta - 1 = 0$

$(3\cos\theta + 1)(2\cos\theta - 1) = 0$

$3\cos\theta + 1 = 0 \qquad 2\cos\theta - 1 = 0$

$3\cos\theta = -1 \qquad 2\cos\theta = 1$

$\cos\theta = -\frac{1}{3} \qquad \cos\theta = \frac{1}{2}$

ii) when $\cos\theta = -\frac{1}{3}$ $\sin^2\theta = \frac{8}{9}$ $(\cos^2\theta + \sin^2\theta = 1)$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\pm\sqrt{\frac{8}{9}}}{-\frac{1}{3}}$ (we want $+2\sqrt{2}$ which can only happen if $- \div -$)

$\tan\theta = \dfrac{-\sqrt{\frac{8}{9}}}{-\frac{1}{3}} = \sqrt{8}$ $\sqrt{8} = 2\sqrt{2}$ as required

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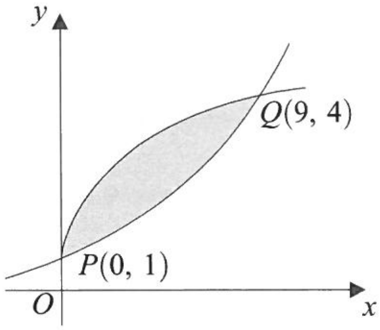
7 The diagram shows a sketch of two curves.



The equations of the two curves are $y = 1 + \sqrt{x}$ and $y = 4^{\frac{x}{9}}$.

The curves meet at the points $P(0,\ 1)$ and $Q(9,\ 4)$.

(a) (i) Describe the geometrical transformation that maps the graph of $y = \sqrt{x}$ onto the graph of $y = 1 + \sqrt{x}$.

[2 marks]

(ii) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^{\frac{x}{9}}$.

[2 marks]

(b) (i) Given that $\int_0^9 \sqrt{x}\,\mathrm{d}x = 18$, find the value of $\int_0^9 (1 + \sqrt{x})\,\mathrm{d}x$.

[1 mark]

(ii) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^9 4^{\frac{x}{9}}\,\mathrm{d}x$. Give your answer to one decimal place.

[4 marks]

(iii) **Hence** find an approximate value for the area of the shaded region bounded by the two curves and state, with an explanation, whether your approximation will be an overestimate or an underestimate of the true value for the area of the shaded region.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 7**
7ai)	translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
ii)	stretch in the x-axis by a scale factor $\left(\frac{1}{\frac{1}{9}} = \right) 9$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7

bi) $\int_0^9 1+\sqrt{x}\,dx = 9 + 18 = 27$

ii) $h = \frac{b-a}{n}$ $\quad h = \frac{9-0}{4} = 2.25$

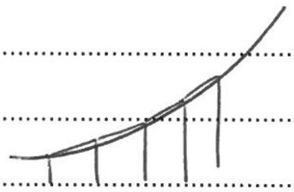
x	0	2.25	4.5	6.75	9
y	1	1.41421....	2	2.8284....	4

$\frac{2.25}{2}\left[1 + 4 + 2(1.414... + 2 + 2.828...)\right]$

$= 19.6709....$

$= 19.7$ (1 dp)

iii) Area $= 27 - 19.7 = 7.3$ (1 dp)



area of trapezia was an overestimate so when this is taken from 27 it will be an underestimate

8 The point A lies on the curve with equation $y = x^{\frac{1}{2}}$. The tangent to this curve at A is parallel to the line $3y - 2x = 1$. Find an equation of this tangent at A.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 8

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$3y - 2x = 1$

$3y = 2x + 1$

$y = \frac{2}{3}x + \frac{1}{3}$

$\frac{2}{3} = \frac{1}{2}x^{-\frac{1}{2}}$

$\frac{4}{3} = x^{-\frac{1}{2}}$

$\frac{4}{3} = \frac{1}{x^{\frac{1}{2}}}$

$\frac{4}{3}x^{\frac{1}{2}} = 1$

$x^{\frac{1}{2}} = \frac{1}{\frac{4}{3}}$

$x^{\frac{1}{2}} = \frac{3}{4}$

$x = \frac{9}{16}$

so $y = \frac{3}{4}$ when $x = \frac{9}{16}$

$y - y_1 = m(x - x_1)$

$y - \frac{3}{4} = \frac{2}{3}(x - \frac{9}{16})$

Do not write outside the box

9 (a) Use logarithms to solve the equation $2^{3x} = 5$, giving your value of x to three significant figures.

[2 marks]

(b) Given that $\log_a k - \log_a 2 = \frac{2}{3}$, express a in terms of k.

[4 marks]

(c) (i) By using the binomial expansion, or otherwise, express $(1 + 2x)^3$ in ascending powers of x.

[3 marks]

(ii) It is given that

$$\log_2[(1+2n)^3 - 8n] = \log_2(1+2n) + \log_2[4(1+n^2)]$$

By forming and solving a suitable quadratic equation, find the possible values of n.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 9

9a)

$$3x \log 2 = \log 5$$

$$3x = \frac{\log 5}{\log 2}$$

$$x = \left(\frac{\log 5}{\log 2}\right) \div 3 = 0.7739760316$$

$$= 0.774 \ (3 sf)$$

b)

$$\log_a \left(\frac{k}{2}\right) = \frac{2}{3}$$

$$a^{\frac{2}{3}} = \frac{k}{2}$$

$$a^{\frac{1}{3}} = \left(\frac{k}{2}\right)^{\frac{1}{2}}$$

$$a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 9

ci)

$${}^3C_0 \times 1^3 \times (2x)^0 = 1 \times 1 \times 1 = 1$$
$${}^3C_1 \times 1^2 \times (2x)^1 = 3 \times 1 \times 2x = 6x$$
$${}^3C_2 \times 1^1 \times (2x)^2 = 3 \times 1 \times 4x^2 = 12x^2$$
$${}^3C_3 \times 1^0 \times (2x)^3 = 1 \times 1 \times 8x^3 = 8x^3$$

$$1 + 6x + 12x^2 + 8x^3$$

ii)

$$\log_2\left[(1+2n)^3 - 8n\right] = \log_2(1+2n) + \log_2\left[4(1+n^2)\right]$$
$$\log_2\left[(1+2n)^3 - 8n\right] = \log_2(1+2n)(4)(1+n^2)$$
$$(1+2n)^3 - 8n = (1+2n)(4+4n^2)$$
$$(1+2n)^3 - 8n = 4 + 4n^2 + 8n + 8n^3$$
$$1 + 6n + 12n^2 + \cancel{8n^3} - 8n = 4 + 4n^2 + 8n + \cancel{8n^3}$$
$$12n^2 - 2n + 1 = 4n^2 + 8n + 4$$
$$8n^2 - 10n - 3 = 0$$

$8n^2 - 10n - 3$

$8 \times -3 = -24$

$\times$ to make -24

$+$ to make -10

$$(2n-3)(4n+1) = 0$$

$(8n-12)(8n+2)$

$(2n-3)(4n+1)$

$2n - 3 = 0$ or $4n + 1 = 0$

$2n = 3$ $\quad$ $4n = -1$

$n = \frac{3}{2}$ $\quad$ $n = \frac{-1}{4}$

Turn over ▶