Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature	WRITTEN SOLUTIONS		

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics

MPC2

Unit Pure Core 2

Thursday 22 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

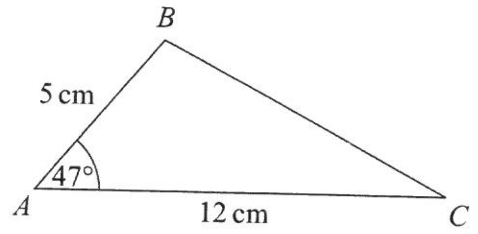
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The diagram shows a triangle ABC.



The size of angle BAC is 47° and the lengths of AB and AC are 5 cm and 12 cm respectively.

(a) Calculate the area of the triangle ABC, giving your answer to the nearest cm^2.

[2 marks]

(b) Calculate the length of BC, giving your answer, in cm, to one decimal place.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 1
1a	$\boxed{\text{Area} = \frac{1}{2}ab\sin C}$ $a = 5$, $b = 12$, $\hat{C} = 47$
	$\text{Area} = \frac{1}{2} \times 5 \times 12 \times \sin 47$
	$= 21.94$
	$= \underline{22\text{cm}^2}$ (nearest cm^2)
b	$BC^2 = 5^2 + 12^2 - (2 \times 5 \times 12 \times \cos 47)$ cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$
	$= 25 + 144 - 81.839\ldots$
	$BC^2 = 87.16\ldots$
	$BC = \sqrt{87.16\ldots}$
	$BC = 9.359\ldots$, $= \underline{9.3\text{cm}}$ (One decimal place)

Do not write outside the box

2 (a) Find $\int \left(1 + 3x^{\frac{1}{2}} + x^{\frac{3}{2}}\right) dx$.

[3 marks]

(b) (i) The expression $(1+y)^3$ can be written in the form $1 + 3y + ny^2 + y^3$. Write down the value of the constant n.

[1 mark]

(ii) Hence, or otherwise, expand $(1+\sqrt{x})^3$.

[1 mark]

(c) Hence find the exact value of $\int_0^1 (1+\sqrt{x})^3 \, dx$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

2a) $\int (1 + 3x^{1/2} + x^{3/2}) \, dx$

$= x + \dfrac{3x^{3/2}}{3/2} + \dfrac{x^{5/2}}{5/2} + C$

$= x + 2x^{3/2} + \dfrac{2}{5}x^{5/2} + C$

bi) $(1+y)^3$

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
\end{array}
$$

$\boxed{1 \quad 3 \quad 3 \quad 1}$

OR ${}^nC_r$

$= ({}^3C_0)(1)^3(y)^0 = 1$

$+ ({}^3C_1)(1)^2(y)^1 = 3y$

$+ ({}^3C_2)(1)^1(y)^2 = 3y^2$

$+ ({}^3C_3)(1)^0(y)^3 = y^3$

$= 1 + 3y + 3y^2 + y^3$ $\quad$ $\underline{n = 3}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

ii) $(1+\sqrt{x})^3$ replace 'y' with $\sqrt{x}$:-

$1+3y+3y^2+y^3$

$= 1+3\sqrt{x}+3(\sqrt{x})^2+(\sqrt{x})^3$

$= \underline{1+3\sqrt{x}+3x+\sqrt{x}^3}$

c) $\int_0^1 (1+\sqrt{x})^3\,dx$

$= \int_0^1 (1+3\sqrt{x}+3x+\sqrt{x}^3)\,dx$

$= \int_0^1 (1+3x^{1/2}+3x+x^{3/2})\,dx$

$= \left[x+\frac{3x^{3/2}}{3/2}+\frac{3x^2}{2}+\frac{x^{5/2}}{5/2}\right]_0^1$

$= \left[x+2x^{3/2}+\frac{3x^2}{2}+\frac{2}{5}x^{5/2}\right]_0^1$

$= \left(1+2(1)^{3/2}+\frac{3(1)^2}{2}+\frac{2}{5}(1)^{5/2}\right)-0$

$= \left(1+2+\frac{3}{2}+\frac{2}{5}\right)-0$

$= \underline{\underline{\frac{49}{10}}}$ OR 4.9

Turn over ▶

3 The first term of a geometric series is 54 and the common ratio of the series is $\frac{8}{9}$.

(a) Find the sum to infinity of the series.

[2 marks]

(b) Find the second term of the series.

[1 mark]

(c) Show that the 12th term of the series can be written in the form $\frac{2^p}{3^q}$, where p and q are integers.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 3
3a)	$a = 54 \qquad r = 8/9$
	$S_\infty = \frac{a}{1-r}$
	$= \frac{54}{1-8/9} = \underline{\underline{486}}$
b)	$U_n = ar^{n-1}$
	$U_2 = 54 \times \left(\frac{8}{9}\right)^1 = \underline{\underline{48}}$
c)	$U_{12} = 54 \times \left(\frac{8}{9}\right)^{11}$
	$= 2 \times 3 \times 3 \times 3 \times \left(\frac{2^3}{3^2}\right)^{11}$
	$= \frac{2 \times 3^3 \times 2^{33}}{3^{22}} = \underline{\underline{\frac{2^{34}}{3^{19}}}}$
	$p = 34, \quad q = 19$

Do not write outside the box

4 A curve has equation $y = \dfrac{1}{x^2} + 4x$.

(a) Find $\dfrac{dy}{dx}$.

[3 marks]

(b) The point $P(-1, -3)$ lies on the curve. Find an equation of the normal to the curve at the point P.

[3 marks]

(c) Find an equation of the tangent to the curve that is parallel to the line $y = -12x$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 4

4) $y = \dfrac{1}{x^2} + 4x$

a) $y = x^{-2} + 4x$

$\dfrac{dy}{dx} = -2x^{-3} + 4$

b) equation of normal need gradient and coordinate

$P(-1, -3)$

grad of tangent $\Rightarrow$

when $x = -1$, $\dfrac{dy}{dx} = -2(-1)^{-3} + 4$

$= 6$

grad of normal $\rightarrow -\dfrac{1}{6}$ (m)

$y - y_1 = m(x - x_1)$

$y + 3 = -\dfrac{1}{6}(x + 1)$

$6y + 18 = -x - 1$

$6y + x + 19 = 0$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

c) equation of tangent parallel to $y = -12x$

gradient $(m) = -12$

$\frac{dy}{dx} = -12$

$-2x^{-3} + 4 = -12 \quad (-4)$

$-2x^{-3} = -16 \quad (\div -2)$

$x^{-3} = 8$

$\frac{1}{x^3} = 8$

$x^3 = \frac{1}{8} \quad (\sqrt[3]{\ })$

$x = \frac{1}{2}$

when $x = \frac{1}{2}$, $y = \frac{1}{(\frac{1}{2})^2} + 4(\frac{1}{2})$

$y = \frac{1}{\frac{1}{4}} + 2 = 6$

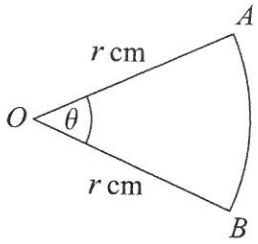
$(\frac{1}{2}, 6)$, $m = -12$

$y - 6 = -12(x - \frac{1}{2})$

$y - 6 = -12x + 6$

$y = -12x + 12$

5 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is θ radians.

The area of the sector is $12\,\text{cm}^2$.

The perimeter of the sector is four times the length of the arc AB.

Find the value of r.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 5

5) Area of sector $= \frac{1}{2}r^2\theta$

$\frac{1}{2}r^2\theta = 12$

Arc length $= r\theta$

$r + r + r\theta = 4r\theta \quad (-r\theta)$

$2r = 3r\theta \quad (\div 3r)$

$\theta = \frac{2r}{3r} = \frac{2}{3}$

$\frac{1}{2}r^2\theta = 12, \quad \theta = 2/3$

$\frac{1}{2}r^2 \times \frac{2}{3} = 12$

$\frac{1}{3}r^2 = 12\ (\times 3), \quad r^2 = 36 \quad (\sqrt{\ })$

$r = 6$

Do not write outside the box

6 (a) Sketch, on the axes given below, the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

[2 marks]

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = \sin 5x$.

[2 marks]

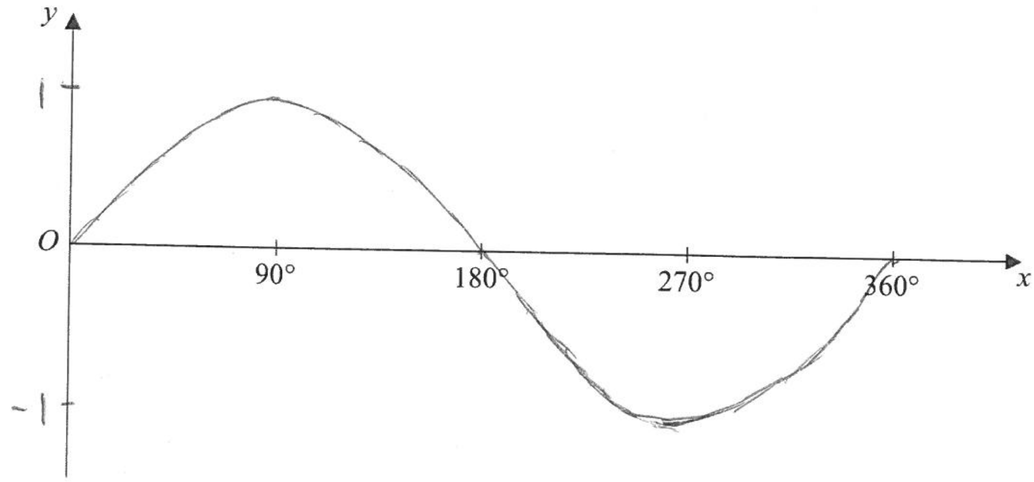
(c) Describe the single geometrical transformation that maps the graph of $y = \sin 5x$ onto the graph of $y = \sin(5x + 10°)$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

(a)



6a) graph

b) $y = \sin x \rightarrow f(x) = \sin x$

so, $\sin 5x = f(5x)$

stretch, scale factor $1/5$ in x direction

c) $y = \sin(5x + 10)$

$y = \sin 5x \rightarrow y = \sin(5x + 10)$
$f(x)$

$y = \sin(5(x+2))$
$f(x+2)$

translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$

Do not write outside the box

7 (a) Given that $\dfrac{\cos^2 x + 4\sin^2 x}{1 - \sin^2 x} = 7$, show that $\tan^2 x = \dfrac{3}{2}$.

[3 marks]

(b) **Hence** solve the equation $\dfrac{\cos^2 2\theta + 4\sin^2 2\theta}{1 - \sin^2 2\theta} = 7$ in the interval $0° < \theta < 180°$, giving your values of θ to the nearest degree.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 7

7a) $\dfrac{\cos^2 x + 4\sin^2 x}{1-\sin^2 x} = 7$

$\boxed{\begin{array}{l}\cos^2 x + \sin^2 x = 1 \\ \text{so } \cos^2 x = 1 - \sin^2 x\end{array}}$

$\dfrac{\cos^2 x + 4\sin^2 x}{\cos^2 x} = 7$ (separate fraction)

$\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{4\sin^2 x}{\cos^2 x} = 7$

$1 + 4\tan^2 x = 7$ (-1)

$4\tan^2 x = 6$ $(\div 4)$

$\tan^2 x = \dfrac{6}{4}$ (cancel down)

$\tan^2 x = \dfrac{3}{2}$ (as required)

1 4

Do not write outside the box

QUESTION PART REFERENCE | **Answer space for question 7**

b) Hence

so, solve $\tan^2 x = \frac{3}{2}$

But replace x with 2θ

change range $\rightarrow$ $0^\circ < \theta < 180^\circ$

$0^\circ < 2\theta < 360^\circ$

$\tan^2 2\theta = \frac{3}{2}$ (5)

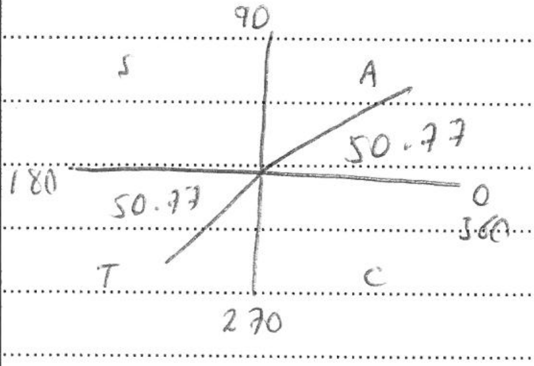
$\tan 2\theta = \pm\sqrt{\frac{3}{2}}$

$\tan 2\theta = \pm\sqrt{3/2}$ OR $\tan 2\theta = -\sqrt{3/2}$

$2\theta = \tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$

$2\theta = 50.768...,\ 230.768...$

$\theta = 25.384...,\ 115.384...$

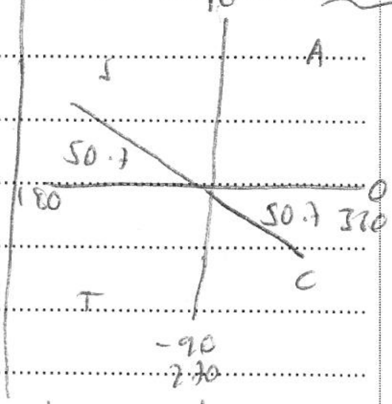


$2\theta = \tan^{-1}\left(-\sqrt{\frac{3}{2}}\right)$

$2\theta = -50.768...$ ✗ not in range

$129.232,\ 309.232.$

$\theta = 64.62.\ \ 154.62$



$\theta = 25^\circ,\ 65^\circ,\ 115^\circ,\ 155^\circ$ (nearest degree)

Do not write outside the box

8 An arithmetic series has first term a and common difference d.

The sum of the first 5 terms of the series is 575.

(a) Show that $a + 2d = 115$.

[3 marks]

(b) Given also that the 10th term of the series is 87, find the value of d.

[3 marks]

(c) The nth term of the series is u_n. Given that $u_k > 0$ and $u_{k+1} < 0$, find the value of $\sum_{n=1}^{k} u_n$.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 8
8)	$a = ?$, $d = ?$ $S_5 = 575$, $S_n = \frac{n}{2}(2a + (n-1)d)$
a)	$n = 5$, $S_5 = 575$
	$S_n = \frac{n}{2}(2a + (n-1)d)$
	$575 = \frac{5}{2}(2a + 4d)$ $(\times 2)$
	$1150 = 5(2a + 4d)$ $(\div 5)$
	$230 = 2a + 4d$ $(\div 2)$
	$115 = a + 2d$ (as req)
b.)	$U_n = a + (n-1)d$ $n = 10$
	$87 = a + 9d$ $U_{10} = 87$
	solve $a + 2d = 115$ (1) (2) - (1)
	$a + 9d = 87$ (2)
	$- \; a + 2d = 115$
	$7d = -28$ $(\div 7)$
	$d = -4$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

c) $U_n = a + (n-1)d \qquad d = -4$

$a + 2d = 115 \quad$ if $d = -4$

$a + 2(-4) = 115$

$a - 8 = 115$

$\underline{a = 123}$

$U_n = a + (n-1)d \qquad a = 123, \quad d = -4$

$U_k > 0$

$U_k = 123 + (k-1)(-4)$

$123 + (k-1)(-4) > 0$

$123 - 4k + 4 > 0$

$127 - 4k > 0$

$4k < 127 \quad (\div 4)$

$k < 31.75$

$U_{k+1} < 0$

$U_{k+1} = 123 + k(-4)$

$123 + k(-4) < 0$

$123 - 4k < 0$

$4k > 123 \quad (\div 4)$

$k > 30.75$

$30.75 < k < 31.75 \quad \therefore \underline{k = 31}$

$\sum_{n=1}^{k} U_n = S_k \rightarrow S_{31} = \frac{31}{2}\left(2(123) + 30(-4)\right)$

$= \underline{1953}$

Turn over ▶

9 A curve has equation $y = 3 \times 12^x$.

(a) The point $(k, 6)$ lies on the curve $y = 3 \times 12^x$. Use logarithms to find the value of k, giving your answer to three significant figures.

[3 marks]

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^{1.5} 3 \times 12^x \, dx$, giving your answer to two significant figures.

[4 marks]

(c) The curve $y = 3 \times 12^x$ is translated by the vector $\begin{bmatrix} 1 \\ p \end{bmatrix}$ to give the curve $y = f(x)$. Given that the curve $y = f(x)$ passes through the origin $(0, 0)$, find the value of the constant p.

[3 marks]

(d) The curve with equation $y = 2^{2-x}$ intersects the curve $y = 3 \times 12^x$ at the point T. Show that the x-coordinate of T can be written in the form $\frac{2 - \log_2 3}{q + \log_2 3}$, where q is an integer. State the value of q.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 9**
9)	$y = 3 \times 12^x$
a)	$(k, 6)$ $x = k$, $y = 6$
	so, $6 = 3 \times 12^k$
	$12^k = 2$ (logs)
	$k \log 12 = \log 2$
	$k = \frac{\log 2}{\log 12}$
	$k = 0.27894.. = \underline{\underline{0.279}}$ (3sf)

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 9

$n = 3$ (strips)

b) $\int_0^{1.5} 3 \times 12^x \, dx$ $h = \frac{1.5 - 0}{3} = \underline{\underline{0.5}}$

x	0	0.5	1	1.5
y	3	10.3923	36	124.707..

$\int_0^{1.5} 3 \times 12^x dx = \frac{0.5}{2}\left(3 + 124.707.. + 2(10.39.. + 36)\right)$

$= 0.25(220.49...)$

$= 55.1230...$

$= \underline{\underline{55}}$ (2 significant figures)

c) $y = 3 \times 12^x$ translated $\begin{pmatrix} 1 \\ p \end{pmatrix}$

$f(x) = 3 \times 12^{x-1} + p$ if passes thro $(0, 0)$

$0 = 3 \times 12^{-1} + p$ $x = 0, y = 0$ $(f x)$

$\underline{\underline{p = -0.25}}$

d) $2^{2-x} = 3 \times 12^x$ (intersecting means "make equal")

$(2-x)\log_2 2 = \log_2(3 \times 12^x)$

$(2-x)\log_2 2 = \log_2 3 + \log_2 12^x$

$(2-x)\log_2 2 = \log_2 3 + x\log_2 12$

$(2-x)\log_2 2 = \log_2 3 + x(\log_2 3 + \log_2 4)$ $\boxed{\log_2 4 = 2}$

$(2-x)\log_2 2 = \log_2 3 + x(\log_2 3 + 2)$

$(2-x)\log_2 2 = \log_2 3 + x\log_2 3 + 2x$ $\boxed{\log_2 2 = 1}$

$2 - x = \log_2 3 + x\log_2 3 + 2x$ $(+x)$

$2 - \log_2 3 = x\log_2 3 + 3x$

$2 - \log_2 3 = x(\log_2 3 + 3)$

$x = \frac{2 - \log_2 3}{\log_2 3 + 3}$ $(q = 3)$

Turn over ▶