Centre Number						Candidate Number				
Surname										
Other Names	WRITTEN									
Candidate Signature	SOLUTIONS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC2

Unit Pure Core 2

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The arithmetic series

$$23 + 32 + 41 + 50 + \ldots + 2534$$

has 280 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 100th term of the series. *(2 marks)*

(c) Find the sum of the 280 terms of the series. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

1a) $d = \underline{9}$

b) $U_{100} = a + (n-1)d$

$= 23 + 99(9)$

$= \underline{914}$

c) $S_{280} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{280}{2}(2(23) + 279(9))$

$= \underline{357980}$

Do not write outside the box

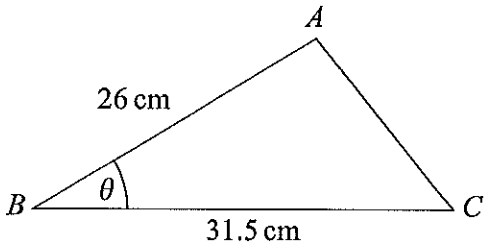
QUESTION PART REFERENCE

Answer space for question 1

Turn over ▶

Do not write outside the box

2 The triangle ABC, shown in the diagram, is such that $AB = 26$ cm and $BC = 31.5$ cm.



The acute angle ABC is θ, where $\sin\theta = \frac{5}{13}$.

(a) Calculate the area of triangle ABC. *(2 marks)*

(b) Find the exact value of $\cos\theta$. *(1 mark)*

(c) Calculate the length of AC. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 2

2a) Area $= \frac{1}{2} ab \sin C$ $\boxed{\sin C = \frac{5}{13}}$

$= \frac{1}{2} \times 26 \times 31.5 \times \frac{5}{13}$

$= \underline{157.5 \text{ cm}^2}$

b) $\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1$

$\frac{25}{169} + \cos^2\theta = 1$

$\cos^2\theta = \frac{144}{169}$

$\cos\theta = \underline{\frac{12}{13}}$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

c) $AC^2 = 26^2 + 31.5^2 - (2 \times 26 \times 31.5 \times \frac{12}{13})$

$= 676 + 992.25 - (1512)$

$= 156.25$

$AC = \sqrt{156.25}$

$= 12.5\text{cm}$

Do not write outside the box

3 (a) Expand $\left(x^{\frac{3}{2}} - 1\right)^2$. *(2 marks)*

(b) Hence find $\int \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(3 marks)*

(c) Hence find the value of $\int_1^4 \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 3

3a) $(x^{3/2} - 1)(x^{3/2} - 1)$

$x^{6/2} - x^{3/2} - x^{3/2} + 1$

$x^3 - 2x^{3/2} + 1$

b) $\int x^3 - 2x^{3/2} + 1 \, dx$

$\frac{x^4}{4} - \frac{2x^{5/2}}{5/2} + x + c$

$\frac{x^4}{4} - \frac{4}{5}x^{5/2} + x + c$

c) $\left[\frac{x^4}{4} - \frac{4}{5}x^{5/2} + x\right]_1^4$

$= \left(\frac{4^4}{4} - \frac{4}{5}(4)^{5/2} + 4\right) - \left(\frac{1^4}{4} - \frac{4}{5}(1)^{5/2} + 1\right)$

$= 41.95$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

Turn over ▶

Do not write outside the box

4 The nth term of a geometric series is u_n, where $u_n = 48\left(\frac{1}{4}\right)^n$.

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Find the value of the common ratio of the series. *(1 mark)*

(c) Find the sum to infinity of the series. *(2 marks)*

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 4

4a) $U_1 = 48\left(\frac{1}{4}\right)^1 = \underline{12}$

$U_2 = 48\left(\frac{1}{4}\right)^2 = \underline{3}$

b) $r = \frac{3}{12} = \frac{1}{4}$

c) $S_\infty = \frac{a}{1-r}$

$= \frac{12}{1-1/4} = \underline{16}$

d) $\sum_{n=4}^{\infty} U_n = S_\infty - S_3$

$S_3 = U_1 + U_2 + U_3$ $\quad U_3 = 48\left(\frac{1}{4}\right)^3 = \frac{3}{4}$

$= 12 + 3 + 3/4$

$= 15\,3/4$

$\sum_{n=4}^{\infty} U_n = 16 - 15\,3/4 = \underline{1/4}$

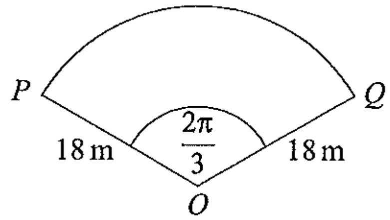
Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

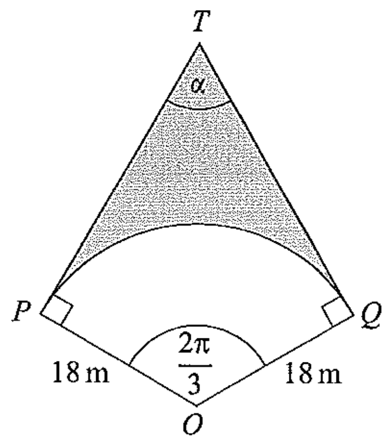
5 The diagram shows a sector OPQ of a circle with centre O.



The radius of the circle is 18 m and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) The tangents to the circle at the points P and Q meet at the point T, and the angles TPO and TQO are both right angles, as shown in the diagram below.



(i) Angle $PTQ = \alpha$ radians. Find α in terms of π. *(1 mark)*

(ii) Find the area of the shaded region bounded by the arc PQ and the tangents TP and TQ, giving your answer to three significant figures. *(6 marks)*

QUESTION PART REFERENCE	Answer space for question 5
5a)	arc length $= r\theta$
	$= r \times \frac{2\pi}{3}$
	$= 18 \times \frac{2\pi}{3} \quad = \underline{12\pi}$

QUESTION PART REFERENCE	Answer space for question 5
b) i)	$\alpha = 180 - \frac{2\pi}{3}$
	$= \pi - \frac{2\pi}{3} = \frac{\pi}{3}$
ii)	Area of sector $= \frac{1}{2} r^2 \theta$
	$= \frac{1}{2} \times 18^2 \times \frac{2\pi}{3}$
	$= 108\pi$
	Area of kite = area of $\Delta TPO + \Delta TQO$
	$\Delta TPO = \Delta TPQ$
	$\Delta TPO = \frac{1}{2} \times 18 \times TP$
	$TP = 18 \times \tan \frac{\pi}{3}$
	$= 18 \times \tan \frac{\pi}{3} = 31.1769\ldots$
	$\Delta TPO = \frac{1}{2} \times 18 \times 31.1769\ldots$
	kite $= 2 \times (\frac{1}{2} \times 18 \times 31.1769\ldots)$
	$= 561.18\ldots$
	Shaded = kite − sector
	$= 561.18\ldots - 108\pi = 222\,m^2$ (3sf)

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 5

Turn over ▶

Do not write outside the box

6 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$$

The point $P(2, 1)$ lies on the curve.

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 2$. *(1 mark)*

(ii) Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$. *(4 marks)*

(iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. *(1 mark)*

(b) Find the equation of the curve. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6ai) when $x = 2$,

$\frac{dy}{dx} = 3(2)^2 - \frac{4}{2^2} - 11$

$= 12 - 1 - 11 = 0$ (as req.)

ii) $\frac{d^2y}{dx^2} = 6x + \frac{8}{x^3}$

when $x = 2$, $\frac{d^2y}{dx^2} = 6(2) + \frac{8}{2^3} = 13$

iii) $\frac{d^2y}{dx^2} > 0$, $\therefore$ minimum

b) $y = x^3 + \frac{4}{x} - 11x + c$ sub in $(2, 1)$

$1 = 2^3 + \frac{4}{2} - 11(2) + c$

$y = x^3 + \frac{4}{x} - 11x + 13$

$1 = 8 + 2 - 22 + c$

$c = 13$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

Do not write outside the box

7 It is given that $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$.

(a) Find the possible values of $\tan\theta$. *(4 marks)*

(b) Hence solve the equation $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$, giving all solutions for θ, in degrees, in the interval $0° \leqslant \theta \leqslant 180°$. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 7

7a) $\tan\theta + 1 = 0$ $\quad$ $\sin^2\theta - 3\cos^2\theta = 1$

$\tan\theta = -1$ $\quad$ $\sin^2\theta = 3\cos^2\theta$

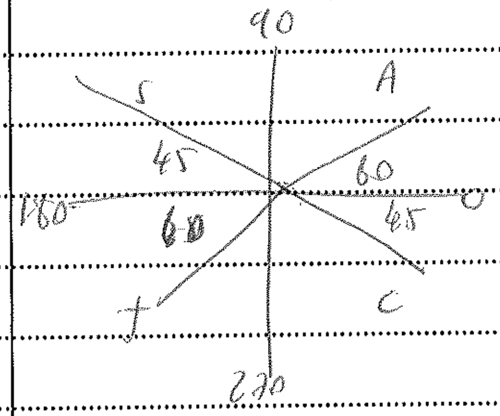
$\frac{\sin^2\theta}{\cos^2\theta} = 3$

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

b) $\tan\theta = -1$ $\quad$ $\tan\theta = -\sqrt{3}$

$\theta = 135°$ $\quad$ $\theta = 120$



$\tan\theta = \sqrt{3}$

$\theta = 60$

$60°, 120°, 135°$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7

8 (a) Sketch the curve with equation $y = 7^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) The curve C_1 has equation $y = 7^x$.
The curve C_2 has equation $y = 7^{2x} - 12$.

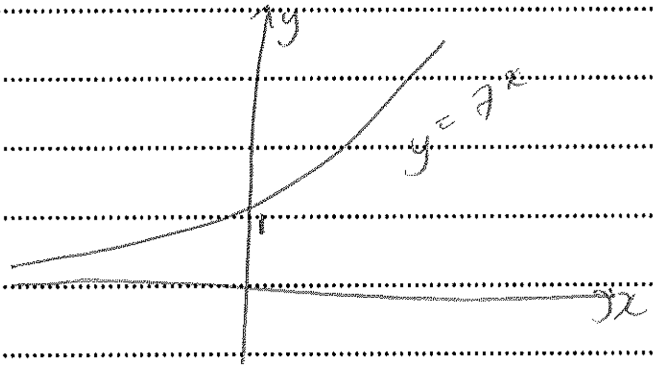
(i) By forming and solving a quadratic equation, prove that the curves C_1 and C_2 intersect at exactly one point. State the y-coordinate of this point. *(4 marks)*

(ii) Use logarithms to find the x-coordinate of the point of intersection of C_1 and C_2, giving your answer to three significant figures. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 8

8a)



b) $7^x = 7^{2x} - 12$

$7^{2x} - 7^x - 12 = 0$

$(7^x - 4)(7^x + 3) = 0$

(OR let $y = 7^x$)

$y^2 - y - 12$

$(y-4)(y+3)$

$7^x - 4 = 0$ OR $7^x + 3 = 0$

$7^x = 4$ $\quad y = 4$ $\quad\quad 7^x = -3$ X

c) $x \log 7 = \log 4$

$x = \dfrac{\log 4}{\log 7}$

$x = 0.712$ (3sf)

Do not write outside the box

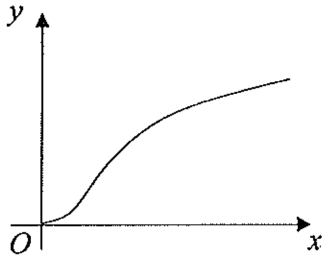
QUESTION PART REFERENCE

Answer space for question 8

Turn over ▶

Do not write outside the box

9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1)\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) The graph of $y = 2\log_{10} x$ can be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a translation. Write down the vector of the translation. *(1 mark)*

(c) (i) Show that $\log_{10}(10x^2) = 1 + 2\log_{10} x$. *(2 marks)*

(ii) Show that the graph of $y = 2\log_{10} x$ can also be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a **stretch**, and describe the stretch. *(4 marks)*

(iii) The curve with equation $y = 1 + 2\log_{10} x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P. Given that the x-coordinate of P is positive, find the gradient of the line OP, where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 9

9a) $h = \frac{1-0}{4} = 1/4$

x	0	$1/4$	$1/2$	$3/4$	1
y	$\log 1$	$\log \frac{17}{16}$	$\log \frac{5}{4}$	$\log \frac{25}{16}$	$\log 2$

$\frac{1/4}{2}\left(\log 1 + 2(\log 17/16 + \log 5/4 + \log 25/16) + \log 2\right)$

$= 0.117$ (3sf)

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 9
b)	$f(x) = 2\log_{10} x$
	$1 + 2\log_{10} x = f(x) + 1$
	Translation $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
c i)	$\log_{10}(10x^2) = \log_{10} 10 + \log_{10} x^2$
	$= 1 + 2\log_{10} x$ (as req)
ii)	$2\log_{10} x \rightarrow \log_{10}(10x^2)$
	$\log_{10} x^2 \rightarrow \log_{10}(\sqrt{10}x)^2$
	$2\log_{10} x = \log_{10} x^2 \rightarrow \log_{10}(\sqrt{10}x)^2$
	Stretch, in x direction SF $1/\sqrt{10}$
iii)	$\log_{10}(10x^2) = \log_{10}(x^2+1)$
	$10x^2 = x^2 + 1$
	$9x^2 = 1$
	$x^2 = 1/9$
	$x = \pm\sqrt{1/9}$
	$x = +\sqrt{1/9}$
	$x = 1/3$ (+ve)
	$y = \log_{10}(10(1/3)^2$
	$= \log_{10}\left(\frac{10}{9}\right)$
	Gradient $= \frac{\log_{10}\frac{10}{9}}{1/3} = 3\log_{10}\frac{10}{9} = \log_{10}\left(\frac{10}{9}\right)^3$
	$= \log_{10}\left(\frac{1000}{729}\right)$

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 9

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 9

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED