Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature	ANSWERS									

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC2

Unit Pure Core 2

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

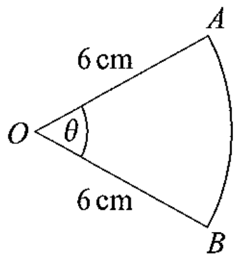
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The diagram shows a sector OAB of a circle with centre O and radius 6 cm.



The angle between the radii OA and OB is θ radians.

The area of the sector OAB is $21.6\,\text{cm}^2$.

(a) Find the value of θ. *(2 marks)*

(b) Find the length of the arc AB. *(2 marks)*

QUESTION PART REFERENCE

1a) Area $= \frac{1}{2} r^2 \theta$

$21.6 = \frac{1}{2} \times 6^2 \times \theta$

$21.6 = 18 \times \theta$

$\theta = \frac{21.6}{18} = \underline{1.2}$

b) Arc length $= r\theta$

$= 6 \times 1.2$

$= \underline{7.2\text{cm}}$

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{2^x}{x+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

QUESTION PART REFERENCE

2a) $\int_0^4 \frac{2^x}{x+1}\,dx$ $h = \frac{4-0}{4}$

$h = 1$

x	0	1	2	3	4
y	1	1	$4/3$	$8/4$	$16/5$

$I = \frac{1}{2}\left(1 + 16/5 + 2\left(1 + 4/3 + 8/4\right)\right)$

$= \underline{6.43}$ (3sf)

b) using more ordinates / strips

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

3 (a) Write $\sqrt[4]{x^3}$ in the form x^k. *(1 mark)*

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. *(2 marks)*

QUESTION PART REFERENCE

3a) $\sqrt[4]{x^3} = x^{3/4}$

b) $\dfrac{1-x^2}{\sqrt[4]{x^3}} = \dfrac{1-x^2}{x^{3/4}}$

$= \dfrac{1}{x^{3/4}} - \dfrac{x^2}{x^{3/4}}$

$= x^{-3/4} - x^{5/4}$

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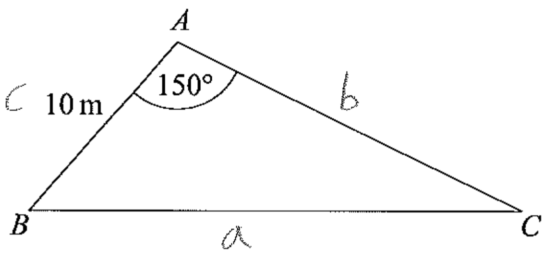
QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is $40\,\text{m}^2$.

(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

QUESTION PART REFERENCE

4a) $\text{Area} = \frac{1}{2} bc \sin A$

$40 = \frac{1}{2}(AC)(10)(\sin 150)$

$40 = 5\sin 150 \times AC$

$AC = \frac{40}{5\sin 150}$

$AC = \underline{16\text{m}}$ (as required)

b) $BC^2 = 10^2 + 16^2 - (2 \times 10 \times 16 \times \cos 150)$

$BC^2 = 100 + 256 - (-277.128\ldots)$

$BC^2 = 633.128\ldots$

$BC = \sqrt{633.128\ldots}$ $\qquad BC = \underline{25.16\text{m}}$ (2dp)

Do not write outside the box

QUESTION PART REFERENCE

c) $\frac{\sin C}{10} = \frac{\sin 150}{25.16}$

$\sin C = \frac{10 \sin 150}{25.16}$

$\sin C = 0.1987\ldots$

$C = \sin^{-1}(0.1987\ldots)$

$C = \underline{11.5^\circ}$ (1 dp)

Turn over ▶

5 (a) (i) Describe the geometrical transformation that maps the graph of $y = \left(1 + \frac{x}{3}\right)^6$ onto the graph of $y = (1 + 2x)^6$. *(2 marks)*

(ii) The curve $y = \left(1 + \frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the curve $y = g(x)$. Find an expression for $g(x)$, simplifying your answer. *(2 marks)*

(b) The first four terms in the binomial expansion of $\left(1 + \frac{x}{3}\right)^6$ are $1 + ax + bx^2 + cx^3$.

Find the values of the constants a, b and c, giving your answers in their simplest form. *(4 marks)*

QUESTION PART REFERENCE

5ai) $f(x) = \left(1 + \frac{x}{3}\right)^6$

$f(6x) = (1+2x)^6$

stretch scale factor $1/6$ in x direction

ii) $f(x) = \left(1 + \frac{x}{3}\right)^6$

$f(x-3) \rightarrow$ translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$\llcorner g(x)$

replace x with $x-3$:-

$\left(1 + \frac{x-3}{3}\right)^6 = \left(1 + \frac{x}{3} - \frac{3}{3}\right)^6$

$g(x) = \left(1 + \frac{x}{3} - 1\right)^6$

$g(x) = \left(\frac{x}{3}\right)^6$

b) $\left(1 + \frac{x}{3}\right)^6 = (1)^6 + {}^6C_1(1)^5\left(\frac{x}{3}\right)^1 + {}^6C_2(1)^4\left(\frac{x}{3}\right)^2 + {}^6C_3(1)^3\left(\frac{x}{3}\right)^3$

$= 1 + 6\left(\frac{x}{3}\right) + 15\left(\frac{x^2}{9}\right) + 20\left(\frac{x^3}{27}\right)$

$= 1 + 2x + \frac{15}{9}x^2 + \frac{20}{27}x^3$, $a = 2$

$b = 15/9$, $c = \frac{20}{27}$

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

QUESTION PART REFERENCE

6a) $S_{25} = 3500$

$\frac{25}{2}(2a + (25-1)d) = 3500$

$\frac{25}{2}(2a + 24d) = 3500$

$25(2a+24d) = 7000$

$2a + 24d = \frac{7000}{25}$

$2a + 24d = 280 \quad (\div 2)$

$a + 12d = 140$ (as required)

Do not write outside the box

QUESTION PART REFERENCE

b) $U_5 = 100$

$100 = a + 4d$ ①

$140 = a + 12d$ ②   ② - ①

$140 = a + 12d$

$-\ 100 = a + 4d$

$40 = 8d$

$\underline{d = 5}$

sub $d = 5$ into ①

$100 = a + 4(5)$

$100 = a + 20$

$\underline{a = 80}$

c) let $\sum_{n=1}^{k} U_n = x$, $\sum_{n=1}^{25} U_n = 3500$

$33\left(\sum_{n=1}^{25} U_n - \sum_{n=1}^{k} U_n\right) = 33(3500 - x)$

so, $33(3500 - x) = 67x$

$115500 - 33x = 67x$

$115500 = 100x$

$x = 1155$

$\therefore \sum_{n=1}^{k} U_n = \underline{1155}$

Turn over ▶

7 (a) Sketch the graph of $y = \dfrac{1}{2^x}$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $\dfrac{1}{2^x} = \dfrac{5}{4}$, giving your answer to three significant figures. *(3 marks)*

(c) Given that

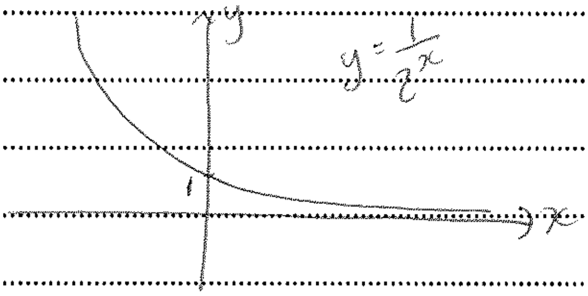
$$\log_a(b^2) + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms. *(5 marks)*

QUESTION PART REFERENCE

7a)



b) $\dfrac{1}{2^x} = \dfrac{5}{4}$

$2^x = \dfrac{4}{5}$

$x \log 2 = \log 0.8$

$x = \dfrac{\log 0.8}{\log 2}$

$x = -0.321928$

$x = -0.322$ (3sf)

Do not write outside the box

QUESTION PART REFERENCE

c) $\log_a(b^2) + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$

$\log_a b^2 + 3\log_a y = 3 + 2(\log_a y - \log_a a)$

$\log_a b^2 + 3\log_a y = 3 + 2\log_a y - 2\log_a a$

$\log_a b^2 + 3\log_a y - 2\log_a y = 3 - 2(1)$

$\log_a b^2 + \log_a y = 1$

$\log_a b^2 y = 1$

$b^2 y = a^1$

$\underline{b^2 y = a} \rightarrow \underline{y = \frac{a}{b^2}}$ OR $\underline{y = ab^{-2}}$

Do not write outside the box

8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

QUESTION PART REFERENCE

8a) $2\sin\theta = 7\cos\theta$ $(\div \cos\theta)$

$\frac{2\sin\theta}{\cos\theta} = 7$ $(\div 2)$

$\frac{\sin\theta}{\cos\theta} = \frac{7}{2}$

$\tan\theta = \frac{7}{2}$

bi) $6\sin^2 x = 4 + \cos x$ $\boxed{\sin^2 x = 1 - \cos^2 x}$

$6(1 - \cos^2 x) = 4 + \cos x$

$6 - 6\cos^2 x = 4 + \cos x$

$6\cos^2 x + \cos x - 2 = 0$ (as required)

ii) $6\cos^2 x + \cos x - 2 = 0$ $0 < x < 360$

$(3\cos x + 2)(2\cos x - 1) = 0$

$3\cos x + 2 = 0$ or $2\cos x - 1 = 0$

$\cos x = -2/3$ $\cos x = 1/2$

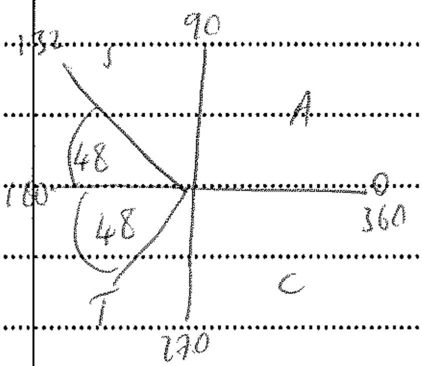
$x = \cos^{-1}(-2/3)$ $x = \cos^{-1}(1/2)$

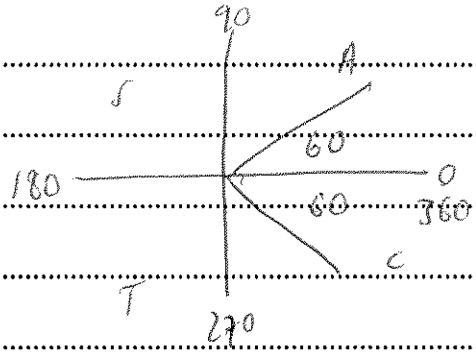
Do not write outside the box

QUESTION PART REFERENCE

$\cos x = -2/3$ (negative) OR $\cos x = 1/2$ (positive)

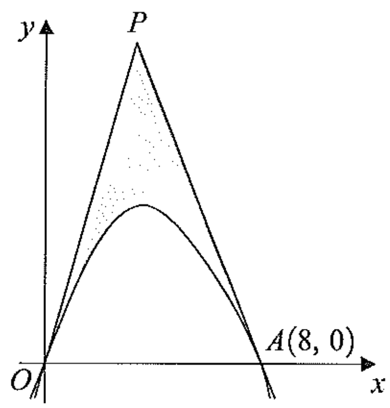
$x = 132, 228^\circ$ $x = 60^\circ, 300^\circ$





Turn over ▶

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\displaystyle\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

QUESTION PART REFERENCE

9a) $y = 12x - 3x^{5/3}$

$\dfrac{dy}{dx} = 12 - \dfrac{15}{3}x^{2/3}$

$= 12 - 5x^{2/3}$

Do not write outside the box

QUESTION PART REFERENCE

b i) when $x = 0$, $\frac{dy}{dx} = 12 - 5(0)^{2/3}$
$= 12$

equation of tangent is $y = 12x$

ii) when $x = 8$, $\frac{dy}{dx} = 12 - 5(8)^{2/3}$

$= -8$

coordinate is $(8, 0)$ gradient is -8 (m)

$y - y_1 = m(x - x_1)$

$y - 0 = -8(x - 8)$

$y = -8x + 64$

$y + 8x = 64$ (as required)

c) $\int (12x - 3x^{5/3})\,dx = \frac{12x^2}{2} - \frac{3x^{8/3}}{8/3} + c$

$= 6x^2 - \frac{9}{8}x^{8/3} + c$

d) shaded area = area of $\triangle$ - area under curve

area of $\triangle$ $= \frac{1}{2}bh$ $b = 8$, h = y value at P

tangents intersect at P :-

$y = 12x$ ① and $y + 8x = 64$ ② sub ① into ②

$12x + 8x = 64$

$20x = 64$, $x = 3.2 \rightarrow$ sub in ①

$y = 12(3.2)$, $y = 38.4$

Turn over ▶

QUESTION PART REFERENCE

area of $\Delta = \frac{1}{2}bh$ $b = 8$, $h = 38.4$

$= \frac{1}{2}(8)(38.4)$

$= \underline{153.6}$

Area under curve $= \int_0^8 (12x - 3x^{5/3})\,dx$

$\left[6x^2 - \frac{9}{8}x^{8/3}\right]_0^8$

$\left(6(8)^2 - \frac{9}{8}(8)^{8/3}\right) - 0$

$384 - 288 = \underline{96}$

shaded area $= \Delta$ - area under curve

$= 153.6 - 96$

$= \underline{57.6}$

END OF QUESTIONS