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1a) arc length $= r\theta$

$4 = r\theta$

$4 = 5\theta$

$\theta = \frac{4}{5} = 0.8$

b) area $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 5^2 \times 0.8$

$= 10\,cm^2$

2ai) $8 = 2^p$, $p = 3$

ii) $\frac{1}{8} = 2^q$, $q = -3$

iii) $\sqrt{2} = 2^r$, $r = 1/2$

b) $\sqrt{2} \times 2^x = \frac{1}{8}$

$2^{1/2} \times 2^x = 2^{-3}$ (add powers when multiplying)

$1/2 + x = -3$

$x = -3\,1/2$

3a) $\cos A = \frac{5^2 + 8^2 - 10^2}{2 \times 5 \times 8}$

$\theta = \cos^{-1}\left(-\frac{11}{80}\right)$

$\theta = 97.9032\ldots \Rightarrow \theta = 97.9°$ (1dp) (accuracy)

bi) area $= \frac{1}{2} ab \sin C$ (must be included angle)

$= \frac{1}{2} \times 5 \times 8 \times \sin 97.9$

$= 19.8101\ldots \Rightarrow$ area $= 19.8\,cm^2$ (3sf)

ii) area $= \frac{1}{2} \times b \times h$ $h = AD$ $b = 10$

$19.8 = \frac{1}{2} \times 10 \times AD$

$AD = \frac{19.8}{5} = 3.9620378 5\ldots \Rightarrow AD = 3.96\,cm$ (3sf)

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4a) $\int_0^{1.5} \sqrt{27x^3+4}\,dx$ $h = \frac{1.5-0}{3} = 0.5$

x	0	0.5	1	1.5
y	2	$\sqrt{7.375}$	$\sqrt{31}$	$\sqrt{95.125}$

$I = \frac{0.5}{2}\left(2 + 2(\sqrt{7.375} + \sqrt{31}) + \sqrt{95.125}\right)$

$= 7.0880308973$

$= 7.08$ (3sf)

b) $g(x) = f(\frac{1}{3}x)$ (scale factor 3 in x axis)

$\rightarrow$ Replace x with $\frac{1}{3}x$

$g(x) = \sqrt{27\left(\frac{1}{3}x\right)^3 + 4}$

$= \sqrt{x^3+4}$

5a) $(1-x)^3 = \left({}^3C_0\right)(1)^3(-x)^0 + \left({}^3C_1\right)(1)^2(-x)^1 + \left({}^3C_2\right)(1)^1(-x)^2 + \left({}^3C_3\right)(1)^0(-x)^3$

$= 1 - 3x + 3x^2 - x^3$

b) $(1+y)^4 = \left({}^4C_0\right)(1)^4(y)^0 + \left({}^4C_1\right)(1)^3(y)^1 + \left({}^4C_2\right)(1)^2(y)^2 + \left({}^4C_3\right)(1)(y)^3$
$+ \left({}^4C_4\right)(1)^0(y)^4$

$(1+y)^4 = 1 + 4y + 6y^2 + 4y^3 + y^4$ $-$
$(1-y)^3 = 1 - 3y + 3y^2 - y^3$

$0 + 7y + 3y^2 + 5y^3 + y^4$

$p = 3,\ q = 5$

$7y + 3y^2 + 5y^3 + y^4$

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5c) $\int [(1+\sqrt{x})^4 - (1-\sqrt{x})^3]\,dx$ $\qquad y = \sqrt{x}$

$\int [(1+y)^4 - (1-y)^3]\,dx$

$= \int 7y + 3y^2 + 5y^3 + y^4\,dx$ $\qquad$ (from b))

$= \int 7\sqrt{x} + 3(\sqrt{x})^2 + 5(\sqrt{x})^3 + (\sqrt{x})^4\,dx$

$= \int (7x^{1/2} + 3x + 5x^{3/2} + x^2)\,dx$

$= \dfrac{7x^{3/2}}{3/2} + \dfrac{3x^2}{2} + \dfrac{5x^{5/2}}{5/2} + \dfrac{x^3}{3} + C$

$= \dfrac{14}{3}x^{3/2} + \dfrac{3}{2}x^2 + 2x^{5/2} + \dfrac{1}{3}x^3 + C$

6ai) $U_3 = 36$ $\qquad U_6 = 972$

$ar^2 = 36$ ① $\quad ar^5 = 972$ ② $\qquad$ ② ÷ ①

$\dfrac{ar^5}{ar^2} = \dfrac{972}{36}$

$r^3 = 27$

$r = 3$ (as req)

ii) $ar^2 = 36$ (3rd term)

$a \times 3^2 = 36$

$a \times 9 = 36$

$a = 4$

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6bi) $a = 4, \quad r = 3$

$$\sum_{n=1}^{20} U_n = S_{20} = \frac{a(1-r^n)}{1-r}$$

$$= \frac{4(1-3^{20})}{1-3}$$

$$= \frac{4(1-3^{20})}{-2}$$

$$= -2(1-3^{20})$$

$$= \underline{2(3^{20}-1)} \quad \text{(as req)}$$

ii) $U_n = ar^{n-1}$

$U_n = 4 \times 3^{n-1}$

$U_n > 4 \times 10^{15}$

$4 \times 3^{n-1} > 4 \times 10^{15} \quad (\div 4)$

$3^{n-1} > 10^{15} \quad (\log s)$

$n-1 \log 3 > \log 10^{15}$

$n-1 > \frac{15 \log 10}{\log 3}$

$n-1 > \frac{15}{\log 3}$

$n-1 > 31.4385...$

$n > 32.4385...$

n is an integer, so $\underline{n = 33}$

7a) $y = x + 3 + 8x^{-4}$

$\frac{dy}{dx} = \underline{1 - 32x^{-5}}$

b) when $x = 1$,

$\frac{dy}{dx} = 1 - 32(1)^{-5} = -31$

$y = 1 + 3 + 8(1)^{-4} = 12$

$y - 12 = -31(x-1)$

$y - 12 = -31x + 31$

$\underline{y + 31x = 43}$

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7c) at M, $\frac{dy}{dx} = 0$

$1 - 32x^{-5} = 0$

$\frac{32}{x^5} = 1$

$x^5 = 32$

$\underline{x = 2}$

$y = 2 + 3 + \frac{8}{2^4}$

$y = 5.5$

$\underline{\underline{M(2, 5.5)}}$

7di) $\int (x + 3 + 8x^{-4})\,dx$

$= \frac{x^2}{2} + 3x + \frac{8x^{-3}}{-3} + c$

$= \frac{x^2}{2} + 3x - \frac{8}{3}x^{-3} + c$

ii) $\left[\frac{x^2}{2} + 3x - \frac{8x^{-3}}{3}\right]_1^2 = \left(\frac{2^2}{2} + 3(2) - \frac{8(2)^{-3}}{3}\right) - \left(\frac{1^2}{2} + 3(1) - \frac{8(1)^{-3}}{3}\right)$

$= \left(2 + 6 - \frac{1}{3}\right) - \left(\frac{1}{2} + 3 - \frac{8}{3}\right)$

$= \underline{\frac{41}{6}}$

e) minimum at $M \rightarrow y = 5.5$

$\therefore$ translation 5.5 down to make x axis a tangent

$\underline{k = -5.5}$

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8a) $2\log_k x - \log_k 5 = 1$

$\log_k x^2 - \log_k 5 = 1$

$\log_k \left(\frac{x^2}{5}\right) = 1$ (eliminate logs)

$\frac{x^2}{5} = k^1$

$k = \frac{x^2}{5}$

b) $\log_a y = \frac{3}{2}$ $\qquad$ $\log_4 a = b+2$

$a = 4^{b+2}$ (2)

$y = a^{3/2}$ (1)

sub (2) into (1)

$y = (4^{b+2})^{3/2}$

$y = ((2^2)^{b+2})^{3/2}$

$y = (2^{2b+4})^{3/2}$

$y = (2^{b+2})^3$

$y = 2^{3b+6}$

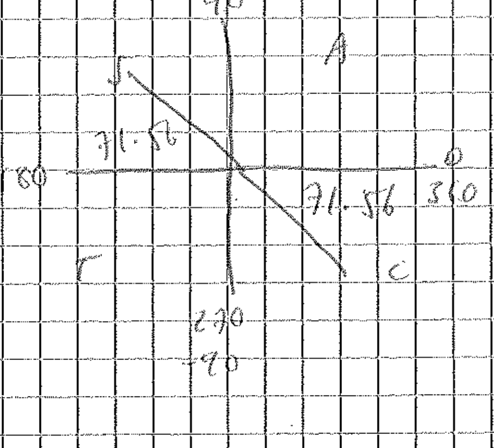
9a) $\tan x = -3$ $\qquad$ $0 \leq x \leq 360°$

$x = \tan^{-1}(-3)$

$x = -71.565051...$

$x = 108.434...., 288.4349...$

$x = 108°, 288°$



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9b i) $7\sin^2\theta + \sin\theta\cos\theta = 6(\cos^2\theta + \sin^2\theta)$ $(\div\cos^2\theta)$

$$\frac{7\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta\cos\theta}{\cos^2\theta} = 6\left(\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$7\tan^2\theta + \frac{\sin\theta}{\cos\theta} = 6 + 6\tan^2\theta \quad (-6\tan^2\theta)$$

$\tan^2\theta + \tan\theta - 6 = 0$ (as required)

ii) $\tan^2\theta + \tan\theta - 6 = 0$

$(\tan\theta + 3)(\tan\theta - 2) = 0$

$\tan\theta = -3$ $\qquad$ $\tan\theta = 2$

$\theta = 108°, 288°$ $\qquad$ $\theta = 63.4349, 243.4349$

$\approx 63°, 243°$