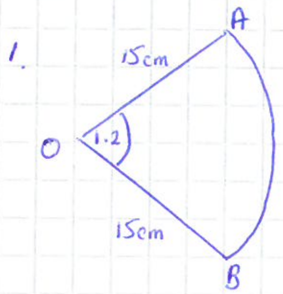
January 2010

1.



(a) (i) Area of Sector $= \frac{1}{2} r^2 \theta$ 1

$= \frac{1}{2} \times 15^2 \times 1.2$

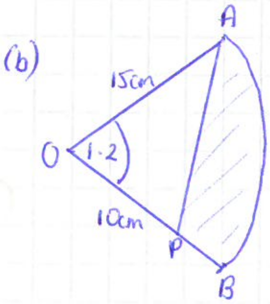
$= 135 \text{ cm}^2$ 1 as required

(ii) Length of Arc $= r\theta$ 1

$= 15 \times 1.2$

$= 18\text{cm}$ 1

(b)



Perimeter $= AB + PB + AP$

$= 18 + 5 + AP$

$= 18 + 5 + 14.7$

$= 37.7\text{cm}$ to 3sf

PB $15 - 10 = 5\text{cm}$

AP Cosine Rule

$AP^2 = 15^2 + 10^2 - 2(15)(10)\cos 1.2$

$AP^2 = 216.2926737$

$AP = 14.7068\ldots$ 1

2. $\frac{dy}{dx} = 7\sqrt{x^5} - 4, \quad x > 0$

(a) $\sqrt{x^5} = x^{5/2}$ 1

(b) $\int (7\sqrt{x^5} - 4)\, dx$

$\int 7x^{5/2} - 4 \quad dx$

$\frac{7x^{7/2}}{7/2} - 4x + c$ 1 1 1

$\frac{2}{7} \times 7x^{7/2} - 4x + c$

$2x^{7/2} - 4x + c$ 1 Part (c)

(c) Eqn of Curve passing through $(1,3)$

$y = 2x^{7/2} - 4x + c$

$3 = 2(1)^{7/2} - 4(1) + c$

$3 = 2 - 4 + c$ 1

$3 = -2 + c, \quad c = 5$

Eqn of the Curve $y = 2x^{7/2} - 4x + 5$ 1

3. (a) $\log_9 x = 0$

$9^0 = 1 \qquad x = 1$

(ii) $\log_9 x = \frac{1}{2}$

$9^{1/2} = 3 \qquad x = 3$

Log Rules:

$\log m + \log n = \log mn$

$\log m - \log n = \log \frac{m}{n}$

$\log m^n = n \log m$

(b) $2 \log_a n = \log_a 18 + \log_a (n-4)$

$\log_a n^2$ (1) $= \log_a 18(n-4)$ (1)

$n^2 = 18(n-4)$

$n^2 - 18n + 72 = 0$ (1)

$(n-12)(n-6) = 0$ (1)

$n = 12$ or $n = 6$ (1)

4. Arithmetic Series

$S_{31} = 310$

(a) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{31} = \frac{31}{2}[2a + (31-1)d]$ (1)

$= \frac{31}{2}[2a + 30d]$

$= 31[a + 15d] = 310$ (1)

$= [a + 15d] = \frac{310}{31} = 10$ (1)

$\therefore \quad a + 15d = 10$ as required

(b) $u_{21} = 2 \times u_{16}$

$a + (21-1)d = 2 \times [a + (16-1)d]$ (1)

$a + 20d = 2(a + 15d)$

$a + 20d = 2a + 30d$

$a + 10d = 0$

Then,

$$\begin{aligned} a + 15d &= 10 \\ -\quad a + 10d &= 0 \\ \hline 5d &= 10 \end{aligned}$$ (1)

$d = 2$ (1)

$a + 15(2) = 10$

$a + 30 = 10$

$a = -20$

(c) $\sum_{n=1}^{k} u_n = 0$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = 0$ (1) $a = -20$ (1) $d = 2$

$\frac{n}{2}\left[-40 + (n-1)\times 2\right] = 0$ (1)

$n\left[-20 + (n-1)\right] = 0$

$-20n + n^2 - n = 0$

$n^2 - 21n = 0$

$n(n-21) = 0$

$\therefore n = 0$ or $n = 21$ (1)

5. $y = \frac{1}{x^3} + 48x$

$= x^{-3} + 48x$ (1)

(a) $\frac{dy}{dx} = -3x^{-4}$ (1) $+ 48$ (1)

(b) Gradient of tangents parallel to the x-axis $= 0$ (since they are horizontal)

$\therefore \frac{dy}{dx} = 0$ $\quad -3x^{-4} + 48 = 0$ (1)

$-3x^{-4} = -48$

$\div 3$

$x^{-4} = 16$

$\frac{1}{x^4} = 16$

$\frac{1}{16} = x^4$ (1)

$x = \sqrt[4]{\frac{1}{16}} = \pm\frac{1}{2}$ (1)

At $x = \frac{1}{2}$, $y = \frac{1}{(\frac{1}{2})^3} + 48(\frac{1}{2})$ AND $y = \frac{1}{(-\frac{1}{2})^3} + 48(-\frac{1}{2})$

$= \frac{1}{\frac{1}{8}} + 24$ $\qquad = \frac{1}{-\frac{1}{8}} - 24$

$= 8 + 24 = 32$ $\qquad = -8 - 24 = -32$

$y = 32$ $\quad y = -32$ (1)

(c) Eqn of normal at $(1, 49)$

Gradient of tangent at $x = 1$

$$-3(1)^{-4} + 48$$

$$-3(1) + 48$$

$$-3 + 48 = 45 \quad 1$$

Gradient of normal at $x = 1$ is $-\frac{1}{45}$ 1 $(m_1 m_2 = -1)$

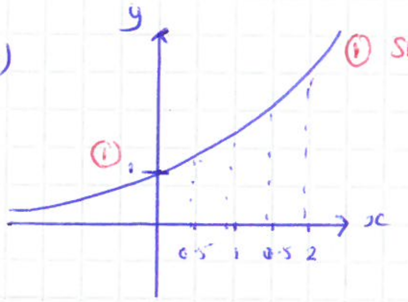
Eqn of normal at $(1, 49)$

$$y - y_1 = m(x - x_1)$$

$$y - 49 = -\frac{1}{45}(x - 1) \quad 1$$

$$y - 49 = -\frac{x}{45} + \frac{1}{45}$$

6 (a)



(b) $\int_0^2 2^x \, dx$

x	0	0.5	1.0	1.5	2.0
y	1	1.414	2	2.828	4

$$\text{Area} = \frac{h}{2}\{y_0 + y_4 + 2(y_1 + y_2 + y_3)\}$$

$$= \frac{0.5}{2} \; 1 \; \{1 + 4 + 2(1.414 + 2 + 2.828)\} \quad 1\,M \; 1\,A$$

$$= 4.37132034 4$$

$$= 4.37 \; 1 \text{ to } 3sf$$

(ii) Increase the number of ordinates 1

(c) $y = 2^x \rightarrow y = 2^{x+7} + 3$

$\uparrow$ 7 left $\quad \uparrow$ 3 up

Translation 1 $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$ 1

(d) $y = 2^{x+k} + 3$ intersects y-axis at $(0,8)$

$8 = 2^{0+k} + 3$

$8 = 2^{k} + 3$

$2^{k} = 5$

Since $a^{m} = n$, $\log_a n = m$

$k = \log_2 5$

7 (a) $(1+2x)^7 = {}^7C_0 (1)^7 (2x)^0 + {}^7C_1 (1)^6 (2x)^1 + {}^7C_2 (1)^5 (2x)^2 + {}^7C_3 (1)^4 (2x)^3$ 1

$= 1 \times 1 \times 1 + 7 \times 1 \times 2x + 21 \times 1 \times 4x^2 + 35 \times 1 \times 8x^3$

$= 1 + 14x + 84x^2 + 280x^3 \ldots$

$a = 14$ 1 $b = 84$ 1 $c = 280$ 1

(b) $(1 - \frac{1}{2}x)^2 (1+2x)^7$

$(1 - \frac{1}{2}x)(1 - \frac{1}{2}x)(1+2x)^7$

$(1 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{4}x^2)(1+2x)^7$

$(1 - x + \frac{1}{4}x^2)(1 + 14x + 84x^2 + 280x^3 + \ldots)$ 1

Coefficient of x^3 $(1 \times 280x^3) + (-x \times 84x^2) + (\frac{1}{4}x^2 \times 14x)$ 1

$280x^3 - 84x^3 + 3.5x^3$ 1 $= 199.5x^3$ 199.5 1

8. (a) $\tan(x + 52) = \tan 22$

$x + 52 = 22$ (1)

$x = -30°$

In the range $0 \le x \le 360°$

$-30 + 180 = \underline{150°}$

(1) (1) Both answers

and $150 + 180 = \underline{330°}$

(b) (i) $3 \tan\theta = \dfrac{8}{\sin\theta}$

Since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$3\dfrac{\sin\theta}{\cos\theta} = \dfrac{8}{\sin\theta}$ (1)

$3\sin^2\theta = 8\cos\theta$

Since $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$3(1 - \cos^2\theta) = 8\cos\theta$ (1)

$3 - 3\cos^2\theta = 8\cos\theta$

$3\cos^2\theta + 8\cos\theta - 3 = 0$ (1)

(ii) $(3\cos\theta - 1)(\cos\theta + 3) = 0$ (1)

$\cos\theta = \dfrac{1}{3}$ (1) or $\cos\theta = -3$

NOT A VALID SOLUTION

(iii) $\cos 2x = \dfrac{1}{3}$ (1)

$2x = \cos^{-1}\dfrac{1}{3}$

$= 70.52877937$ (1)

$\underline{\text{or}}$ $360 - 70.5 = 289.4712206$ (1)

if $2x = 70.5$ OR if $2x = 289.5$

$x = 35°$ $x = 145°$

(1) Both answers to the nearest degree